# Mersenne prime

In mathematics, a **Mersenne prime** is a prime number that is one less than a power of two. That is, it is a prime number that can be written in the form $Mn = 2^n - 1$ for some integer $n$. They are named after Marin Mersenne, a French Minim friar, who studied them in the early 17th century. The first four Mersenne primes (sequence A000668 in the OEIS) are 3, 7, 31, and 127.

If $n$ is a composite number then so is $2^n - 1$. ($2^{ab} - 1$ is divisible by both $2^a - 1$ and $2^b - 1$.) The definition is therefore unchanged when written $Mp = 2^p - 1$ where $p$ is assumed prime.

More generally, numbers of the form $Mn = 2^n - 1$ without the primality requirement are called **Mersenne numbers**. Mersenne numbers are sometimes defined to have the additional requirement that $n$ be prime, equivalently that they be pernicious Mersenne numbers, namely those numbers whose binary representation contains a prime number of ones and no zeros. The smallest composite pernicious Mersenne number is $2^{11} - 1 = 2047 = 23 \times 89$.

Mersenne primes $Mp$ are also noteworthy due to their connection to perfect numbers.

As of January 2016, 49 Mersenne primes are known. The largest known prime number $2^{74,207,281} - 1$ is a Mersenne prime.[2][3][4]

Since 1997, all newly found Mersenne primes have been discovered by the "Great Internet Mersenne Prime Search" (GIMPS), a distributed computing project on the Internet.

## 1 About Mersenne primes

Many fundamental questions about Mersenne primes remain unresolved. It is not even known whether the set of Mersenne primes is finite or infinite. The Lenstra–Pomerance–Wagstaff conjecture asserts that there are infinitely many Mersenne primes and predicts their order of growth. It is also not known whether infinitely many Mersenne numbers with prime exponents are composite, although this would follow from widely believed conjectures about prime numbers, for example, the infinitude of Sophie Germain primes congruent to 3 (mod 4), for these primes $p$, $2p + 1$ (which is also prime) will divide $Mp$, e.g., $23 \mid M_{11}$, $47 \mid M_{23}$, $167 \mid M_{83}$, $263 \mid M_{131}$, $359 \mid M_{179}$, $383 \mid M_{191}$, $479 \mid M_{239}$, and $503 \mid M_{251}$. (sequence A002515 in the OEIS). Since for these primes $p$, $2p + 1$ is congruent to 7 mod 8, so 2 is a quadratic residue mod $2p + 1$, and the multiplicative order of 2 mod $2p + 1$ must divide $\frac{(2p+1)-1}{2} = p$. Since $p$ is a prime, it must be $p$ or 1. However, it cannot be 1 since $\Phi_1(2) = 1$ and 1 has no prime factors, so it must be $p$. Hence, $2p + 1$ divides $\Phi_p(2) = 2^p - 1$ and $2^p - 1 = M_p$ cannot be prime.

The first four Mersenne primes are

$$M_2 = 3,\ M_3 = 7,\ M_5 = 31 \text{ and } M_7 = 127.$$

A basic theorem about Mersenne numbers states that if $Mp$ is prime, then the exponent $p$ must also be prime. This follows from the identity

$$\begin{aligned} 2^{ab} - 1 &= (2^a - 1) \cdot \left(1 + 2^a + 2^{2a} + 2^{3a} + \cdots + 2^{(b-1)a}\right) \\ &= (2^b - 1) \cdot \left(1 + 2^b + 2^{2b} + 2^{3b} + \cdots + 2^{(a-1)b}\right). \end{aligned}$$

This rules out primality for Mersenne numbers with composite exponent, such as $M_4 = 2^4 - 1 = 15 = 3 \times 5 = (2^2 - 1) \times (1 + 2^2)$.

Though the above examples might suggest that $Mp$ is prime for all primes $p$, this is not the case, and the smallest counterexample is the Mersenne number

$$M_{11} = 2^{11} - 1 = 2047 = 23 \times 89.$$

The evidence at hand does suggest that a randomly selected Mersenne number is much more likely to be prime than an arbitrary randomly selected odd integer of similar size. Nonetheless, prime $Mp$ appear to grow increasingly sparse as $p$ increases. In fact, of the 2,270,720 prime numbers $p$ up to 37,156,667,[5] $Mp$ is prime for only 45 of them.

The lack of any simple test to determine whether a given Mersenne number is prime makes the search for Mersenne primes a difficult task, since Mersenne numbers grow very rapidly. The Lucas–Lehmer primality test (LLT) is an efficient primality test that greatly aids this task. The search for the largest known prime has somewhat of a cult following. Consequently, a lot of computer power has been expended searching for new Mersenne primes, much of which is now done using distributed computing.

Mersenne primes are used in pseudorandom number generators such as the Mersenne twister, Park–Miller random number generator, Generalized Shift Register and Fibonacci RNG.

## 2 Perfect numbers

Main article: Euclid–Euler theorem

Mersenne primes $Mp$ are also noteworthy due to their connection to perfect numbers. In the 4th century BC, Euclid proved that if $2^p - 1$ is prime, then $2^{p-1}(2^p - 1)$ is a perfect number. This number, also expressible as $Mp(Mp + 1)/2$, is the $Mp$th triangular number and the $2^{p-1}$th hexagonal number. In the 18th century, Leonhard Euler proved that, conversely, all even perfect numbers have this form.[6] This is known as the Euclid–Euler theorem. It is unknown whether there are any odd perfect numbers.

## 3 History

Mersenne primes take their name from the 17th-century French scholar Marin Mersenne, who compiled what was supposed to be a list of Mersenne primes with exponents up to 257, as follows:

> 2, 3, 5, 7, 13, 17, 19, 31, 67, 127, 257

His list was completely accurate until 31, but then becomes largely incorrect, as Mersenne mistakenly included $M_{67}$ and $M_{257}$ (which are composite), and omitted $M_{61}$, $M_{89}$, and $M_{107}$ (which are prime). Mersenne gave little indication how he came up with his list.[7] (sequence A109461 in the OEIS)

Édouard Lucas proved in 1876 that $M_{127}$ is indeed prime, as Mersenne claimed. This was the largest known prime number for 75 years, and the largest ever found by hand. $M_{61}$ was determined to be prime in 1883 by Ivan Mikheevich Pervushin, though Mersenne claimed it was composite, and for this reason it is sometimes called Pervushin's number. This was the second-largest known prime number, and it remained so until 1911. Lucas had shown another error in Mersenne's list in 1876. Without finding a factor, Lucas demonstrated that $M_{67}$ is actually composite. No factor was found until a famous talk by Frank Nelson Cole in 1903.[8] Without speaking a word, he went to a blackboard and raised 2 to the 67th power, then subtracted one. On the other side of the board, he multiplied 193,707,721 × 761,838,257,287 and got the same number, then returned to his seat (to applause) without speaking.[9] He later said that the result had taken him "three years of Sundays" to find.[10] A correct list of all Mersenne primes in this number range was completed and rigorously verified only about three centuries after Mersenne published his list.

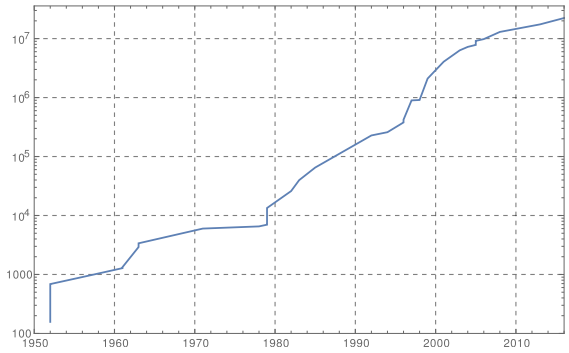
## 4 Searching for Mersenne primes

Fast algorithms for finding Mersenne primes are available, and as of 2016 the eleven largest known prime numbers are Mersenne primes.

The first four Mersenne primes $M_2 = 3$, $M_3 = 7$, $M_5 = 31$ and $M_7 = 127$ were known in antiquity. The fifth, $M_{13} = 8191$, was discovered anonymously before 1461; the next two ($M_{17}$ and $M_{19}$) were found by Pietro Cataldi in 1588. After nearly two centuries, $M_{31}$ was verified to be prime by Leonhard Euler in 1772. The next (in historical, not numerical order) was $M_{127}$, found by Édouard Lucas in 1876, then $M_{61}$ by Ivan Mikheevich Pervushin in 1883. Two more ($M_{89}$ and $M_{107}$) were found early in the 20th century, by R. E. Powers in 1911 and 1914, respectively.

The best method presently known for testing the primality of Mersenne numbers is the Lucas–Lehmer primality test. Specifically, it can be shown that for prime $p > 2$, $Mp = 2^p - 1$ is prime if and only if $Mp$ divides $Sp_{-2}$, where $S_0 = 4$ and $Sk = (Sk_{-1})^2 - 2$ for $k > 0$.

During the era of manual calculation, all the exponents up to and including 257 were tested with the Lucas–Lehmer test and found to be composite. A notable contribution was made by retired Yale physics professor Horace Scudder Uhler, who did the calculations for exponents 157, 167, 193, 199, 227, and 229.[11] Unfortunately for those investigators, the interval they were testing contains the largest known gap between Mersenne primes, in relative terms: the next prime exponent would turn out to be more than four times larger than the previous record of 127.




*Graph of number of digits in largest known Mersenne prime by year – electronic era. Note that the vertical scale, the number of digits, is doubly logarithmic in the value of the prime.*

The search for Mersenne primes was revolutionized by the introduction of the electronic digital computer. Alan Turing searched for them on the Manchester Mark 1 in 1949,[12] but the first successful identification of a Mersenne prime, $M_{521}$, by this means was achieved at 10:00 pm on January 30, 1952 using the U.S. National Bureau of Standards Western Automatic Computer (SWAC) at the Institute for Numerical Analysis at the University of California, Los Angeles, under the direction of Lehmer, with a computer search program

written and run by Prof. R. M. Robinson. It was the first Mersenne prime to be identified in thirty-eight years; the next one, $M_{607}$, was found by the computer a little less than two hours later. Three more — $M_{1279}$, $M_{2203}$, $M_{2281}$ — were found by the same program in the next several months. $M_{4253}$ is the first Mersenne prime that is titanic, $M_{44,497}$ is the first gigantic, and $M_{6,972,593}$ was the first megaprime to be discovered, being a prime with at least 1,000,000 digits.[13] All three were the first known prime of any kind of that size. The number of digits in the decimal representation of $Mn$ equals $\lfloor n \times \log_{10} 2 \rfloor + 1$, where $\lfloor x \rfloor$ denotes the floor function (or equivalently $\lfloor \log_{10} Mn \rfloor + 1$).

In September 2008, mathematicians at UCLA participating in GIMPS won part of a $100,000 prize from the Electronic Frontier Foundation for their discovery of a very nearly 13-million-digit Mersenne prime. The prize, finally confirmed in October 2009, is for the first known prime with at least 10 million digits. The prime was found on a Dell OptiPlex 745 on August 23, 2008. This was the eighth Mersenne prime discovered at UCLA.[14]

On April 12, 2009, a GIMPS server log reported that a 47th Mersenne prime had possibly been found. The find was verified on June 12, 2009. The prime is $2^{42,643,801} - 1$. Although it is chronologically the 47th Mersenne prime to be discovered, it is smaller than the largest known at the time, which was the 45th to be discovered.

On January 25, 2013, Curtis Cooper, a mathematician at the University of Central Missouri, discovered a 48th Mersenne prime, $2^{57,885,161} - 1$ (a number with 17,425,170 digits), as a result of a search executed by a GIMPS server network.[15]

On January 19, 2016, Cooper published his discovery of a 49th Mersenne prime, $2^{74,207,281} - 1$ (a number with 22,338,618 digits), as a result of a search executed by a GIMPS server network.[2] This was the fourth Mersenne prime discovered by Cooper and his team in the past ten years.

# 5 Theorems about Mersenne numbers

1. If $a$ and $p$ are natural numbers such that $a^p - 1$ is prime, then $a = 2$ or $p = 1$.
   - **Proof**: $a \equiv 1 \pmod{a-1}$. Then $a^p \equiv 1 \pmod{a-1}$, so $a^p - 1 \equiv 0 \pmod{a-1}$. Thus $a - 1 \mid a^p - 1$. However, $a^p - 1$ is prime, so $a - 1 = a^p - 1$ or $a - 1 = \pm 1$. In the former case, $a = a^p$, hence $a = 0,1$ (which is a contradiction, as neither $-1$ nor 0 is prime) or $p = 1$. In the latter case, $a = 2$ or $a = 0$. If $a = 0$, however, $0^p - 1 = 0 - 1 = -1$ which is not prime. Therefore, $a = 2$.
2. If $2^p - 1$ is prime, then $p$ is prime.
   - **Proof**: suppose that $p$ is composite, hence can be written $p = ab$ with $a$ and $b > 1$. Then $2^p - 1 = 2^{ab} - 1 = (2^a)^b - 1 = (2^a - 1)((2^a)^{b-1} + (2^a)^{b-2} + \ldots + 2^a + 1)$ so $2^p - 1$ is composite contradicting our assumption that $2^p - 1$ is prime.
3. If $p$ is an odd prime, then every prime $q$ that divides $2^p - 1$ must be 1 plus a multiple of $2p$. This holds even when $2^p - 1$ is prime.
   - **Examples**: Example I: $2^5 - 1 = 31$ is prime, and $31 = 1 + 3 \times (2 \times 5)$. Example II: $2^{11} - 1 = 23 \times 89$, where $23 = 1 + (2 \times 11)$, and $89 = 1 + 4 \times (2 \times 11)$.
   - **Proof**: By Fermat's little theorem, $q$ is a factor of $2^{q-1} - 1$. Since $q$ is a factor of $2^p - 1$, for all positive integers $c$, $q$ is also a factor of $2^{pc} - 1$. Since $p$ is prime and $q$ is not a factor of $2^1 - 1$, $p$ is also the smallest positive integer $x$ such that $q$ is a factor of $2^x - 1$. As a result, for all positive integers $x$, $q$ is a factor of $2^x - 1$ if and only if $p$ is a factor of $x$. Therefore, since $q$ is a factor of $2^{q-1} - 1$, $p$ is a factor of $q - 1$ so $q \equiv 1 \pmod{p}$. Furthermore, since $q$ is a factor of $2^p - 1$, which is odd, $q$ is odd. Therefore, $q \equiv 1 \pmod{2p}$.
   - **Note**: This fact provides a proof of Euclid's theorem, which asserts the infinitude of primes, distinct from the proof written by Euclid: for every odd prime $p$, all primes dividing $2^p - 1$ are larger than $p$; thus there are always larger primes than any particular prime.
   - **Note**: In conjunction with the next theorem below, certain multiples of $2p$ are impossible, namely when $2p$ is multiplied by twice an odd number. Thus for instance $4p+1$, $12p+1$, $20p+1$, and so forth cannot be factors of $2^p - 1$. **Proof**: Each factor must have $2kp + 1 = 8n \pm 1$ for some $k$ and $n$, so if we assume $k = 2(2m+1)$ then we get either $p = 2(n-mp)$ or $2p + 1 = 4(n-mp)$, each of which is a contradiction as one side of the equation is odd and the other is even. More generally we can show that $(p \bmod 4) \equiv 1 \Rightarrow k \equiv 0$ or $3 \pmod 4$, and $(p \bmod 4) \equiv 3 \Rightarrow k \equiv 0$ or $1 \pmod 4$.
4. If $p$ is an odd prime, then every prime $q$ that divides $2^p - 1$ is congruent to $\pm 1 \pmod 8$.
   - **Proof**: $2^{p+1} \equiv 2 \pmod q$, so $2^{1/2(p+1)}$ is a square root of 2 mod $q$. By quadratic reciprocity, every prime modulo in which the number 2 has a square root is congruent to $\pm 1 \pmod 8$.
5. A Mersenne prime cannot be a Wieferich prime.
   - **Proof**: We show if $p = 2^m - 1$ is a Mersenne prime, then the congruence $2^p - 1 \equiv 1 \pmod$

$p^2$) does not hold. By Fermat's little theorem, $m \mid p-1$. Now write, $p-1 = m\lambda$. If the given congruence is satisfied, then $p^2 \mid 2^{m\lambda}-1$,therefore $0 \equiv 2^{m\lambda}-1/2^m-1 = 1+2^m+2^{2m}+\ldots+2^{(\lambda-1)m} \equiv -\lambda \bmod (2^m-1)$. Hence $2^m-1 \mid \lambda$,and therefore $\lambda \geq 2^m-1$. This leads to $p-1 \geq m(2^m-1)$, which is impossible since $m \geq 2$.

6. If $m$ and $n$ are natural numbers then $m$ and $n$ are coprime if and only if $2^m-1$ and $2^n-1$ are coprime. Consequently, a prime number divides at most one prime-exponent Mersenne number,[16] so in other words the set of pernicious Mersenne numbers is pairwise coprime.

7. If $p$ and $2p+1$ are both prime (meaning that $p$ is a Sophie Germain prime), and $p$ is congruent to 3 (mod 4), then $2p+1$ divides $2^p-1$.[17]

   - **Example**: 11 and 23 are both prime, and $11 = 2 \times 4 + 3$, so 23 divides $2^{11}-1$.
   - **Proof**: Let $q$ be $2p+1$. By Fermat's little theorem, $2^{2p} \equiv 1 \pmod q$, so either $2^p \equiv 1 \pmod q$ or $2^p \equiv -1 \pmod q$. Supposing latter true, then $2^{p+1} = (2^{1/2(p+1)})^2 \equiv -2 \pmod q$, so $-2$ would be a quadratic residue mod $q$. However, since $p$ is congruent to 3 (mod 4), $q$ is congruent to 7 (mod 8) and therefore 2 is a quadratic residue mod $q$. Also since $q$ is congruent to 3 (mod 4), $-1$ is a quadratic nonresidue mod $q$, so $-2$ is the product of a residue and a nonresidue and hence it is a nonresidue, which is a contradiction. Hence, the former congruence must be true and $2p+1$ divides $Mp$.

8. All composite divisors of prime-exponent Mersenne numbers pass the Fermat primality test for the base 2.

# 6 List of known Mersenne primes

The table below lists all known Mersenne primes (sequence A000043 ($p$) and A000668 ($Mp$) in OEIS):

[1] It is not verified whether any undiscovered Mersenne primes exist between the 45th ($M_{37,156,667}$) and the 49th ($M_{74,207,281}$) on this chart; the ranking is therefore provisional.

[2] $M_{42,643,801}$ was first found by a machine on April 12, 2009; however, no human took notice of this fact until June 4. Thus, either April 12 or June 4 may be considered the 'discovery' date.

[3] Strindmo also uses the alias Stig M. Valstad.

[4] $M_{74,207,281}$ was first found by a machine on September 17, 2015; however, no human took notice of this fact until January 7, 2016. Thus, either date may be considered the 'discovery' date. GIMPS considers the January 2016 date to be the official one.

All Mersenne numbers below the 48th Mersenne prime ($M_{57,885,161}$) have been tested at least once but some have not been double-checked. Primes are not always discovered in increasing order. For example, the 29th Mersenne prime was discovered *after* the 30th and the 31st. Similarly, $M_{43,112,609}$ was followed by two smaller Mersenne primes, first 2 weeks later and then 8 months later.[73]

The largest known Mersenne prime ($2^{74,207,281}-1$) is also the largest known prime number.[2] To help visualize its size, displaying the number in base 10 would require 5,957 pages with 75 digits per line and 50 lines per page. $M_{43,112,609}$ was the first discovered prime number with more than 10 million decimal digits.

In modern times, the largest known prime has almost always been a Mersenne prime.[74]

# 7 Factorization of composite Mersenne numbers

The factors of a prime number are by definition one, and the number itself – this section is about composite numbers. Mersenne numbers are very good test cases for the special number field sieve algorithm, so often the largest number factorized with this algorithm has been a Mersenne number. As of August 2016, $2^{1,193}-1$ is the record-holder,[75] using a variant on the special number field sieve allowing the factorisation of several numbers at once. See integer factorization records for links to more information. The special number field sieve can factorize numbers with more than one large factor. If a number has only one very large factor then other algorithms can factorize larger numbers by first finding small factors and then making a primality test on the cofactor. As of August 2016, the largest factorization with probable prime factors allowed is $2^{5,240,707}-1 = 75{,}392{,}810{,}903 \times q$, where $q$ is a 1,577,600-digit probable prime.[76]

(sequence A244453 in the OEIS) (or A089162 with both prime and composite Mersenne numbers) (for the primes $p$, see A054723)

# 8 Mersenne primitive part

The primitive part of Mersenne number $Mn$ is $\Phi n(2)$, the $n$th cyclotomic polynomial at 2, they are

1, 3, 7, 5, 31, 3, 127, 17, 73, 11, 2047, 13, 8191, 43, 151, 257, 131071, 57, 524287, 205, 2359, 683, 8388607, 241, 1082401, 2731, 262657, 3277, 536870911, 331, ... (sequence A019320 in the OEIS)

Besides, if we notice those prime factors, and delete "old prime factors", for example, 3 divides the 2nd, 6th, 18th, 54th, 162nd, ... terms of this sequence, we only allow the 2nd term divided by 3, if we do, they are

> 1, 3, 7, 5, 31, 1, 127, 17, 73, 11, 2047, 13, 8191, 43, 151, 257, 131071, 19, 524287, 41, 337, 683, 8388607, 241, 1082401, 2731, 262657, 3277, 536870911, 331, ... (sequence A064078 in the OEIS)

The numbers $n$ for which $\Phi n(2)$ is prime are

> 2, 3, 4, 5, 6, 7, 8, 9, 10, 12, 13, 14, 15, 16, 17, 19, 22, 24, 26, 27, 30, 31, 32, 33, 34, 38, 40, 42, 46, 49, 56, 61, 62, 65, 69, 77, 78, 80, 85, 86, 89, 90, 93, 98, 107, 120, 122, 126, 127, 129, 133, 145, 150, ... (sequence A072226 in the OEIS)

The numbers $n$ for which $2^n - 1$ has an only primitive prime factor are

> 2, 3, 4, 5, 7, 8, 9, 10, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 24, 26, 27, 30, 31, 32, 33, 34, 38, 40, 42, 46, 49, 54, 56, 61, 62, 65, 69, 77, 78, 80, 85, 86, 89, 90, 93, 98, 107, 120, 122, 126, 127, 129, 133, 145, 147, 150, ... (sequence A161508 in the OEIS) (Differ from last sequence, this sequence does not have the term 6, but has the terms 18, 20, 21, 54, 147, 342, 602, and 889, and it is conjectured that no others)

# 9 Mersenne numbers in nature and elsewhere

In computer science, unsigned $n$-bit integers can be used to express numbers up to $Mn$. Signed $(n + 1)$-bit integers can express values between $-(Mn + 1)$ and $Mn$, using the two's complement representation.

In the mathematical problem Tower of Hanoi, solving a puzzle with an $n$-disc tower requires $Mn$ steps, assuming no mistakes are made.[77]

The asteroid with minor planet number 8191 is named 8191 Mersenne after Marin Mersenne, because 8191 is a Mersenne prime (3 Juno, 7 Iris, 31 Euphrosyne and 127 Johanna having been discovered and named during the 19th century).[78]

# 10 Mersenne–Fermat primes

A **Mersenne–Fermat number** is defined as $2^{p^r} - 1/2^{p^{r-1}} - 1$, with $p$ prime, $r$ natural number, and can be written as **MF(*p*, *r*)**, when $r = 1$, it is a Mersenne number, and when $p = 2$, it is a Fermat number, the only known Mersenne–Fermat prime with $r > 1$ are

> MF(2, 2), MF(3, 2), MF(7, 2), MF(59, 2), MF(2, 3), MF(3, 3), MF(2, 4), and MF(2, 5).[79]

In fact, MF($p$, $r$) = $\Phi p^r(2)$, where $\Phi$ is the cyclotomic polynomial.

# 11 Generalizations

The simplest generalized Mersenne primes are prime numbers of the form $f(2^n)$, where $f(x)$ is a low-degree polynomial with small integer coefficients.[80] An example is $2^{64} - 2^{32} + 1$, in this case, $n = 32$, and $f(x) = x^2 - x + 1$; another example is $2^{192} - 2^{64} - 1$, in this case, $n = 64$, and $f(x) = x^3 - x - 1$.

It is also natural to try to generalize primes of the form $2^n - 1$ to primes of the form $b^n - 1$ (for $b \neq 2$ and $n > 1$). However (see also theorems above), $b^n - 1$ is always divisible by $b - 1$, so unless the latter is a unit, the former is not a prime. There are two ways to deal with that:

## 11.1 Complex numbers

In the ring of integers (on real numbers), if $b - 1$ is a unit, then $b$ is either 2 or 0. But $2^n - 1$ are the usual Mersenne primes, and the formula $0^n - 1$ does not lead to anything interesting (since it is always $-1$ for all $n > 0$). Thus, we can regard a ring of "integers" on complex numbers instead of real numbers, like Gaussian integers and Eisenstein integers.

### 11.1.1 Gaussian Mersenne primes

If we regard the ring of Gaussian integers, we get the case $b = 1 + i$ and $b = 1 - i$, and can ask (WLOG) for what $n$ the number $(1 + i)^n - 1$ is a *Gaussian prime* which will then be called a **Gaussian Mersenne prime**.[81]

$(1 + i)^n - 1$ is a Gaussian prime for the following $n$:

> 2, 3, 5, 7, 11, 19, 29, 47, 73, 79, 113, 151, 157, 163, 167, 239, 241, 283, 353, 367, 379, 457, 997, 1367, 3041, 10141, 14699, 27529, 49207, 77291, 85237, 106693, 160423, 203789, 364289, 991961, 1203793, 1667321, 3704053, 4792057, ... (sequence A057429 in the OEIS)

This sequence is in many ways similar to the list of exponents of ordinary Mersenne primes.

The norms (i.e. squares of absolute values) of these Gaussian primes are rational primes:

> 5, 13, 41, 113, 2113, 525313, 536903681, 140737471578113, ... (sequence A182300 in the OEIS).

### 11.1.2 Eisenstein Mersenne primes

We can also regard the ring of Eisenstein integers, we get the case $b = 1 + \omega$ and $b = 1 - \omega$, and can ask for what $n$ the number $(1 + \omega)^n - 1$ is an *Eisenstein prime* which will then be called a **Eisenstein Mersenne prime**.

$(1 + \omega)^n - 1$ is an Eisenstein prime for the following $n$:

> 2, 5, 7, 11, 17, 19, 79, 163, 193, 239, 317, 353, 659, 709, 1049, 1103, 1759, 2029, 5153, 7541, 9049, 10453, 23743, 255361, 534827, 2237561, ... (sequence A066408 in the OEIS)

The norms (i.e. squares of absolute values) of these Eisenstein primes are rational primes:

> 7, 271, 2269, 176419, 129159847, 1162320517, ... (sequence A066413 in the OEIS)

## 11.2 Divide an integer

### 11.2.1 Repunit primes

Main article: Repunit

The other way to deal with the fact that $b^n - 1$ is always divisible by $b - 1$, it is to simply take out this factor and ask which values of $n$ make

$$\frac{b^n - 1}{b - 1}$$

be prime. (The integer $b$ can be either positive or negative.) If for example we take $b = 10$, we get $n$ values of:

> 2, 19, 23, 317, 1031, 49081, 86453, 109297, 270343, ... (sequence A004023 in the OEIS), corresponding to primes 11, 1111111111111111111, 11111111111111111111111, ... (sequence A004022 in the OEIS).

These primes are called **repunit primes**. Another example is when we take $b = -12$, we get $n$ values of:

> 2, 5, 11, 109, 193, 1483, 11353, 21419, 21911, 24071, 106859, 139739, ... (sequence A057178 in the OEIS), corresponding to primes −11, 19141, 57154490053, ....

It is a conjecture that for every integer $b$ which is not a perfect power, there are infinitely many values of $n$ such that $b^n - 1/b - 1$ is prime. (When $b$ is a perfect power, it can be shown that there is at most one $n$ value such that $b^n - 1/b - 1$ is prime)

Least $n$ such that $b^n - 1/b - 1$ is prime are (starting with $b = 2$)

> 2, 3, 2, 3, 2, 5, 3, 0, 2, 17, 2, 5, 3, 3, 2, 3, 2, 19, 3, 3, 2, 5, 3, 0, 7, 3, 2, 5, 2, 7, 0, 3, 13, 313, 2, 13, 3, 349, 2, 3, 2, 5, 5, 19, 2, 127, 19, 0, 3, 4229, 2, 11, 3, 17, 7, 3, 2, 3, 2, 7, 3, 5, 0, 19, 2, 19, 5, 3, 2, 3, 2, ... (sequence A084740 in the OEIS)

For negative bases $b$, they are (starting with $b = -2$)

> 3, 2, 2, 5, 2, 3, 2, 3, 5, 5, 2, 3, 2, 3, 3, 7, 2, 17, 2, 3, 3, 11, 2, 3, 11, 0, 3, 7, 2, 109, 2, 5, 3, 11, 31, 5, 2, 3, 53, 17, 2, 5, 2, 103, 7, 5, 2, 7, 1153, 3, 7, 21943, 2, 3, 37, 53, 3, 17, 2, 7, 2, 3, 0, 19, 7, 3, 2, 11, 3, 5, 2, ... (sequence A084742 in the OEIS) (notice this OEIS sequence does not allow $n = 2$)

Least base $b$ such that $b^{\text{prime}(n)} - 1/b - 1$ is prime are

> 2, 2, 2, 2, 5, 2, 2, 2, 10, 6, 2, 61, 14, 15, 5, 24, 19, 2, 46, 3, 11, 22, 41, 2, 12, 22, 3, 2, 12, 86, 2, 7, 13, 11, 5, 29, 56, 30, 44, 60, 304, 5, 74, 118, 33, 156, 46, 183, 72, 606, 602, 223, 115, 37, 52, 104, 41, 6, 338, 217, ... (sequence A066180 in the OEIS)

For negative bases $b$, they are

> 3, 2, 2, 2, 2, 2, 2, 2, 2, 7, 2, 16, 61, 2, 6, 10, 6, 2, 5, 46, 18, 2, 49, 16, 70, 2, 5, 6, 12, 92, 2, 48, 89, 30, 16, 147, 19, 19, 2, 16, 11, 289, 2, 12, 52, 2, 66, 9, 22, 5, 489, 69, 137, 16, 36, 96, 76, 117, 26, 3, ... (sequence A103795 in the OEIS)

### 11.2.2 Other generalized Mersenne primes

Another generalized Mersenne number is

$$\frac{a^n - b^n}{a - b}$$

with $a$, $b$ any coprime integers, $a > 1$ and $-a < b < a$. (Since $a^n - b^n$ is always divisible by $a - b$, the division is necessary for there to be any chance of finding prime numbers. In fact, this number is the same as the Lucas number $Un(a + b, ab)$, since $a$ and $b$ are the roots of the quadratic equation $x^2 - (a + b)x + ab = 0$, and this number equals 1 when $n = 1$) We can ask which $n$ makes this number prime. It can be shown that such $n$ must be primes themselves or equal to 4, and $n$ can be 4 if and only if $a + b = 1$ and $a^2 + b^2$ is prime. (Since $a^4 - b^4/a - b = (a + b)(a^2 + b^2)$. Thus, in this case the pair $(a, b)$ must be $(x + 1, -x)$ and $x^2 + (x + 1)^2$ must be prime. That is, $x$ must be in OEIS A027861.) It is a conjecture that for any pair $(a, b)$ such that for every natural number $r > 1$, $a$ and $b$ are not both perfect $r$th powers, and $-4ab$ is not a perfect fourth power. there are infinitely many values of $n$ such that $a^n - b^n/a - b$ is prime. (When $a$ and $b$ are both perfect $r$th powers for an $r > 1$ or when $-4ab$ is a perfect fourth power, it can be shown that there are at most two $n$ values with this property, since if so, then $a^n - b^n/a - b$ can be factored algebraically) However, this has not been proved for any single value of $(a, b)$.

*Note: if $b < 0$ and $n$ is even, then the numbers $n$ are not included in the corresponding OEIS sequence.

A conjecture related to the generalized Mersenne primes:[90][91] (the conjecture predicts where is the next generalized Mersenne prime, if the conjecture is true, then there are infinitely many primes for all such $(a,b)$ pairs)

For any integers $a$ and $b$ which satisfy the conditions:

1. $a > 1, -a < b < a$.
2. $a$ and $b$ are coprime. (thus, $b$ cannot be 0)
3. For every natural number $r > 1$, $a$ and $b$ are not both perfect $r$th powers. (since when $a$ and $b$ are both perfect $r$th powers, it can be shown that there are at most two $n$ value such that $a^n - b^n/a - b$ is prime, and these $n$ values are $r$ itself or a root of $r$, or 2)
4. $-4ab$ is not a perfect fourth power (if so, then the number has aurifeuillean factorization).

has prime numbers of the form

$$R_p(a,b) = \frac{a^p - b^p}{a - b}$$

for prime $p$, the prime numbers will be distributed near the best fit line

$$Y = G \cdot \log_a(\log_a(R_{(a,b)}(n))) + C$$

where

$$\lim_{n\to\infty} G = \frac{1}{e^\gamma} = 0.561459483566\ldots$$

and there are about

$$(\log_e(N) + m \cdot \log_e(2) \cdot \log_e\left(\log_e(N)\right) + \frac{1}{\sqrt{N}} - \delta\Big) \cdot \frac{e^\gamma}{\log_e(a)}$$

prime numbers of this form less than $N$.

- $e$ is the base of the natural logarithm.
- $\gamma$ is Euler–Mascheroni constant.
- $\log a$ is the logarithm in base $a$.
- $R_{(a,b)}(n)$ is the $n$th prime number of the form $a^p - b^p/a - b$ for prime $p$.
- $C$ is a data fit constant which varies with $a$ and $b$.
- $\delta$ is a data fit constant which varies with $a$ and $b$.
- $m$ is the largest natural number such that $a$ and $-b$ are both $2^{m-1}$th powers.

We also have the following three properties:

1. The number of prime numbers of the form $a^p - b^p/a - b$ (with prime $p$) less than or equal to $n$ is about $e^\gamma \log a(\log a(n))$.
2. The expected number of prime numbers of the form $a^p - b^p/a - b$ with prime $p$ between $n$ and $an$ is about $e^\gamma$.
3. The probability that number of the form $a^p - b^p/a - b$ is prime (for prime $p$) is about $e^\gamma/p \log e(a)$.

If this conjecture is true, then for all such $(a,b)$ pairs, let $q$ be the $n$th prime of the form $a^p - b^p/a - b$, the graph of $\log a(\log a(q))$ versus $n$ is almost linear. (See [90])

When $a = b + 1$, it is $(b + 1)^n - b^n$, a difference of two consecutive perfect $n$th powers, and if $a^n - b^n$ is prime, then $a$ must be $b + 1$, because it is divisible by $a - b$.

Least $n$ such that $(b + 1)^n - b^n$ is prime are

2, 2, 2, 3, 2, 2, 7, 2, 2, 3, 2, 17, 3, 2, 2, 5, 3, 2, 5, 2, 2, 229, 2, 3, 3, 2, 3, 3, 2, 2, 5, 3, 2, 3, 2, 2, 3, 3, 2, 7, 2, 3, 37, 2, 3, 5, 58543, 2, 3, 2, 2, 3, 2, 2, 3, 2, 5, 3, 4663, 54517, 17, 3, 2, 5, 2, 3, 3, 2, 2, 47, 61, 19, ... (sequence A058013 in the OEIS)

Least $b$ such that $(b + 1)^{\text{prime}(n)} - b^{\text{prime}(n)}$ is prime are

1, 1, 1, 1, 5, 1, 1, 1, 5, 2, 1, 39, 6, 4, 12, 2, 2, 1, 6, 17, 46, 7, 5, 1, 25, 2, 41, 1, 12, 7, 1, 7, 327, 7, 8, 44, 26, 12, 75, 14, 51, 110, 4, 14, 49, 286, 15, 4, 39, 22, 109, 367, 22, 67, 27, 95, 80, 149, 2, 142, 3, 11, ... (sequence A222119 in the OEIS)

## 12 See also

- Repunit
- Fermat prime
- Power of 2
- Erdős–Borwein constant
- Mersenne conjectures
- Mersenne twister
- Double Mersenne number
- Prime95 / MPrime
- Great Internet Mersenne Prime Search (GIMPS)
- Largest known prime number
- Titanic prime
- Gigantic prime
- Megaprime
- Wieferich prime
- Wagstaff prime
- Cullen prime
- Woodall prime
- Proth prime
- Solinas prime
- Gillies' conjecture

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half of the book, he elegantly expounds linear numbers [i.e. prime numbers], polygonal numbers and all sorts of plane numbers, solid numbers and the five figures which are assigned to the elements of the universe, discussing both their individual attributes and their shared features, and their proportionality and reciprocity." *Iamblichus* The Theology of Arithmetic translated by Robin Waterfiled, 1988, p. 112f. [Retrieved 2012-11-11]. Iamblichus also gives us a direct quote from Speusippus' book where Speusippus among other things writes: "Secondly, it is necessary for a perfect number [the concept "perfect number" is not used here in a modern sense] to contain an equal amount of prime and incomposite numbers, and secondary and composite numbers." *Iamblichus* The Theology of Arithmetic translated by Robin Waterfiled, 1988, p. 113. [Retrieved 2012-11-11]. For the Greek original text, see Speusippus of Athens: A Critical Study with a Collection of the Related Texts and Commentary by Leonardo Tarán, 1981, p. 140 line 21–22 [Retrieved 2012-11-11] In his comments to Nicomachus of Gerasas's Introduction to Arithmetic, Iamblichus also mentions that Thymaridas, ca. 400 BC – ca. 350 BC, uses the term *rectilinear* for prime numbers, and that Theon of Smyrna, fl. AD 100, uses *euthymetric* and *linear* as alternative terms. Nicomachus of Gerasa, Introduction to Arithmetic, 1926, p. 127 [Retrieved 2012-11-11] It is unclear though when this said Thymaridas lived. "In a highly suspect passage in Iamblichus, Thymaridas is listed as a pupil of Pythagoras himself." Pythagoreanism [Retrieved 2012-11-11] Before Philolaus, c. 470–c. 385 BC, we have no proof of any knowledge of prime numbers.

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[83] $(x, 1)$ and $(x, -1)$ for $x = 2$ to 50

[84] $(x, 1)$ for $x = 2$ to 152

[85] $(x, -1)$ for $x = 2$ to 151

[86] $(x + 1, x)$ for $x = 1$ to 150

[87] $(x + 1, -x)$ for $x = 1$ to 40

[88] $(x + 2, x)$ for odd $x = 1$ to 107

[89] $(x, -1)$ for $x = 2$ to 200

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# 14 External links

- Hazewinkel, Michiel, ed. (2001), "Mersenne number", *Encyclopedia of Mathematics*, Springer, ISBN 978-1-55608-010-4
- GIMPS home page
- GIMPS status — status page gives various statistics on search progress, typically updated every week, including progress towards proving the ordering of primes 42–47
- GIMPS, known factors of Mersenne numbers
- $Mq = (8x)^2 - (3qy)^2$ Property of Mersenne numbers with prime exponent that are composite (PDF)
- $Mq = x^2 + d \cdot y^2$ math thesis (PS)
- Grime, James. "31 and Mersenne Primes". *Numberphile*. Brady Haran.
- Mersenne prime bibliography with hyperlinks to original publications
- report about Mersenne primes — detection in detail (German)
- GIMPS wiki
- Will Edgington's Mersenne Page — contains factors for small Mersenne numbers
- Known factors of Mersenne numbers
- Decimal digits and English names of Mersenne primes
- Prime curios: 2305843009213693951
- Factorization of Mersenne numbers $Mn$, with $n$ odd, $n$ up to 1199
- Factorization of Mersenne numbers $M_2n$, $2n$ up to 2398 ($n$ up to 1199) or $2n$ is in the form $8k + 4$ up to 4796 ($n$ is on the form $4k + 2$ up to 2398)
- Factorization of Mersenne numbers $Mn$ ($n$ up to 1280)
- Factorization of completely factored Mersenne numbers
- The Cunningham project, factorization of $b^n \pm 1$, $b = 2, 3, 5, 6, 7, 10, 11, 12$
- Factorization of $b^n \pm 1$, $2 \le b \le 12$
- Factorization of $a^n \pm b^n$, with coprime $a$, $b$, $2 \le b < a \le 12$

## 14.1 MathWorld links

- Weisstein, Eric W. "Mersenne number". *MathWorld*.
- Weisstein, Eric W. "Mersenne prime". *MathWorld*.
- 47th Mersenne Prime Found

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