Appendix A Basic formulas

A.1 Expectation

$$\mathbb{E}_{(a)}\left[f(a)+g(a)\right]=\mathbb{E}_{(a)}\left[f(a)\right]+\mathbb{E}_{(a)}\left[g(a)\right], \tag{A.1}$$

$$\mathbb{E}_{(a)}\left[bf(a)\right]=b\mathbb{E}_{(a)}\left[f(a)\right]. \tag{A.2}$$

A.2 Delta function

A.2.1 Kronecker delta function

$$f_b=\sum_a \delta(a,b)f_a. \tag{A.3}$$

A.2.2 Product formula of Kronecker delta function

$$f_b=\prod_a f_a^{\delta(a,b)}. \tag{A.4}$$

(Proof)

$$\begin{aligned}\prod_a f_a^{\delta(a,b)} &= \exp\left(\log\left(\prod_a f_a^{\delta(a,b)}\right)\right)\\ &= \exp\left(\sum_a \delta(a,b)\log(f_a)\right)\\ &= \exp(\log(f_b))=f_b.\end{aligned} \tag{A.5}$$

A.2.3 Dirac delta function

$$f(y)=\int \delta(x-y)f(x)dx. \tag{A.6}$$

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A.3 Jensen's inequality

For a concave function f, where X is a probabilistic random variable sampled from a distribution function, and an arbitrary function $g(X)$, we have the following inequality:

$$f\left(\mathbb{E}_{(X)}[g(X)]\right) \geq \mathbb{E}_{(X)}[f(g(X))]. \tag{A.7}$$

In the special case of $f(\cdot) = \log(\cdot)$, (A.7) is rewritten as follows:

$$\log\left(\mathbb{E}_{(X)}[g(X)]\right) \geq \mathbb{E}_{(X)}[\log(g(X))]. \tag{A.8}$$

A.4 Gamma function

$$\Gamma(x+1) = x\Gamma(x), \tag{A.9}$$

$$\Gamma\left(\frac{1}{2}\right) = \pi^{\frac{1}{2}}. \tag{A.10}$$

A.4.1 Stirling's approximation

$$\log\Gamma\left(\frac{x}{2}\right) \to \frac{x}{2}\log\left(\frac{x}{2}\right) - \frac{x}{2} - \frac{1}{2}\log\left(\frac{x}{2\pi}\right), \qquad x \to \infty. \tag{A.11}$$

A.4.2 Di-gamma function

$$\begin{aligned}\Psi(x) &\triangleq \frac{\partial}{\partial x}\log\Gamma(x) \\ &= \frac{\frac{\partial\Gamma(x)}{\partial x}}{\Gamma(x)}.\end{aligned} \tag{A.12}$$

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