

10

Low noise circuits

10.1 Introduction

As Wilmshurst [1] has written, noise in electronics has, today, come to mean ‘almost any kind of unwanted signal in an electronic system’. This is in contrast to the classical picture of noise as being a problem area which is only concerned with the fact that electronic circuits operate at a finite temperature and also have to operate with electric currents that are really made up of a flow of discrete charged particles. Further evidence for the wider view which is now taken of noise problems in electronics can be taken from the use of the term ‘electromagnetic compatibility (EMC) [2]’.

For the above reason, this chapter is really in two parts. To begin with, circuit shapes and circuit ideas that attempt to minimise the effects of the intrinsic thermal and shot noise of electronic devices will be considered. This calls for a brief summary of some well-known theory which will be given first. The topic then divides fairly naturally into low and high frequency amplifiers, and some interesting experimental circuits can be proposed for both fields. After this look at these classical kinds of noise problems, the chapter concludes by considering some of the circuit ideas which have been proposed to eliminate very special noise problems in various signal processing systems.

10.2 Intrinsic thermal noise sources

An excellent reference for the fundamentals of noise in electronic circuit design is chapter 11 of the book by Gray and Meyer [3]. For a deeper treatment of noise in solid state devices, both Bell [4] and Buckingham [5] will be found valuable.

The origin of thermal noise, not only in electronic systems, is the finite temperature and the discrete particle nature of the world we live in. There must be a noise power, kTB , associated with any signal channel, which is added to the signal power that channel may carry. Here k is the Boltzmann constant, 1.38×10^{-23} J/K, T is the absolute temperature in Kelvin, and B is the channel bandwidth in hertz. For audio frequency work, kTB is extremely small, and is, of course, a measure of the sensitivity of the human ear. Taking T to be 290 K and B to be 10 kHz, $kTB = 4 \times 10^{-17}$ W. In this example, the noise is due to the finite temperature of the air and the fact that air is made up of discrete molecules. The remarkable sensitivity of the human ear, estimated above, can be confirmed experimentally [6].

Applied to electrical systems, this fundamental idea of a noise power source, kTB , associated with any signal channel, leads to the result that any resistor, value R , may be represented by a noiseless resistor in series with a thermal noise voltage source of rms value

$$e_n = (4kTBR)^{\frac{1}{2}}. \quad (10.1)$$

Alternatively, the representation may be a noiseless resistor in parallel with a thermal noise current source of rms value

$$i_n = (4kTB/R)^{\frac{1}{2}}. \quad (10.2)$$

The rms noise voltage, given by equation (10.1), and the rms noise current, given by equation (10.2), both increase with the square root of the system bandwidth. This follows, of course, from the fact that the available noise power, kTB , increases with bandwidth directly. This is why the units $\text{nV}/\text{Hz}^{\frac{1}{2}}$ and $\text{pA}/\text{Hz}^{\frac{1}{2}}$ are frequently used in noise data as measures for e_n and i_n . An actual measurement of noise is nearly always a power measurement (the spectral density) taken over a well-defined bandwidth, and then this power, referred to unit bandwidth for convenience, is converted into an equivalent voltage or current.

10.3 Intrinsic shot noise sources

The fact that an electrical current is really a flow of discrete charges means that there must be a statistical component associated with any current. This means that the collector current, or drain current, of a transistor must have a shot noise component. For the bipolar transistor there will also be a shot noise component associated with the base current, and a thermal noise source associated with the unavoidable bulk resistance of the base.

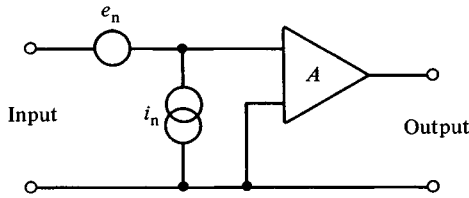


Fig. 10.1. The equivalent circuit for noise in a solid state amplifying device. The ideal device, A , is noise-free and the noise of the real device is all referred to the input where it is represented by the two generators e_n and i_n .

At low frequencies, the audio frequencies, the noise in solid state devices can become much greater than the simple shot noise model would predict. This will be discussed briefly in the next section. For the moment, it is only necessary to note that it has become standard practice to represent the noise in a solid state device by means of the equivalent circuit shown in Fig. 10.1. Using this simple model, it is easy to show [7] that there is an optimum source impedance

$$R_{s(\text{opt})} = e_n/i_n \quad (10.3)$$

which should be used with the device to give a minimum noise figure

$$F_{\text{min}} = 10 \log_{10} (1 + e_n i_n / 2kT). \quad (10.4)$$

Noise figure is a measure of how much noise an amplifying device adds to the thermal noise which is intrinsic to the signal channel anyway, and this would be zero for a noiseless amplifier. Equation (10.4) shows that the really important noise parameter for an amplifying device is the *product* of the two generators, e_n and i_n , shown in Fig. 10.1. This product gives a measure of the amplifier noise power per unit bandwidth, while kT gives a measure of the thermal noise power over the same unit bandwidth.

10.4 Low frequency noise and the integrated circuit process

Very remarkable improvements in the noise performance of solid state devices have been achieved since the mid-1960s. These improvements are clearly due to far better processing technique.

Considering bipolar devices first, the main problem with the early devices of the 1960s was so-called $1/f$ noise, or flicker noise, which plagued circuit performance in the audio frequency range. Fig. 10.2 illustrates the problem. If the noise spectral density of a device is measured at its output, and then referred back to the input, as shown in Fig. 10.1, to relate to the

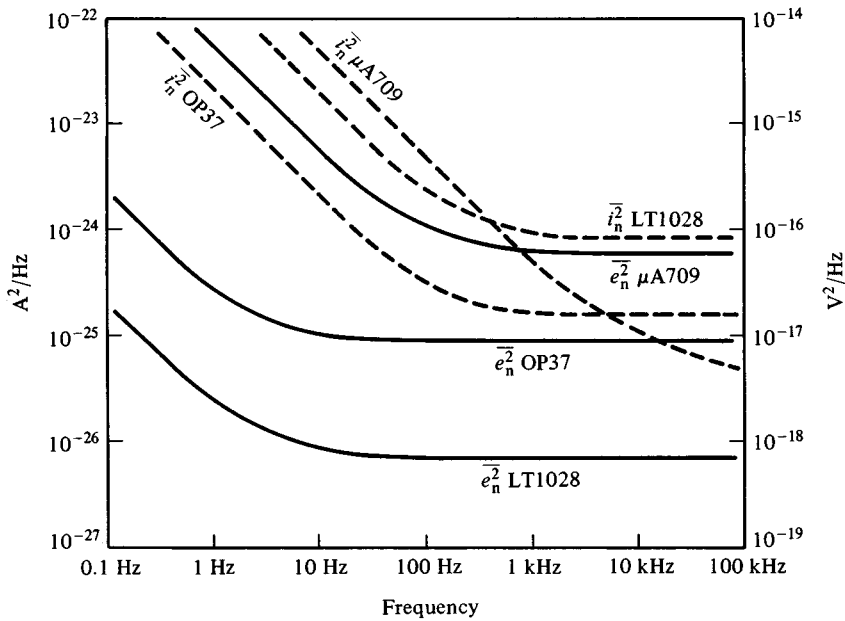


Fig. 10.2. The spectral densities of the two generators shown in Fig. 10.1 for a classical op-amp of 1965, the μA709 , for a low noise op-amp of 1981, the OP37, and for a more recent ultra-low noise op-amp, the LT1028.

noise sources $\overline{e_n^2}$ and $\overline{i_n^2}$, a $1/f$ variation is always found when measurements are taken at a low enough frequency. What has changed dramatically since the mid-1960s, is the frequency at which this $1/f$ noise, or flicker noise, sets in. Improvements in processing technique have pushed this frequency lower and lower, as well as reducing the noise overall.

The μA709 was an early high gain bipolar operational amplifier, first described in 1965 [8]. As Fig. 10.2 shows the spectral density $\overline{i_n^2}$ never really flattens out with this device as measurements are taken over the audio range. Even at 100 kHz, $\overline{i_n^2}$ is still falling quite rapidly. In contrast, $\overline{e_n^2}$ for the μA709 may be treated as a constant from 1 kHz to 100 kHz, but it shows the $1/f$ behaviour at frequencies below 1 kHz.

It is this poor low frequency noise performance which more recent developments in integrated circuit processing, and design, have put right. Fig. 10.2 shows $\overline{e_n^2}$ and $\overline{i_n^2}$ for a well-established low noise operational amplifier: the OP37 [9]. For the OP37, both $\overline{e_n^2}$ and $\overline{i_n^2}$ are really constant down to 1 kHz, and $\overline{e_n^2}$ does not begin to show the $1/f$ variation until the measurement frequency is below 10 Hz. A more recent ultra-low noise bipolar operational amplifier, the LT1028 [10], is very similar as far as the

Table 10.1. *Three recent op-amps, the OP37, LT1028 and OP15, are compared with the classical μ A709 for low frequency noise performance.*

Device	Type	Test freq. (kHz)	e_n (nV/Hz ^{1/2})	i_n (pA/Hz ^{1/2})	$R_{s(\text{opt})}$	Noise (fig. dB)
μ A709	Bipolar	1	7.75	0.71	10.9 k Ω	2.27
OP37	Bipolar	1	3.0	0.4	7.5 k Ω	0.61
LT1028	Bipolar	1	0.85	1.0	850 Ω	0.44
OP15	JFET	1	15.0	0.01	1.5 M Ω	0.08

frequency dependence of $\overline{e_n^2}$ and $\overline{i_n^2}$ is concerned. What is special about the LT1028 is its remarkably low $\overline{e_n^2}$.

Table 10.1 attempts to summarise these results. Going back to equations (10.3) and (10.4), the values of optimum source impedance and the resulting noise figure may be calculated for the three bipolar devices which were the subject of Fig. 10.2. These calculations are made at a frequency of 1 kHz, where the modern devices are showing constant levels of $\overline{e_n^2}$ and $\overline{i_n^2}$, an essential requirement for equations (10.3) and (10.4) to be valid. The calculations for the μ A709 are only included for a rough comparison. The results shown in Table 10.1 indicate the very low noise figures which should be obtained with the OP37 and the LT1028 when these devices are used as audio amplifiers. The values obtained for $R_{s(\text{opt})}$ are also interesting because a low value of $R_{s(\text{opt})}$ can be a very useful feature. This follows because most signal sources, like microphones, sensors, magnetic recording heads, etc., have quite low impedance, and, even when this is not so, the signal source may have to be connected to its amplifier through a fairly low impedance cable.

Looking again at equations (10.3) and (10.4) shows that a good design strategy for a low noise operational amplifier is to aim to minimise the product $i_n e_n$, and thus minimise the noise figure given by equation (10.4), and also try to reduce e_n as much as possible, without being concerned if this implies an increase in i_n , because this will then reduce e_n/i_n and so reduce the value of $R_{s(\text{opt})}$. Precisely this strategy has been adopted by the designers of the LT1028: the collector current in the input devices is made quite high, giving good wide-band performance, low e_n , but quite high i_n .

Not all amplifiers work from a low signal source impedance, however, and that is why Table 10.1 includes the OP15, an operational amplifier which has JFET input devices. The JFET is the best solid state device of all for very low frequency work because the $1/f$, or flicker, noise is very

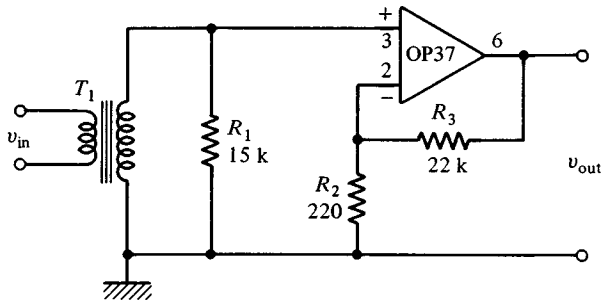


Fig. 10.3. An experimental low noise audio pre-amplifier. The OP37 operates on $\pm 15\text{ V}$ supplies to pins 7 and 4, which must be well decoupled to ground. T_1 is a high performance audio transformer (STC-66121B or RS-217-804) connected 1:12.9 by wiring the two primaries in parallel and the two secondaries in series. The transformer should be mounted in its mu-metal can.

low [11]. As Table 10.1 shows, the JFET has a very low noise figure indeed for audio work, but a very high value of $R_{s(\text{opt})}$. An amplifier like the OP15 is thus ideal as a pre-amplifier for a crystal microphone, where it would be mounted inside the microphone itself, or for the input stage of an oscilloscope amplifier of the kind discussed in section 6.4. Working from a low source impedance, however, the OP15 would appear to have high noise, compared with a bipolar device, because of its very high value of e_n .

10.5 An experimental audio pre-amplifier

If a low noise operational amplifier like the OP37 is to be used as an audio pre-amplifier, its optimum source impedance for minimum noise figure, given in Table 10.1 at $7.5\text{ k}\Omega$, is unlikely to be suitable directly. For example, a dynamic microphone will present a source impedance of perhaps $100\ \Omega$ while a crystal microphone will present a source impedance far higher than $7.5\text{ k}\Omega$.

Fig. 10.3 shows the kind of circuit which will overcome this problem and cause the OP37 to work from its optimum source impedance of $7.5\text{ k}\Omega$. This is done by using a step-up input transformer with a secondary load of $2R_{s(\text{opt})}$. The input to the amplifier then presents an input impedance of $2R_{s(\text{opt})}/N^2$, where N is the turns ratio of the transformer.

Fig. 10.3 is the first experimental circuit for this chapter. T_1 must be a high quality audio transformer, because magnetic components can have noise problems themselves at very small signal levels, due to the properties of the magnetic material used for the core [12], and also because of their

sensitivity to stray fluctuating magnetic fields. The transformer in the circuit shown in Fig. 10.3 provides some voltage gain, $N = 12.9$ in this case, and a further voltage gain of 100 is provided by the OP37 with its feedback network consisting of R_2 and R_3 . Gray and Meyer deal with the problem of the additional noise that comes from the feedback resistors in a case like this [13], and show that the parallel combination of R_2 and R_3 , in Fig. 10.3, is effectively added to the signal source impedance. For this reason, R_2 and R_3 are kept low. Another reason for keeping R_3 low is to exploit the very wide-band properties of the OP37. Connected as a feedback amplifier with a gain of 100, the OP37 should give this gain up to well over 100 kHz, provided stray capacitance across R_3 does not begin to increase the feedback.

The source impedance presented to the OP37 is 7.5 k Ω because, looking back into R_1 , the 15 k Ω shown in Fig. 10.3, this is seen to be in parallel with $N^2 R_s$, where R_s is now the source impedance connected to the true input terminals, v_{in} . When $N = 12.9$, this true source impedance should be 90 Ω : a typical dynamic microphone impedance.

Now equation (10.1) shows that the rms noise voltage which should be measured across a 90 Ω resistor at 290 K, with a bandwidth of 100 kHz, is just under 0.4 μ V. As the circuit shown in Fig. 10.3 has a total voltage gain of 1290, it follows that a noise voltage of about 0.5 mV rms should be observed at the output, if the circuit itself adds no additional noise. In fact, the experimentalist may well see a lower noise level at the output, when the input is simply connected to a 90 Ω resistor, because the overall bandwidth of the circuit shown in Fig. 10.3 will almost certainly be less than 100 kHz because it will be limited by the transformer to be about 50 kHz.

The noise output of 0.5 mV is at a level where it may be conveniently observed on an oscilloscope, when this is on its most sensitive range: 2–5 mV/div. The noise will be seen as a diffuse band across the screen with no random spikes or granularity. This is characteristic of a low noise device like the OP37. If the OP37 is replaced with an ordinary op-amp, like a 741, with which it is pin compatible, a difference in the appearance of the noise will be seen. Measured with an rms voltmeter, the noise at the output of the circuit with a 741 may well be less than with an OP37; this is simply because the bandwidth is now down to nearer 10 kHz because of the poor frequency response of the 741. The real contrast between the two devices will be apparent if the output from the circuit shown in Fig. 10.3 is taken to a simple audio amplifier and loudspeaker arrangement, and the noise is actually listened to. The OP37 produces a featureless hiss, the sound of true white noise. Every 741, in contrast, seems to have its own

particular variety of noise. This subjective test is, of course, picking out the noise in the 1 kHz region, where the human ear is most sensitive. It is here that a device like the 741 may produce the most outrageous noise, or may be fairly quiet. It all depends upon the quality of the manufacturing process.

10.6 What happens to the input power?

With the circuit shown in Fig. 10.3 still in mind, it is interesting to consider a really classical problem of electronic circuit design and ask what really happens to the very small power input that such a circuit accepts. The dynamic microphone which would, in practice, be connected to the input, would produce a signal of, say, $10\ \mu\text{V}$ rms across the $90\ \Omega$ input impedance when the sound level was well above the circuit noise level. This is a power input of just over 1 pW. It might be thought that such a small quantity of power should be carefully led into the amplifying device, but this is not what happens at all. Virtually all of this input power is simply dissipated in the $15\ \text{k}\Omega$ resistor, R_1 .

This is because the power gain of a device like the OP37 is so big, at frequencies in the audio range, that only a minute amount of power need be delivered to its true input. The function of the input circuit is only to get maximum power transfer from the signal source, the microphone, into the circuit itself. This power is then used to develop a voltage, which is the input signal for the amplifying device itself, and, at the same time, the optimum source impedance for this amplifying device must be arranged.

Things need to be more carefully done if low noise performance is called for at higher frequencies. As the signal frequency goes up, the power gain which may be obtained from an electronic amplifying device gets smaller and smaller until, when the microwave region is entered, it is absolutely essential to see that the device itself presents an impedance match to the signal source. Then all the input power should be usefully employed in operating the amplifying device itself. The idea of using a high input impedance amplifier, and then getting an input match by shunting this high impedance with a resistor, which, essentially, is what has been done in Fig. 10.3, is just not acceptable at high frequency.

Considerations of this kind come into many electronic instrumentation problems where a small high frequency signal must be dealt with, and the signal is so small that the amplifier must have the lowest possible noise figure. It is in this area that some very interesting circuit shapes are found. To illustrate this, the next few sections consider a problem which really belongs to the field of communications engineering: the repeater amplifier.

10.7 The repeater amplifier problem

A repeater amplifier must be designed so that it can be inserted into a cable, every 10 km or so, to make up for the losses in that cable and yet add as little extra noise as possible. The repeater amplifier, as shown in Fig. 10.4, should have an input impedance equal to whatever load impedance is put upon it. In practice, this would be the characteristic impedance of the cable: typically $50\ \Omega$ or $75\ \Omega$. Furthermore, the ideal repeater amplifier should present an output impedance equal to whatever impedance may be connected to its input. Only then can it be simply inserted into any cable.

Nordholt [14] has pointed out that the usual ‘brute force’ approach to the design of a repeater amplifier is first to build an amplifier, with the required gain and bandwidth, which has a high input impedance and a low output impedance. The correct terminating impedance for the cable, R_{in} in Fig. 10.4, is then obtained by putting an impedance in parallel with the high input impedance. Similarly, the correct driving impedance for the cable, R_L in Fig. 10.4, is obtained by adding an impedance in series with the very low input impedance of the amplifier. This approach is clearly not a good one, from a noise point of view, because of the power that must be lost in these added impedances. Exactly the same criticism came up in the previous section, where it was shown that virtually all the power input to the audio amplifier, shown in Fig. 10.3, was dissipated in the resistor R_1 .

It may have been Norton [15] who first proposed some ways of getting around this problem, although the techniques which are really useful can be attributed to earlier work by Chaplin, Candy and Cole [16]. This earlier work will be considered in detail in section 10.10. Norton’s proposals are particularly interesting, however, because they are examples of what has been presented here as the circuit shape approach to electronic circuit design. The first circuit proposed in Norton’s paper is shown in Fig. 10.5.

In Fig. 10.5, an amplifier, A , is shown with negative feedback from output to input, arranged by means of two transformers, T_1 and T_2 . It should be noted that the connection of these two transformers is very similar to that found in a directional coupler: the device considered in chapter 5 and shown in Fig. 5.12.

If the amplifier, A , in Fig. 10.5 has a high voltage gain, the voltage, v , across its input may be assumed negligible. Thus

$$v_{in} + v_{out}/N_1 = 0 \quad (10.5)$$

where N_1 is the turns ratio of T_1 .

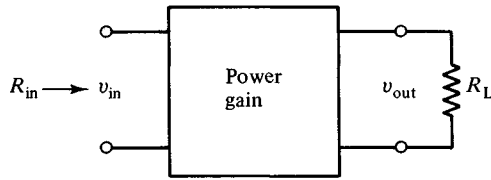


Fig. 10.4. The repeater amplifier. Ideally, R_{in} is equal to R_L so that the amplifier may be inserted into a cable which has $Z_o = R_{in} = R_L$.

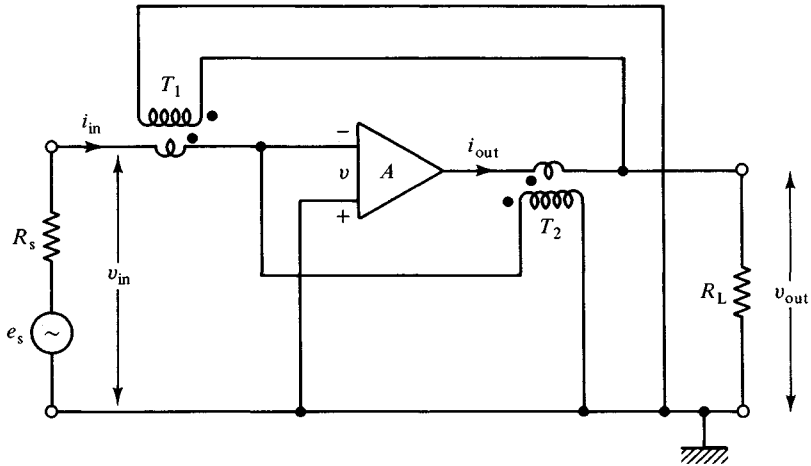


Fig. 10.5. Feedback by means of two transformers.

Similarly, if the amplifier, A , has a very high input impedance, its input current will be negligible, so that

$$i_{in} + i_{out}/N_2 = 0 \quad (10.6)$$

where N_2 is the turns ratio of T_2 . The amplifier system, as shown in Fig. 10.5, is thus an inverting amplifier and, if $N_1 = N_2 = N$, the voltage gain will be N , the input impedance will be equal to the load impedance, R_L , and the output impedance will be equal to the source impedance, R_s . The power gain will be N^2 . All this is just what is required as a solution to the repeater amplifier problem.

The really interesting thing about Norton's proposal is the way in which the power input, $v_{in} i_{in}$ in Fig. 10.5, is nearly all passed on to the load, R_L , by means of the two transformers. The small amount of input power which is not passed on belongs to the terms which were neglected above, to obtain equations (10.5) and (10.6), and is the true input power of the amplifier A .

While very interesting as an idea, or a circuit shape, the amplifier shown

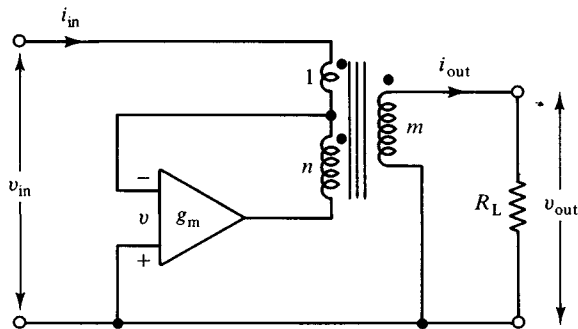


Fig. 10.6. An OTA with transformer feedback.

in Fig. 10.5 is very difficult to make in practice, at the high frequencies where the idea should really be applied, and has also been criticised by Nordholt [14] from a noise point of view. Norton [15] argued that, because feedback is provided by means of the lossless transformers, T_1 and T_2 in Fig. 10.5, the amplifier system is left with only the device noise. Nordholt [14] showed that this was not the case, although it is true that a better noise figure may be obtained with lossless feedback, compared to resistive feedback, and Norton [15] could demonstrate this experimentally.

Norton gave a second circuit in his paper [15] for a repeater amplifier which should avoid loss of power in its input termination. This circuit involved a single transistor in grounded base configuration, with negative feedback again provided through a transformer. As a circuit shape, this second circuit of Norton's suggests the one shown in Fig. 10.6, where an operational transconductance amplifier (OTA) is shown with transformer feedback. The transformer has a third winding to provide an output. This circuit leads to the next experimental circuit for this chapter.

10.8 An analysis of the circuit using an OTA

The OTA shown in Fig. 10.6 is a device which provides an output current, $g_m v$, where v is its true input voltage. A $1:n:m$ transformer is arranged to provide negative feedback and also a voltage output across a load, R_L . Note that 100% negative feedback at d.c. is provided across the OTA when the signal source, v_{in} , is connected to the circuit through a capacitor. This circuit detail will be included in the next section. At the moment it is necessary to analyse just how the circuit shown in Fig. 10.6 is going to work.

Negative feedback at the signal frequency is provided by transformer windings '1' and 'n', because these windings are connected in the sense

shown by the spots in Fig. 10.6. There must be negligible current flow into the inverting input of the OTA, however, so that the ampere-turn balance in the transformer can only be provided when i_{out} , flowing in winding 'm', balances the current i_{in} which flows in windings '1' and 'n'. This means that

$$(1+n)i_{in} = (m)i_{out}. \quad (10.7)$$

The current i_{in} will be given by

$$i_{in} = g_m v \quad (10.8)$$

where v is the very small OTA input voltage. As i_{out} must be given by v_{out}/R_L , where R_L is the load on the circuit shown in Fig. 10.6, it follows that equations (10.7) and (10.8) may be combined to give

$$v = mv_{out}/g_m R_L(1+n). \quad (10.9)$$

The voltages across windings '1' and 'm' must be related so that

$$v_{in} - v = v_{out}/m. \quad (10.10)$$

Combining equations (10.9) and (10.10) gives the voltage gain of the circuit as

$$v_{out}/v_{in} = m/[1 + m^2/g_m R_L(1+n)] \quad (10.11)$$

and this will equal m if the term $m^2/g_m R_L(1+n)$ is very small compared to unity.

Norton's circuit, from which the circuit under discussion has been derived, was intended to have an input impedance equal to the load impedance R_L . When $v_{out}/v_{in} = m$ and $i_{out} = v_{out}/R_L$, equation (10.7) leads to the result

$$R_{in} = v_{in}/i_{in} = (1+n)R_L/m^2 \quad (10.12)$$

so that the condition $R_{in} = R_L$ is obtained when

$$n = m^2 - 1. \quad (10.13)$$

Equation (10.13) shows that some difficulties will be found in applying this circuit as a repeater amplifier at high frequency because, if a reasonable gain is required, 20 db for example, the turns ratios, 1:n:m, will be 1:99:10, and, at high frequency, this will not be easy. At audio frequencies, such high turns ratios present no problems, but audio amplifiers with the repeater property, $R_{in} = R_L$, are not in much demand. From an experimentalist's point of view it is better to look upon the circuit, shown in Fig. 10.6, as one which provides a voltage gain m and an impedance transformation, given by equation (10.12), of $(1+n)/m^2$. These properties are useful at audio frequencies, particularly when coupled with

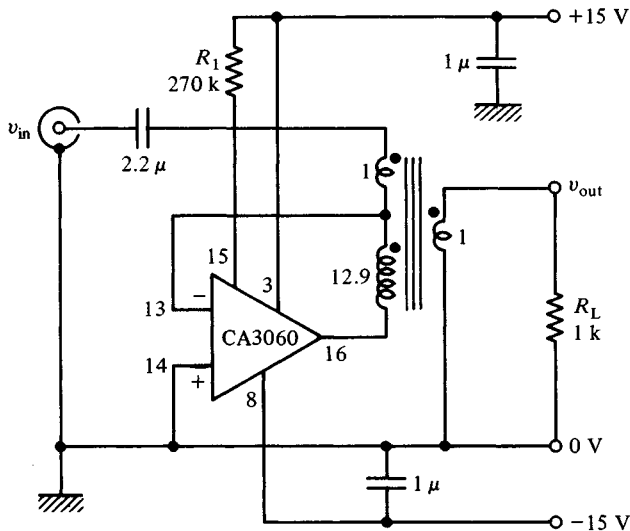


Fig. 10.7. An experimental circuit using the CA3060. The transformer is an STC-66121B or RS-217-804 and is used with the two primary windings quite separate, to give input and output windings, and the two secondary windings in series to give the winding shown across the OTA.

the low noise potential of the circuit that should follow from the lossless nature of its feedback elements. As in the previous circuit, Fig. 10.5, virtually all the input power to the circuit shown in Fig. 10.6 is passed on to the load, R_L , through the transformer.

10.9 An experimental circuit

A version of the circuit, first shown in Fig. 10.6, which can be built for experimental work is shown in Fig. 10.7. This uses the CA3060 OTA with its bias current, into pin 15, at just over $100\ \mu\text{A}$. This is set by R_1 . The g_m of the CA3060 is about 0.1S , at this bias current, and is 3 db down at 100 kHz. This is ideal for audio applications.

The circuit shown in Fig. 10.7 uses the same type of audio transformer which was used in the first experimental circuit of this chapter, Fig. 10.3. This is connected to make $m = 1$ and $n = 12.9$, so that the resulting amplifier has unity voltage gain and a high input impedance. Furthermore, the condition $m^2/g_m R_L(1+n) \ll 1$, suggested by equation (10.11), is now very easy to meet, even when R_L is taken below the value of $1\text{ k}\Omega$ shown in Fig. 10.7.

The voltage gain and input impedance of the experimental circuit should be measured and checked with the theory of the previous section.

The frequency response is determined by the transformer and should be flat across the 10 Hz–30 kHz band. It is interesting to reduce R_L and note how an optimum power output may be obtained when the OTA runs into both voltage and current limitations at the same peak output level.

The circuit shown in Fig. 10.7 has been put forward because it is unusual and is derived from a particularly interesting circuit shape, proposed by Norton [15] for high frequency and low noise work. The low frequency version given here has only limited practical application [17]. Norton's high frequency circuit, using a single transistor, gave a gain of 8 db over the band 5–200 MHz and had a noise figure of only 1.25 db at 100 MHz.

10.10 A repeater amplifier with resistive feedback

Looking back at Fig. 10.5, where feedback through the transformers, T_1 and T_2 , caused the input impedance, v_{in}/i_{in} , to equal the load impedance, R_L , it is necessary to ask some more general questions about why this really happens. It turns out that the essential feature of this kind of feedback is that it involves a transfer of information about v_{in} and i_{in} to the *output*, and, at the same time, information about v_{out} and i_{out} to the *input*. In this way, changes in R_s cause changes in Z_{out} , while changes in R_L cause changes in Z_{in} : the very property required for a repeater amplifier.

This kind of multiple feedback can be done in many ways, and a solution using resistive feedback networks was published in 1959 by Chaplin *et al.* [16]. As a circuit shape, the idea can be put forward by considering the circuit shown in Fig. 10.8, where d.c. levels should be ignored for the moment and only the small signal changes, v_{in} , i_{in} , v_{out} and i_{out} considered. The analysis which follows is adapted from the very clear treatment given by Kovács [18].

Fig. 10.8 shows an amplifier in which the output stage, Q_1 , is a series feedback circuit: the same circuit considered here in chapter 6, Fig. 6.3(a). This circuit, when working into a load, R_L , can provide a signal which is an accurate measure of the current in R_L because of the voltage which is developed across its feedback resistor, R_{E2} . As shown in Fig. 10.8, it is precisely this voltage across R_{E2} which is fed back to the input through resistor R_{F2} .

Similarly, feedback from output to input involving the value of v_{out} is fed back through R_{F1} into R_{E1} . This circuit consequently has the same feature as Fig. 10.5, discussed at the beginning of this section, as far as information about the output condition being fed back to the input. The

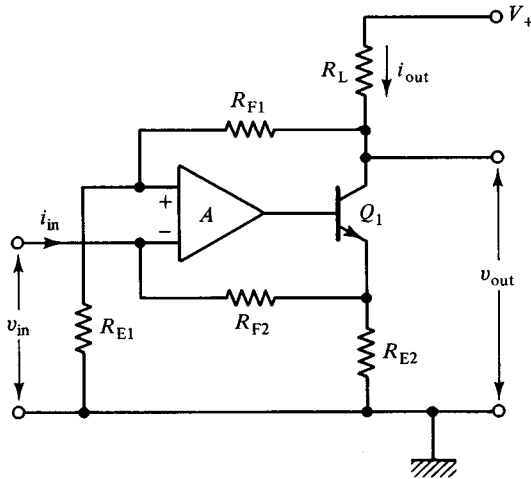


Fig. 10.8. A first step towards a repeater amplifier using resistive feedback networks.

following analysis will show that the same applies for input conditions influencing the output properties of the circuit.

If the gain block, A in Fig. 10.8, is ideal, having high input impedance and low output impedance, the open-loop voltage gain of the circuit shown in Fig. 10.8 will be simply AR_L/R_{E2} , all feedback being removed by open circuiting R_{F1} and R_{F2} . It may be assumed that AR_L/R_{E2} is very high. Obviously, restoration of feedback through R_{F2} will have no effect upon the gain v_{out}/v_{in} , because the components R_{F2} and R_{E2} can only change i_{in} , not v_{in} . Restoration of feedback through R_{F1} does fix the gain v_{out}/v_{in} , however, and this will be

$$v_{out}/v_{in} = (R_{F1} + R_{E1})/R_{E1}. \quad (10.14)$$

The current flowing into the input terminals of the high gain block, A , will be negligible. This means that i_{in} must flow on through R_{F2} , and must be given by

$$i_{in} = [v_{in} + v_{out}(R_{E2}/R_L)]/R_{F2} \quad (10.15)$$

so that, using equation (10.14) and writing $R_{in} = v_{in}/i_{in}$, it follows that

$$R_{in} = [R_{F2} R_{E1}/(R_L R_{E1} + R_{F1} R_{E2} + R_{E1} R_{E2})] R_L. \quad (10.16)$$

Equation (10.16) shows that the repeater amplifier property, $R_{in} = R_L$, will be obtained if the designer can make $R_{F1} R_{E2}$ the dominant term in the denominator, and then also make the ratios $R_{F1}:R_{E1}$ and $R_{F2}:R_{E2}$ equal. This is the exact analogy of the identity, $N_1 = N_2$, between the turns ratios

of the two transformers used for feedback components in the earlier circuit, Fig. 10.5.

To find the output impedance of the amplifier proposed by Fig. 10.8, a resistor, R_s , must be connected across the input terminals and R_L removed. A current, Δi_{out} , must then be injected into the output terminal and the resulting increase in output voltage, Δv_{out} , calculated.

The simplest way of looking at this calculation is to note that the change in the true input voltage to the high gain block, A , must be negligible. In the situation described above, the injection of Δi_{out} will cause the inverting input of A to rise by $\Delta i_{\text{out}} R_{E2} R_s / (R_{F2} + R_s)$, and the resulting increase in output voltage, Δv_{out} , will cause the non-inverting input of A to rise by $\Delta v_{\text{out}} R_{E1} / (R_{F1} + R_{E1})$. Equating these two increases then gives $R_{\text{out}} = \Delta v_{\text{out}} / \Delta i_{\text{out}}$ as

$$R_{\text{out}} = [R_{E2}(R_{F1} + R_{E1}) / R_{E1}(R_{F2} + R_s)] R_s. \quad (10.17)$$

Again, the repeater amplifier property, $R_{\text{out}} = R_s$, is obtained if the designer can make $R_{F1} R_{E2}$ and $R_{F2} R_{E1}$ the dominant terms, and also make $R_{F1} : R_{E1}$ equal $R_{F2} : R_{E2}$.

The above analysis confirms that the circuit shape shown in Fig. 10.8 is a possible basis for a good repeater amplifier design. Noise performance should be good because the terminating impedance for the input cable, R_{in} , and the driving impedance for the output cable, R_{out} , can be made to match the cable characteristic impedance, Z_o , but R_{in} and R_{out} are not found in the circuit as real resistors dissipating signal power. These essential terminating impedances have been determined by means of feedback resistors and, furthermore, the absolute value of these feedback resistors may now be chosen to optimise the active device source impedances from a noise point of view.

Clearly, the detailed design of high frequency amplifiers of this kind involves a great amount of calculation with accurate device modelling. The book by Maclean [19] deals with the design of this kind of amplifier and gives a complete treatment of the theory, with several examples of amplifiers which have been built and tested. The book includes photographs of the hardware and details of the computer aided design programs that were used. Another reference which gives hardware detail, along with the complete theoretical background needed, is the paper by Meyer, Eschenbach and Chin [20].

It is possible, however, to take quite a simple point of view in the design of an amplifier of this kind, provided it is to be used at fairly modest frequencies and intended to have quite high gain. High gain implies that only a small amount of feedback will be used, and this means that it will

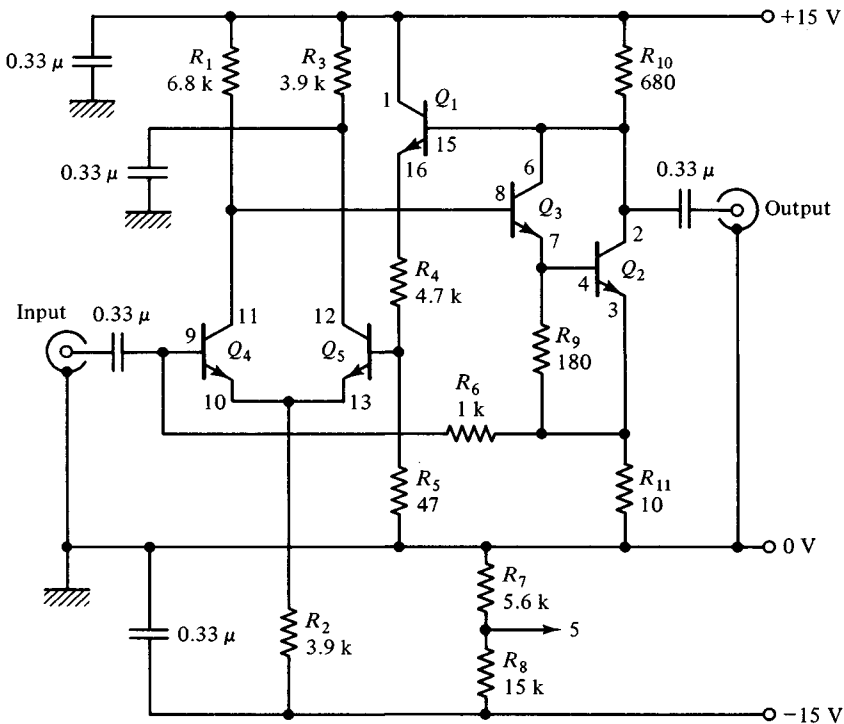


Fig. 10.9. An experimental repeater amplifier built around a CA3127E transistor array. Devices are numbered, and pin numbers given, according to the data sheet [21].

be easier to ensure amplifier stability. This is the approach adopted in the next section, where an experimental version of one of these interesting amplifiers is described.

10.11 An experimental repeater amplifier

The experimental wide-band repeater amplifier, shown in Fig. 10.9, is built around a CA3127E array of five high frequency transistors. These have a value of f_T in excess of 1 GHz, and are best operated with V_{CE} at 6 V. The collector currents should all lie between 1 mA and 10 mA to ensure high gain-bandwidth product.

Comparing the circuits shown in Figs. 10.8 and 10.9, it is clear that the high gain block, *A* in Fig. 10.8, is replaced by the long tailed pair, Q_4 and Q_5 in Fig. 10.9. This long tailed pair provides the inverting and non-inverting inputs that are needed for the two feedback paths shown in Fig. 10.8.

The series feedback output stage, shown as Q_1 in Fig. 10.8, is replaced

by the series feedback output stage of Fig. 10.9, which involves the two transistors, Q_3 and Q_2 . R_{E2} , in Fig. 10.8, is replaced by R_{11} , in Fig. 10.9, but R_L , in Fig. 10.8, is the true output load in Fig. 10.9, R_{10} being made high compared to R_L .

Feedback of the voltage across R_{11} to the inverting input is through R_6 , in Fig. 10.9, so that R_6 plays the same role as R_{F2} in Fig. 10.8. Feedback of the output voltage, which is done through R_{F1} into R_{E1} in the prototype circuit shown in Fig. 10.8, is made through the emitter follower, Q_1 in Fig. 10.9, and then through R_4 into R_5 . Introducing Q_1 avoids the problem of having to supply the d.c. called for by the feedback path, R_4 into R_5 , from the output point.

The pin-out of the CA3127E makes access to Q_3 and Q_2 on one side of the 16 pin package, and access to Q_4 and Q_5 on the other side. For this reason, a choice of Q_4 and Q_5 as the input transistors, and Q_3 and Q_2 as the output transistors, makes the separation of output and input, in the layout of the circuit shown in Fig. 10.9, somewhat easier. Q_1 and Q_2 are the best matched pair of transistors in the CA3127E, but the choice of these as input devices leads to layout problems, and, in any case, good matching of the input devices is not called for in an RC coupled amplifier.

The resistor values shown in Fig. 10.9 are chosen to give a high open-loop gain to the circuit, and set sensible collector current levels and d.c. levels at the same time. High gain in the output stage means getting R_{11} down as low as possible, while high gain in the input stage means as high a value of R_1 as possible, taking into account the d.c. level called for at the base of Q_3 , the collector current in Q_4 , and the V_{CE} of Q_4 . With the values chosen, I_{C4} is 2 mA and V_{CE4} is just under 2 V. For the other input transistor there is no problem about not getting a high value of V_{CE} : the problem is to keep V_{CE} down. The $V_{CE(max)}$ of the transistors in the CA3127E is 15 V, and this circuit is working from a ± 15 V supply.

Returning to equation (10.14), the gain of the experimental circuit, shown in Fig. 10.9, is set by the resistors R_4 and R_5 , these playing the role of R_{F1} and R_{E1} shown in Fig. 10.8. As equations (10.16) and (10.17) showed, the repeater amplifier property of providing an R_{out} equal to the source impedance, and an R_{in} equal to the load impedance, is arranged by making the ratios of the feedback resistors, $R_{F1}:R_{E1}$ and $R_{F2}:R_{E2}$, equal. This means that, in the experimental circuit, $R_4:R_5$ should be made equal to $R_6:R_{11}$.

A gain of 100 has been chosen for the experimental circuit, which is only about one tenth of the open-loop gain so that there should be no problem with stability. Testing the amplifier with a sinusoidal input from a 50 Ω source, and with a 50 Ω load on the output, should confirm this gain of

40 db and show a 3 db drop in gain around 30 MHz, above which the gain should fall rapidly. This will be true up to output levels as high as 50 mV peak, where the output stage is having to supply ± 1 mA peak to the 50 Ω load. At output levels above this, some non-linearity will begin to be observed.

There are, of course, many other possibilities for an experimental circuit of this kind. Different types of transistor array may be used [22], discrete transistors with the same high frequency performance would allow a better layout, and an emitter follower buffer between Q_4 and Q_3 , in Fig. 10.9, could be considered.

10.12 Measurement of input and output impedances

The most important feature of the experimental circuit shown in Fig. 10.9 is its ability to be inserted into a cable. The input impedance of the amplifier should equal its load impedance, and its output impedance should equal whatever source impedance is connected to it.

The input of the circuit shown in Fig. 10.9 works at very low level. The easiest way of checking that the input cable is correctly terminated by the amplifier input is to vary the length of cable in between the source, which should be a good quality 50 Ω or 75 Ω signal generator, and the amplifier itself. The output of the amplifier, measured across a good quality 50 Ω or 75 Ω termination, should be independent of the cable length. This test must, of course, be done at a high enough frequency. At 20 MHz, where the 30 MHz bandwidth amplifier might be expected to begin to show departure from the simple theory given in section 10.10, a $\lambda/4$ length of 50 Ω or 75 Ω cable is only a few metres long, and observing the apparent change in overall gain, as the length of the input cable is changed from $\lambda/4$ to $\lambda/2$, makes it possible to make a rough estimate of Z_{in} .

A more direct measurement of the output impedance is possible because the signal level at the output is high enough for direct observation. An interesting method is to use the directional bridge that was described in chapter 5, and built as the experimental circuit shown in Fig. 5.13. The output of the experimental amplifier, shown in Fig. 10.9, is connected to the load port, shown in Fig. 5.13, the source port is taken to a signal of 50 Ω source impedance, and the detector port is taken to a sensitive RF voltmeter which has a 50 Ω input impedance. This voltmeter will then show the level of mismatch at the amplifier output when the amplifier input has a good quality 50 Ω termination across it. This test is, of course, made with a sinusoidal source and the frequency is varied across the pass-band. The same information may be obtained if a pulse generator is used

as a signal source and the RF voltmeter is replaced with a very sensitive wide bandwidth oscilloscope. It is very important, however, to keep the signal level at the output of the amplifier under test very low, at least below the 50 mV level at which non-linearity was found to appear in the gain measurements discussed in the previous section.

10.13 Noise measurements

The experimental circuit shown in Fig. 10.9 lends itself to simple noise figure measurements because the noise developed at the output is high enough to be observed with simple equipment. This is to be expected in view of equation (10.1). The rms noise voltage across the 50 Ω input termination for a bandwidth of 30 MHz at 290 K is just below 5 μ V, so that a perfect amplifier with a gain of 100 would produce a noise output of 0.5 mV.

The experimental repeater amplifier will, of course, produce more output noise than 0.5 mV when its input is terminated with a good quality 50 Ω or 75 Ω termination, and its output is similarly terminated. Observing the output noise with a sensitive wide-band oscilloscope will show that accurate noise measurements may be far more difficult than the experimentalist expected. Unless the experimental circuit has been mounted inside a good quality metal box, the input and output connections are high quality co-axial connectors, and the power supply cable well decoupled at the point where it enters the circuit box, all kinds of unwanted noise may be observed. In urban environments one of the main problems will be the very high levels of television transmitter signals in the laboratory. While these are at frequencies more than ten times higher than the pass-band of the circuit under test, these television signals can be so large that they will still appear at the output. Such signals may be easily identified because their modulation is synchronised to the line frequency. Other radio signals which are a nuisance come from mobile communication equipment. These can be identified from the random nature of their coming and going.

Having overcome these environmental problems, the noise figure of the experimental circuit may be checked by using a standard noise generator test set. The usual technique is to increase the noise input until the noise power at the output is doubled [23]. Noise generators have improved greatly since the early instruments, which used thermionic diodes working under temperature limited emission. Modern noise generators use a high quality wide-band amplifier to amplify the thermal noise from a resistor at constant temperature, and this is followed by filters and attenuators to provide a test set noise output of variable bandwidth and level [24].

10.14 Noise reduction in special cases

The remaining sections of this chapter will consider some selected ideas that have been proposed to reduce noise in special cases. These are cases where either the thermal noise, the device noise, or some unwanted signal coming from the very nature of the signal processing system itself, can be reduced by using some special circuit. In the following three sections, one example of each of these three cases will be given.

10.15 Electronic cooling

Thermal noise is a problem in any very low level, wide-band, circuit that has to operate at room temperature. An obvious way of reducing thermal noise is to cool the circuit down well below room temperature, but this technique is only used in very special research environments. It is, for example, found in laboratories working in the field of experimental fundamental particle physics, where particle detectors are often used at very low temperatures to reduce the effects of thermal noise. This may be why the idea of ‘electronic cooling’ originated in the field of fundamental particle physics, although it has nothing to do with cryogenics, being a room temperature circuit technique, perhaps first described by Radeka [25].

The fundamental idea of electronic cooling is shown in Fig. 10.10. An amplifier is shown with negative feedback, provided by a single capacitor, C . At this initial stage, no questions are asked about the d.c. stability problems of this circuit idea.

Now suppose that the open-loop gain, $A(j\omega)$, of the amplifier shown in Fig. 10.10 has the form

$$A(j\omega) = A_o / (1 + j\omega/\omega_c) \quad (10.18)$$

which is, for example, typical of any operational amplifier having internal compensation. Well above ω_c the gain falls at a rate of 6 db/octave, and may be written

$$A(j\omega) \approx -jA_o(\omega_c/\omega). \quad (10.19)$$

The current fed back to the input terminal, i in Fig. 10.10, must be given by

$$i = -j\omega C v_{in} A(j\omega) \quad (10.20)$$

so that well above ω_c , where equation (10.19) applies

$$i = -\omega_c A_o C v_{in}. \quad (10.21)$$

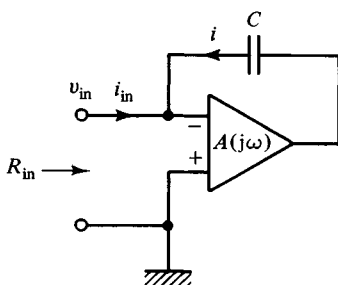


Fig. 10.10. An electronically cooled termination.

This current is independent of frequency because the increasing admittance of the capacitor, $j\omega C$, cancels out the falling gain of the amplifier, equation (10.19). It follows that, looking into the input terminals of the circuit shown in Fig. 10.10 a real positive resistance

$$R_{in} = 1/\omega_c A_o C \quad (10.22)$$

will be seen.

Now it can be argued that this resistor, R_{in} , is 'noiseless' if the amplifier, $A(j\omega)$ shown in Fig. 10.10, is also noiseless. The reasoning is that the feedback through C involves no dissipation. When the amplifier, $A(j\omega)$, does have some intrinsic noise, a return must be made to Fig. 10.1 where the noise of an amplifier is represented by two generators, e_n and i_n . For any practical value of R_{in} the contribution from i_n may be neglected. This means that the resistor, R_{in} , is left looking as though it is in series with a noise voltage generator e_n , the e_n belonging to the amplifier, $A(j\omega)$, in Fig. 10.10. This e_n may be much lower than the $(4kTR_{in})^{1/2}$ which would represent the noise of a real resistor, R_{in} , over unity bandwidth.

From a noise point of view, then, R_{in} looks like a resistor which is not at room temperature. Suppose a practical version of the circuit shown in Fig. 10.10 was made using the LT1028 operational amplifier, which, as Table 10.1 shows, has $e_n = 0.85 \text{ nV/Hz}^{1/2}$. At room temperature, 290 K, this is the noise voltage expected from a resistor with a value of 45Ω . It follows, by simple proportion, that R_{in} appears to be at a temperature of $290(45/R_{in})$. So, if a value of C is chosen to make R_{in} , given by equation (10.22), equal to 450Ω , then this 450Ω would appear to be 'electronically cooled' down to 29 K: well below liquid nitrogen.

What value of C would, in fact, be needed to make $R_{in} = 450 \Omega$? The LT1028 has a gain-bandwidth product of 50 MHz ($A_0 \approx 7 \times 10^6$ and $\omega_c/2\pi \approx 7 \text{ Hz}$). Equation (10.22) shows that $C = 7 \text{ pF}$ would give $R_{in} = 450 \Omega$. This is rather a small value of capacitance in practice, and

the problem of providing d.c. feedback across the LT1028, without spoiling the whole project, is going to be difficult. Nevertheless, this discussion shows that the circuit idea shown in Fig. 10.10 is a valuable one.

Electronic cooling has been applied by Gatti, Manfredi and Marioli [26] to provide a low noise termination for a transformer coupled radiation detector, and they give references to earlier work. The idea is interesting, very similar in its philosophy to the ideas of Norton [15], discussed in section 10.7, and subject to the same criticisms given by Nordholt [14] concerning dissipationless feedback in general.

10.16 Using devices in parallel

A circuit idea which attempts to reduce intrinsic device noise is worth considering briefly because it may be an example of a circuit designer doing the right thing for the wrong reasons. This is the idea that several transistors connected in parallel will give a better noise figure than a single device. In the author's experience, this idea is assumed to be correct by quite a few people, although it appears to have been explicitly published only once [27].

The reasoning behind the idea that several amplifiers in parallel are better than one, is that the noise from each amplifier is random, there is no correlation between the noise from one amplifier and its neighbour, but that the signal output from each amplifier is the same and these outputs may be summed. If there are n amplifiers, perhaps the signal to noise ratio at the output will be improved by $n^{\frac{1}{2}}$, because the signal will have been amplified by n while the noise will have been amplified by only $n^{\frac{1}{2}}$.

This will not happen because it is the noise *power* which matters, not its instantaneous value expressed as a voltage or a current. Looking back at equation (10.4) will confirm this: it is the product $e_n i_n$ which decides the noise figure. Nevertheless, better noise performance may be achieved by using several transistors in parallel for two reasons.

The first reason is that a parallel connection of bipolar transistors gives a composite device with a lower base spreading resistance, usually referred to as $r_{bb'}$ [28, 29]. This resistance contributes to \bar{e}_n^2 by an amount $(4kTr_{bb'}) V^2/\text{Hz}$. In fact, high frequency, low noise, bipolar transistors, of the highest quality, are actually made, internally, as a parallel connection of several individual devices. This has been the case for a very long time [30]. The composite transistor has interdigitated base and emitter contacts which reduce $r_{bb'}$ to a minimum.

The second reason why a parallel combination of bipolar transistors may lead to a lower noise figure is that the designer may then be able to

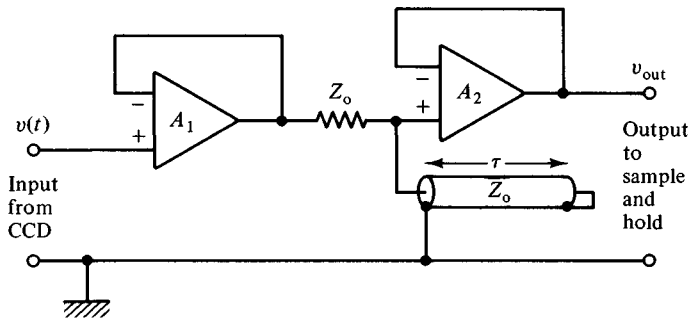


Fig. 10.11. The reflection delayed noise suppression technique due to Ohbo *et al.* [31].

get the input conditions such that the circuit is working closer to the source impedance that gives minimum noise figure: the $R_{s(\text{opt})}$ given above by equation (10.3). A better design would, of course, be made by choosing the correct single device.

10.17. An example from high definition television

Finally, it is interesting to consider an example of reducing an unwanted random signal which turns up in some signal processing system because of the very nature of that system. The example chosen concerns a high definition, solid state, colour television camera.

Completely solid state television cameras use an array of photodiodes which deliver their outputs to charge coupled devices (CCDs) arranged vertically and horizontally, these CCDs giving a serial data output.

Flicker noise and noise from surface states in a solid state device of this kind are considerable. The video output will have a large random low frequency component, and this problem is usually overcome by clamping the output just before the signal level from one pixel is about to arrive at the output, and then measuring only the change in level. This method is very difficult to apply at the very high data rates involved in high definition work. A camera described by Ohbo, Akiyama and Tanaka [31], having 2×10^6 pixels, produced its output on two interlaced channels working at over 37 MHz.

The solution described by Ohbo *et al.* [31] is a beautiful example of a circuit shape or circuit idea. It is shown in Fig. 10.11. The video signal from the CCD is buffered by amplifier A_1 , which must provide a very low output impedance. This low output impedance is then made up, by means of the resistor Z_o in Fig. 10.11, to match the characteristic impedance of a short circuited length of delay line.

The input of the delay line is connected to the input of the second amplifier, A_2 in Fig. 10.11. This amplifier is a high input impedance voltage follower. It follows that the input voltage to amplifier A_2 is the sum of two signals. The first is the video signal at time t , divided by two because it is arriving at the input of A_2 from the potential divider made up from resistor Z_0 loaded by the delay line. The second signal is the *inverted* video signal, delayed by 2τ and again divided by 2. This second signal is inverted because of its reflection at the short circuit, delayed by twice the length of the line, and halved because the reflected signal sees the correct termination, Z_0 , when it arrives back at the input of A_2 .

It follows that, if the video signal is $v(t)$, the output from the circuit shown in Fig. 10.11 is

$$v_{\text{out}} = [v(t) - v(t - 2\tau)]/2. \quad (10.23)$$

If this signal is now made the input to a sample and hold circuit, which samples only during the instant when the CCD is giving the pixel output, then all the low frequency flicker noise and surface state noise from the solid state camera output signal will be removed. Only the *change* in output signal will be sampled and held, this change being precisely the pixel intensity that is needed to make up the true video signal of the scene being televised.

10.18 Conclusions

In the field of electronic instrumentation the circuit designer might bear in mind a remark made by Faulkner, in a paper which was cited above [28], ‘we must avoid the assumption that noise considerations are a sort of “extra” which only needs to be taken into account under exceptional circumstances’. In other words, good circuits are low noise circuits. From this point of view, the EMC considerations, mentioned at the beginning of this chapter, may be the most important. Electronic instruments must work alongside other pieces of equipment, and this means that these instruments must be designed so that their circuits do not pick up unwanted signals. For ideas in this area, the books by Wilmshurst [1] and Ott [2] are invaluable.

Notes

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- 6 Morse, P. M., *Vibration and Sound*, McGraw-Hill, New York, 1948, p. 227. Fig. 52 on this page shows the threshold of hearing to be well below 10^{-16} W/cm² for frequencies between 2 kHz and 5 kHz. The human ear has an aperture of about 1 cm².
- 7 Note 3 above, pp. 695–6. A particularly clear treatment is also found in the book by M. H. Jones: *A Practical Introduction to Electronic Circuits*, Cambridge University Press, Cambridge, second edition, 1985, pp. 44–7.
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