Dynamic Programming Approach for Large Scale Unit Commitment Problem

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Abstract— **In this paper, the large scale Unit Commitment (UC) problem has been solved using Dynamic Programming (DP) and the test results for conventional DP, Sequential DP and Truncation DP are compared with other stochastic techniques. The commitment is such that the total cost is minimal. The total cost includes both the production cost and the costs associated with start-up and shutdown of units. DP is an optimization technique which gives the optimal solution.**

Keywords-unit commitment; dynamic programming; genetic algorithm; lagrange relaxation, simulated annealing, evolutionary programming, binary particle swarm optimization.

I. INTRODUCTION

Unit Commitment (UC) is used to schedule the generators such that the total system production cost over the scheduled time horizon is minimized under the spinning reserve and operational constraints of generator units. UC problem is a nonlinear, mixed integer combinatorial optimization problem. The global optimal solution can be obtained by complete enumeration, which is not applicable to large power systems due to its excessive computational time requirements [1]. Hence, the UC problem is quite difficult due to its inherent high-dimensional, non-convex, discrete and non-linear nature. The UC problem can be considered as two linked optimization problems, namely the unit-scheduled problem, which is a combinatorial optimization problem and the economic dispatch (ED) problem, which is a non-linear programming optimization problem [2]. There are many UC methods such as the dynamic programming which is introduced in this paper, lagrangian relaxation [3], priority list method, branch and bound method [4] and mixed integer linear programming (MILP) [5]. All of these methods have their advantages and disadvantages. By using dynamic programming for unit commitment, we can get optimal solutions. But solution of large scale UC problems using conventional DP is time consuming because it involves complete enumeration of units instead it gives the best optimal solution [6]. Total no. of combinations for Conventional DP = 2^N-1 where 'N' is total number of units.

In this paper, we will illustrate the components of UC and introduce the solution of UC problem using Conventional dynamic programming, Sequential combination (SC)-DP and Truncation Combination (TC)-DP for 10 unit system over 24 hours interval.

The organization of the paper is as follows: Section II introduces the UC formulation. Section III shows the method to solve UC based on DP. In Section IV, Simulation results on a 10-unit power generation system is presented and the results are compared with different heuristic techniques and Section V concludes the paper.

II. UC PROBLEM FORMULATION

The objective of UC problem is to minimize the production cost over the scheduled time horizon (e.g., 24 h) under the generator operational and spinning reserve constraints.

Mathematically, the objective function to be minimized is

$$F(P_i^t, U_{i,t}) = \sum_{t=1}^{T}\sum_{i=1}^{N}\left[F_i(P_i^t) + ST_{i,t}(1-U_{i,t-1})\right]U_{i,t} \qquad (1)$$

subject to following constraints

(a) power balance constraint

$$P_{load}^t - \sum_{i=1}^{N} P_i^t U_{i,t} = 0 \qquad (2)$$

(b) spinning reserve constraint

$$P_{load}^t + R^t - \sum_{i=1}^{N} P_{i,max} U_{i,t} \le 0 \qquad (3)$$

(c) generation limit constraints

$$P_{i,min}U_{i,t} \le P_i^t \le P_{i,max}U_{i,t}, \qquad i = 1,2,\dots N \qquad (4)$$

(d) start-up cost

$$ST_{i,t} = \begin{cases} HST_i, & \text{if } T_{i,down} \le T_{i,off} \le T_{i,cold} + T_{i,down}, \\ CST_i, & \text{if } T_{i,off} > T_{i,cold} + T_{i,down}, \end{cases} \qquad (5)$$

where,

$F_i(P_i^t)$ – Fuel cost function of the i^{th} unit with generation output P_i^t, at hour t. Usually, it is a quadratic polynomial with coefficients a_i, b_i and c_i as follows:

$$F_i(P_i^t) = a_i + b_i P_i^t + c_i (P_i^t)2$$

N – Number of units

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T – Number of hours
P_i^t – The generation output of the i[th] unit at hour t
ST_i – Start-up cost of i[th] unit
$U_{i,t}$ – The on/off status of the i[th] unit at hour t, and $U_{i,t} = 0$ when off, $U_{i,t} = 1$ when on.
P_{load}^t- load demand at hour t (in MW)
R^t - spinning reserve at hour t (in MW)
$P_{i,max}$ - Maximum real power generation of unit i (in MW)
$P_{i,min}$ - Minimum real power generation of unit i (in MW)
HST_i - Hot start-up cost of unit i (in dollars)
CST_i - Cold start-up cost of unit i (in dollars)
$T_{i,down}$ - Minimum down time of unit i (in hours)
$T_{i,off}$ - Continuously off time of unit i (in hours)
$T_{i,cold}$ - Cold start hours of unit i (in hours)

III. Dynamic Programming Approach

First, Dynamic programming is a methodical procedure which systematically evaluates a large number of possible decisions in a multi-step problem. When we utilize the existing conventional dynamic programming method, although its solution is correct and has the optimal value, it takes a lot of memory and spends much time in getting an optimal solution [7]-[11]. For example, assume that there are 4 units which can supply the 24 hour load. So, the total maximum path to satisfy the 24 hour load curve is calculated by:

$$\text{Total Paths} = (2^4 - 1)^{24}$$

Because of this disadvantage, the SC-DP and TC-DP is used to solve the UC problem. The chief advantage of these two methods is the reduction of dimensionality of the problem. Also the calculation of production cost lies near the optimal solution. In SC-DP, the strict priority order of units is imposed. For example, assume the same 4 units, there would be only four combinations to try:

Priority 1 Unit

Priority 1 Unit + Priority 2 Unit

Priority 1 Unit + Priority 2 Unit + Priority 3 Unit

Priority 1 Unit + Priority 2 Unit + Priority 3 Unit + Priority 4 Unit

In TC-DP fixed number of units is allowed to run to satisfy the load demand for each hour.

Recursive algorithm to compute the minimum cost in Kth hour with Ith Combination is,

$$F_{cost}(J,K) = \min_{\{L\}}[P_{cost}(J,K) + S_{cost}(J-1, L: J, K) + F_{cost}(J-1, L)] \quad (6)$$

where,
$F_{cost}(J,K)$ – Least total cost to arrive at state (J, K)
$P_{cost}(J,K)$ – Production cost for state (J, K)
$S_{cost}(J-1, L: J, K)$ – Transition cost from state (J-1,L) to State (J, K)
State (J, K) – K[th] Combination in J[th] hour

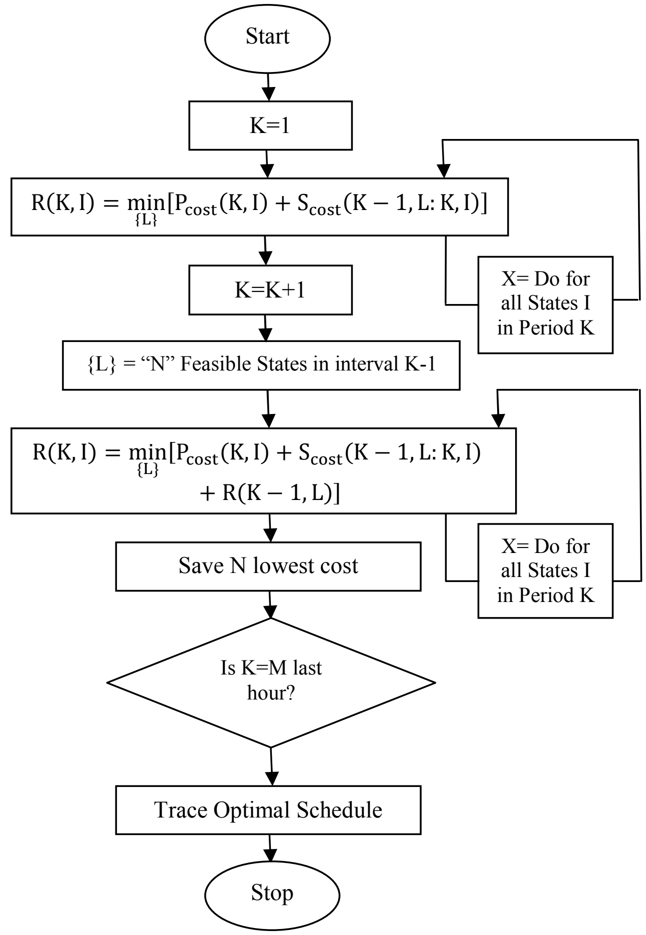

Figure 1. UC via forward Dynamic Programming

IV. Simulation Results

The results for DP are presented and tested on 10 unit base system with a 24-hour time horizon. The program was written in MATLAB. The input data for 10 unit system and load demands for 24 hours are shown in Tables I and Table II. In this section, a 10-generator, 24-hour unit commitment schedule is determined with the help of Conventional DP, SC-DP and TC-DP and their results consist of production cost and CPU time are compared with different stochastic techniques.

TABLE I. Data For 10-Unit System [12]

Parameter	Unit 1	Unit 2	Unit 3	Unit 4	Unit 5
Pmax (MW)	455	455	130	130	162
Pmin (MW)	150	150	20	20	25
a ($/h)	1000	970	700	680	450
b ($/MWh)	16.19	17.26	16.6	16.5	19.7
c ($/MW2-h)	0.00048	0.00031	0.002	0.00211	0.00398
min up (h)	8	8	5	5	6

min down (h)	8	8	5	5	6
hot start cost ($)	4500	5000	550	560	900
cold start cost ($)	9000	10000	1100	1120	1800
cold start hours (h)	5	5	4	4	4
initial status (h)	8	8	-5	-5	-6

Parameter	Unit 6	Unit 7	Unit 8	Unit 9	Unit 10
Pmax (MW)	80	85	55	55	55
Pmin (MW)	20	25	10	10	10
a ($/h)	370	480	660	665	670
b ($/MWh)	22.26	27.74	25.92	27.27	27.79
c ($/MW2-h)	0.00712	0.0079	0.00413	0.00222	0.00173
min up (h)	3	3	1	1	1
min down (h)	3	3	1	1	1
hot start cost ($)	170	260	30	30	30
cold start cost ($)	340	520	60	60	60
cold start hours(h)	2	2	0	0	0
initial status (h)	-3	-3	-1	-1	-1

TABLE II. LOAD DEMAND FOR 24-HOUR [12]

Hour	P_{load}	Hour	P_{load}	Hour	P_{load}	Hour	P_{load}
1	700	**7**	1150	**13**	1400	**19**	1200
2	750	**8**	1200	**14**	1300	**20**	1400
3	850	**9**	1300	**15**	1200	**21**	1300
4	950	**10**	1400	**16**	1050	**22**	1100
5	1000	**11**	1450	**17**	1000	**23**	900
6	1100	**12**	1500	**18**	1100	**24**	800

A. Results for Conventional DP (Complete enumeration)

In this, Table III shows the UC schedule of 10 units over a 24 hours period. Table IV shows the production cost of generators for each hour(P_{cost}).

TABLE III. UNIT COMMITMENT SCHEDULE FOR CONVENTIONAL DP

	Hours →											
	1	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**
Unit1	1	1	1	1	1	1	1	1	1	1	1	1
Unit2	1	1	1	1	1	1	1	1	1	1	1	1
Unit3	0	0	0	0	0	0	0	0	1	1	1	1
Unit4	0	0	0	1	1	1	1	1	1	1	1	1
Unit5	0	0	0	0	0	1	1	1	1	1	1	1
Unit6	0	0	0	0	0	0	0	0	0	1	1	1
Unit7	0	0	0	0	0	0	0	0	0	0	0	0
Unit8	0	0	0	0	0	0	0	0	0	0	1	1
Unit9	0	0	0	0	0	0	0	0	0	0	0	1
Unit10	0	0	0	0	0	0	0	0	0	0	0	0

	Hours →											
	13	**14**	**15**	**16**	**17**	**18**	**19**	**20**	**21**	**22**	**23**	**24**
Unit1	1	1	1	1	1	1	1	1	1	1	1	1
Unit2	1	1	1	1	1	1	1	1	1	1	1	1
Unit3	1	1	0	0	0	0	0	1	1	1	0	0
Unit4	1	1	1	0	0	1	1	1	1	1	0	0
Unit5	1	1	1	1	1	1	1	1	1	0	0	0
Unit6	1	0	0	0	0	0	0	1	0	0	0	0
Unit7	0	0	0	0	0	0	0	0	0	0	0	0
Unit8	0	0	0	0	0	0	0	0	0	0	0	0
Unit9	0	0	0	0	0	0	0	0	0	0	0	0

TABLE IV. PRODUCTION COST FOR EACH HOUR

hrs	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**
P_{cost}	13683.13	14554.50	16328.94	18677.96	19530.90	22219.25	23071.95	23925.64
hrs	**9**	**10**	**11**	**12**	**13**	**14**	**15**	**16**
P_{cost}	26305.11	28842.99	30855.69	32941.87	28842.99	26305.11	23925.64	20723.39
hrs	**17**	**18**	**19**	**20**	**21**	**22**	**23**	**24**
P_{cost}	19870.09	22219.25	23925.64	28842.99	26305.11	21910.82	17182.48	15476.40

B. Results for SC-DP

In this, the strict priority order of generating units is followed. Table V shows the strict priority order of 10 units based on the full load average production cost. Table VI shows the UC schedule of 10 units over a 24 hours period and Table VII shows the production cost of generators for each hour (P_{cost}).

TABLE V. PRIORITY UNIT LIST

Units	**1**	**2**	**3**	**4**	**5**
FLAPC	18.39	19.39	21.98	21.73	22.48
Units	**6**	**7**	**8**	**9**	**10**
FLAPC	26.89	33.39	37.92	39.36	39.97

TABLE VI. UC SCHEDULE FOR SC-DP

	Hours →											
Unit	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**
1	1	1	1	1	1	1	1	1	1	1	1	1
2	1	1	1	1	1	1	1	1	1	1	1	1
3	0	0	0	1	1	1	1	1	1	1	1	1
4	0	0	0	0	0	1	1	1	1	1	1	1
5	0	0	0	0	0	0	0	1	1	1	1	1
6	0	0	0	0	0	0	0	0	0	1	1	1
7	0	0	0	0	0	0	0	0	0	0	1	1
8	0	0	0	0	0	0	0	0	0	0	0	1
9	0	0	0	0	0	0	0	0	0	0	0	0
10	0	0	0	0	0	0	0	0	0	0	0	0

	Hours →											
Unit	**13**	**14**	**15**	**16**	**17**	**18**	**19**	**20**	**21**	**22**	**23**	**24**
1	1	1	1	1	1	1	1	1	1	1	1	1
2	1	1	1	1	1	1	1	1	1	1	1	1
3	1	1	1	1	1	1	1	1	1	1	0	0
4	1	1	1	1	1	1	1	1	1	1	0	0
5	1	1	1	0	0	0	1	1	1	0	0	0
6	1	0	0	0	0	0	0	1	0	0	0	0
7	0	0	0	0	0	0	0	0	0	0	0	0
8	0	0	0	0	0	0	0	0	0	0	0	0
9	0	0	0	0	0	0	0	0	0	0	0	0
10	0	0	0	0	0	0	0	0	0	0	0	0

TABLE VII. PRODUCTION COST FOR EACH HOUR

Hour	1	2	3	4	5	6	7	8
P_{cost}	13683.13	14554.50	16328.94	18677.96	19530.90	21910.82	22764.15	24599.90
Hour	**9**	**10**	**11**	**12**	**13**	**14**	**15**	**16**
P_{cost}	26305.11	28842.99	31140.42	33153.19	28842.99	26305.11	24599.90	21058.47
Hour	**17**	**18**	**19**	**20**	**21**	**22**	**23**	**24**
P_{cost}	20207.11	21910.82	24599.90	28842.99	26305.11	21910.82	17182.48	15476.40

C. Results for TC-DP

In this also the strict priority order is imposed and based on this fixed number of schedulable units are selected to satisfy the load demand for each hour.

Number of Units considered = 8

TABLE VIII. PRODUCTION COST FOR EACH HOUR

Hours	1	2	3	4	5	6	7	8
P_{cost}	13683.13	14554.50	16301.89	18637.68	19512.77	21860.29	22879.12	23917.85
Hours	**9**	**10**	**11**	**12**	**13**	**14**	**15**	**16**
P_{cost}	26184.02	28768.21	30593.51	32550.09	28768.21	26184.02	23917.85	20895.88
Hours	**17**	**18**	**19**	**20**	**21**	**22**	**23**	**24**
P_{cost}	20020.02	21860.29	23917.85	28768.21	26184.02	21860.29	17177.91	15427.42

The optimal results are obtained using conventional DP but the computational time taken is more than that of SC-DP and TC-DP. The comparison of production cost and CPU time with other stochastic methods are shown in Table IX.

The CPU times may not be directly comparable due to different computers used. CPU times of GA [12] and EP [13] are obtained from HP Apollo 720 workstation and HP C160 workstation, respectively whereas CPU times for LR [12] and SA [14] are obtained from a Pentium IV, 1.6 GHz personal computer and the CPU times for DP are obtained from Intel(R) Core(TM) 2 Duo T6600 @ 2.20 GHz. The CPU time for LR is much smaller compared to other methods. So LR method can provide a fast solution but the quality of solution strongly depends on the algorithm used to update the Lagrangian multipliers.

TABLE IX. RESULTS FOR DIFFERENT METHODS FOR 10 UNIT SYSTEM OVER 24 HOUR TIME PERIOD

Methods	Overall Production Cost (in $)	CPU Time (in Sec)
Conventional DP	5,53,507.85	458
SC-DP	5,55,814.11	12
TC-DP	5,50,805.02	63.3
LR [12]	5,65,825.00	2.2
GA [12]	5,65,825.00	221
EP [13]	5,64,551.00	100
SA [14]	5,65,828.00	3
BPSO [15]	5,63,977.00	18

V. CONCLUSION

There are a lot of methods for solving the Unit Commitment problem. Their advantages and disadvantages are studied and described. One of the main problems is that they do not get the optimal solution for performing the Unit Commitment. Therefore, we considered dynamic programming to get an optimal solution despite being impossible to utilize in a large scale power system. This paper presents the three versions of DP to solve UC problem which provide better numerical convergence than other stochastic techniques according to the numerical results. Easy implementation is main attractive feature of all versions of DP.

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