Chapter 13

Follow the Leader Feedback Filters

13.1 Introduction

One of the important issues in filter design is that parameter sensitivity has to be taken into consideration. A doubly terminated ladder is extensively used mainly because of the same reason, that is, parameters of the filter realized through it have low sensitivities. At the same time, an alternate synthesis method using the cascade approach has found favour because of its ability to tune specific pole–zeros in higher-order filters through the utilization of non-interactive second-order sections. However, in the cascade process, sensitivities increase, especially for high-Q filters. Since we know that negative feedback improves the performance of electronic circuits in a number of ways, the same have been applied to obtain what is known as multiple feedback (MF) topologies. An MF topology consists of a network with a single feed-forward path comprising unilateral (active structures are mostly unilateral unlike passive structures which are bilateral) second-order sections, having different kind of feedbacks. The nature of the feedback decides its final topology. One of the topologies is called *leap frog* and was discussed in Chapter 9. While performing operational simulation, the circuit had a topology shown in Figures 9.22 and 9.24, where it is easy to recognize the structure as a leap frog structure. Another topology under the broad area of MF topologies known as *follow the leader feedback* (FLF) is the subject of this chapter.

In Section 13.2, the basic FLF structure and the kind of transfer functions obtained is included. Also included in this section is the derivation of the structure's transfer function when either lossless integrators or lossy integrators are used in the feedback paths. Use of only a feedback block could provide all pole functions. Hence, feed-forward is also included, as will be discussed in Section 13.3, to improve the versatility of the scheme. A slightly modified

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feed-forward scheme, called the *shifted companion structure* is discussed in Section 13.4. It was observed that all these schemes were special cases of a general FLF structure that is given in Section 13.5. Without sacrificing the generality, synthesis of the filter becomes a bit easier if the feedback blocks have BP structures with equal value of quality factor, instead of different quality factors. Such a scheme, known as the *primary resonator block technique* is also discussed in this chapter.

13.2 Structure of the Follow the Leader Feedback Filters

Though the basics remain the same, there are slightly different FLF structures depending on the kind of basic blocks employed and whether the filter is an all-pole type or has finite zeros.

The basic structure of an FLF filter is shown in Figure 13.1, wherein the transfer function $H_i (i = 1,2, \ldots, n)$ decides the nature of the final response. The transfer function of the blocks can be bilinear or biquadratic; it may be made of lossless or lossy integrators.

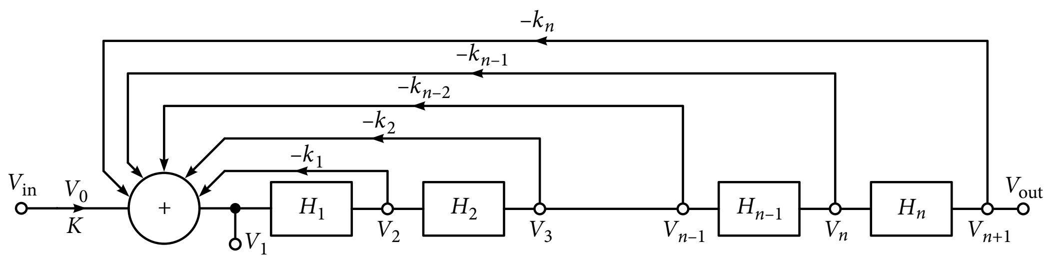

Figure 13.1 Basic structure of the follow the leader feedback filter scheme.

Without specifying the circuitry involved in providing the negative feedbacks $k_1, k_2, \ldots, k_n$ and the summation at the input, application of KVL (Kirchhoff's voltage law) at the input summing junction gives:

$$KV_0 - \{k_n V_{n+1} + k_{n-1} V_n + k_{n-2} V_{n-1} + \ldots \ldots + k_2 V_3 + k_1 V_2\} = V_1 \tag{13.1}$$

As transfer functions of the blocks can be written as:

$$H_n = \frac{V_{n+1}}{V_n} = \frac{V_{out}}{V_n}, \; H_{n-1} = \frac{V_n}{V_{n-1}}, \ldots\ldots, H_2 = \frac{V_3}{V_2}, \; H_1 = \frac{V_2}{V_1} \tag{13.2}$$

$$\begin{aligned} V_n &= \frac{V_{n+1}}{H_n}, \; V_{n-1} = \frac{V_n}{H_{n-1}} = \frac{V_{n+1}}{H_{n-1} \times H_n}, \ldots\ldots, \\ V_2 &= \frac{V_{n+1}}{H_2 \times \ldots \times H_n}, \; V_1 = \frac{V_{n+1}}{H_1 \times \ldots.. \times H_n} \end{aligned} \tag{13.3}$$

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Equation (13.1) can be modified with $V_0 = V_{\text{in}}$ and $V_{n+1} = V_{\text{out}}$ as:

$$KV_{\text{in}} - \left\{ \begin{aligned} & k_n + \frac{k_{n-1}}{H_n} + \frac{k_{n-2}}{H_{n-1} \times H_n} + \ldots\ldots + \frac{k_1}{H_2 \times \ldots \times H_n} \\ & + \frac{1}{H_1 \times \ldots.. \times H_n} \end{aligned} \right\} V_{\text{out}} = 0 \tag{13.4}$$

$$H(s) = \frac{V_{\text{out}}}{V_{\text{in}}} = \frac{K}{\left\{ k_n + \frac{k_{n-1}}{H_n} + \frac{k_{n-2}}{H_{n-1} \times H_n} + \ldots\ldots + \frac{k_1}{H_2 \times \ldots \times H_n} + \frac{1}{H_1 \times \ldots.. \times H_n} \right\}} \tag{13.5}$$

$$= \frac{K(H_1 \times H_2 \times \ldots\ldots. \times H_n)}{\{1 + k_1 H_1 + k_2 H_1 H_2 + \ldots\ldots + k_{n-1}\left(H_1 \times \ldots \times H_{n-1}\right) + k_n (H_1 \times \ldots \times H_n)\}} \tag{13.6}$$

Equation (13.6) can realize a high-order filter in which the poles and zeros will depend on the nature of the transfer function of the blocks H_i and the feedback factors k_i. Zeros of the transfer function $H(s)$ will depend on the zeros of H_i and its denominator will be a polynomial in s. If H_i is a lossless integrator, the realized filter shall be an all-pole filter. However, if equation (13.6) is to be a general function with arbitrary zeros, H_i needs to a general biquad; this gives the reason for discussing different cases.

13.2.1 Use of lossless integrator blocks

One of the simplest cases is the one in which the transfer functions H_i blocks are only lossless integrators ($H_i = 1/s$). With the use of such blocks, equation (13.6) will modify as:

$$\frac{V_{\text{out}}}{V_{\text{in}}} = \frac{K\left(1/s^n\right)}{1 + \frac{k_1}{s} + \frac{k_2}{s^2} + \ldots\ldots + \frac{k_{n-1}}{s^{n-1}} + \frac{k_n}{s^n}} \tag{13.7}$$

$$= \frac{K}{s^n + k_1 s^{n-1} + k_2 s^{n-2} + \ldots\ldots. + k_{n-1} s + k_n} \tag{13.8a}$$

It gives the following relation:

$$\frac{V_{\text{in}}}{1/K} - \left\{ s^n + \frac{s^{n-1}}{1/k_1} + \ldots + \frac{s}{1/k_{n-1}} + \frac{1}{1/k_n} \right\} V_{\text{out}} = 0 \tag{13.8b}$$

Equation (13.8) represents an LP filter section; though by taking the outputs at different points, HP and BP transfer functions can also be obtained.

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In practice, inverting integrators are easier to realize than non-inverting ones. Hence, if all integrators are inverting lossless types, the feedback coefficients are to be multiplied by +1 or −1 to get all the feedbacks as negative. The following example will illustrate the procedure.

Example 13.1: Realize a fourth-order Chebyshev filter with a corner frequency of 20 krad/s and a ripple width of 1 dB.

Solution: The normalized transfer function of a fourth-order LP Chebyshev filter with a corner frequency of 1 rad/s is as follows

$$H(s) = \frac{V_{\text{out}}}{V_{\text{in}}} = \frac{K}{s^4 + b_3 s^3 + b_2 s^2 + b_1 s + b_0} \tag{13.9}$$

For a ripple width of 1 dB, the values of the coefficients in equation (13.9) are as follows:

$$b_0 = 0.2756,\ b_1 = 0.7426,\ b_2 = 1.4538,\ b_3 = 0.9528 \tag{13.10}$$

In order to get a gain of −1 dB at dc for the even order Chebyshev LP filter, value of K will be:

$$b_0 \times (-1\ \text{dB}) = 0.2756 \times 0.8912 = 0.2456.$$

Equation (13.9) can be modified in more than one way so as to be compatible with a circuit realizable in the form of Figure 13.1. The possible forms in which equation (13.9) can be written are as follows:

$$KV_{\text{in}} - (s^4 + b_2 s^2 + b_0)V_{\text{out}} - (b_3 s^3 + b_1 s)V_{\text{out}} = 0 \tag{13.11}$$

$$\frac{V_{\text{in}}}{1/K} - \left(s^4 + \frac{s^2}{1/b_2} + \frac{1}{1/b_0}\right)V_{\text{out}} + \left(-\frac{s^3}{1/b_3} - \frac{s}{1/b_1}\right)V_{\text{out}} = 0 \tag{13.12}$$

Equation (13.12) can be realized using four lossless integrators and two inverting summers as shown in Figure 13.2(a).

Comparing the coefficients in equation (13.12) with that in equation (13.8), we can write $k_1 = 1/b_3 = 1.0495$, $k_2 = 1/b_2 = 0.6878$, $k_3 = 1/b_1 = 1.346$, and $k_4 = 1/b_0 = 3.628$. Coefficients k_4, k_2 and k_0 are to be multiplied by (−1) and coefficients k_3 and k_1 are to multiplied by +1. The resulting circuit is shown in Figure 13.2(a), where frequency de-normalization is done by a factor of 20 krad/s and an impedance scaling factor of 10^3 was used to bring component values within a suitable range. The final element values are also shown in the figure.

The simulated magnitude response of the filter is shown in Figure 13.2(b). Its corner frequency is 3.1464 kHz (19.777 krad/s), maximum gain at ripple peaks is 0.9976 and dc gain is 0.8921, equivalent to 0.973 dB; all simulated parameters are found to be very close to the design values.

As mentioned earlier, equation (13.11) can be modified in other ways as well. In an alternate form, it is realized using three inverting lossless integrators, one inverting integrating summer

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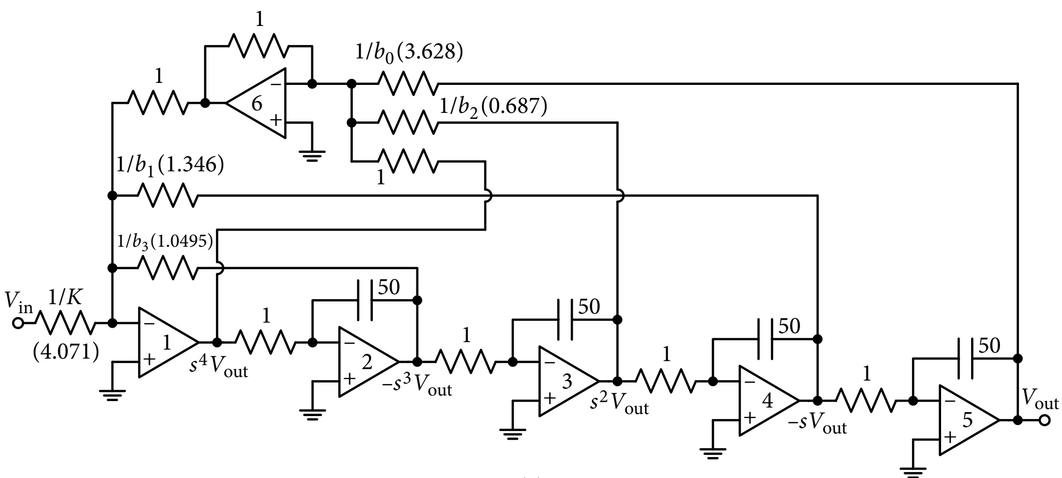
and one inverting summer as shown in Figure 13.3(a). Its simulated response is shown in Figure 13.3(b); the pass band corner frequency is 3.118 kHz (19.6 krad/s) and ripple width is 1.008 dB.



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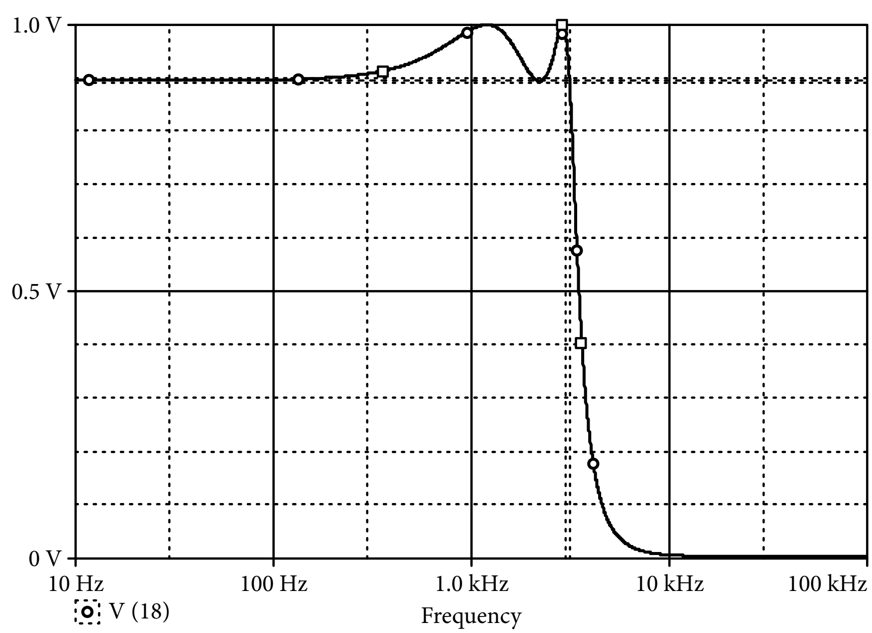

(b)

Figure 13.2 (a) Fourth-order normalized low pass filter realization for Example 13.1. All resistances are in kΩ and capacitors are in nF. (b) Magnitude response of the fourth-order FLF low pass Chebyshev filter using lossless integrator blocks for Example 13.1.

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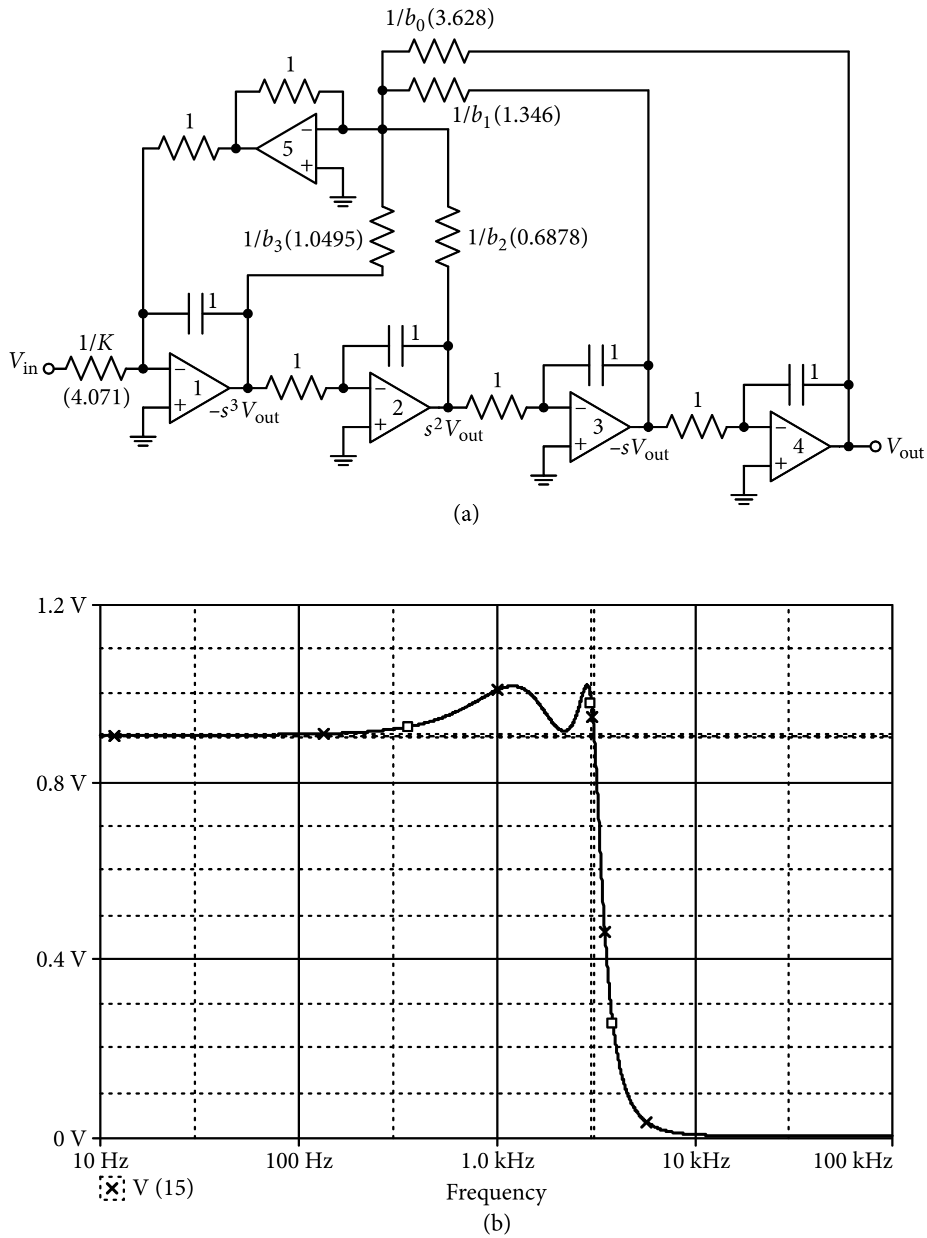

Figure 13.3 (a) Alternate FLF structure for the fourth-order low pass filter shown in Figure 13.2(a). All resistances are in kΩ and capacitors are in nF. (b) Magnitude response of a fourth-order FLF low pass Chebyshev filter using lossless integrator blocks for Example 13.1: an alternate structure.

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13.2.2 Use of lossy integrator blocks

A lossy inverting integrator shown in Figure 13.4(a) has the following form of transfer function:

$$\frac{V_2}{V_1} = -\frac{1}{s+a} \tag{13.13}$$

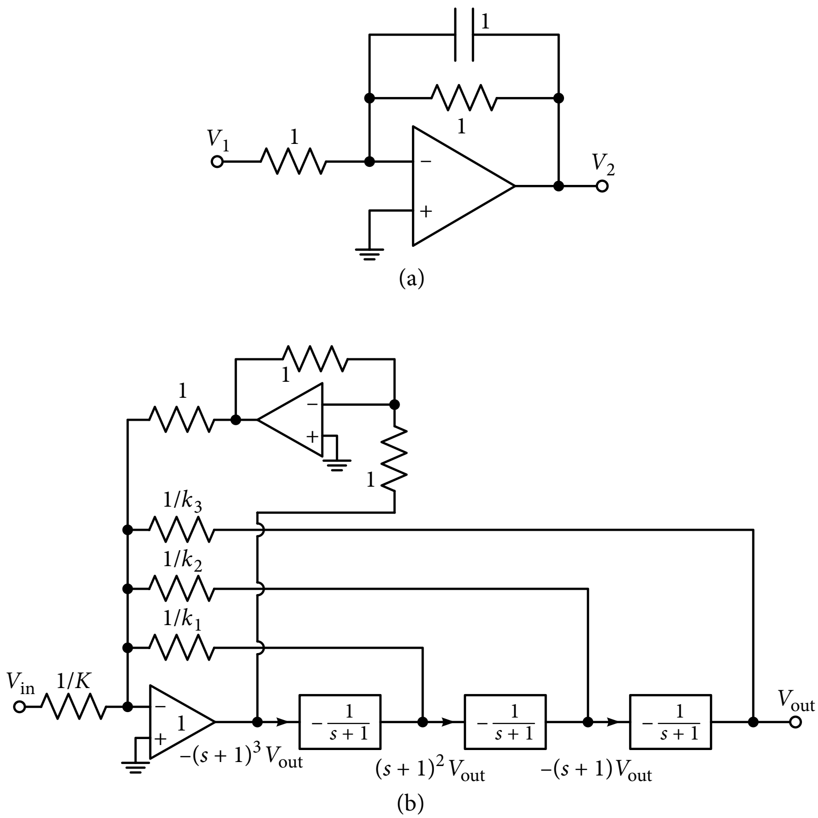

Figure 13.4 (a) Normalized lossy inverting integrator to be used in an FLF structure. (b) FLF structure for a third-order filter using the inverter shown in Figure 13.4(a) for realizing transfer function of equation (13.15).

If such lossy integrators are used to realize FLF structured filters, equation (13.6) will be written in the following form:

$$\frac{V_{out}}{V_{in}} = \frac{K}{(s+a)^n + k_1(s+a)^{n-1} + k_2(s+a)^{n-2} + \dots\dots\dots + k_{n-1}(s+a) + k_n} \tag{13.14}$$

Equation (13.14) represents an all-pole LP filter of order n. If it is compared with the general all-pole LP response of equation (13.15) given in the following equation, coefficient matching can be done.

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$$\frac{V_{\text{out}}}{V_{\text{in}}}=\frac{K}{s^n+b_{n-1}s^{n-1}+b_{n-2}s^{n-2}+\ldots\ldots..+b_2s^2+b_1s+b_0} \tag{13.15}$$

With coefficient matching between equations (13.14) and (13.15), the general relations between b_i and k_i can be written as:

$$b_{n-1} = na + k_1$$

$$b_{n-2} = \{n(n-1)/2\}a^2 + (n-1)ak_1 + k_2$$

$$b_0 = a^n + a^{n-1}k_1 + \ldots\ldots + a^2k_{n-2} + ak_{n-1} + k_n \tag{13.16}$$

However, for comparatively lower-order filters, the relations will be simpler and can be used directly. For filter realization, a suitable value of a is selected and coefficients k_i are found in terms of b_i in equation (13.16). The transfer function of equation (13.15) is then realized through the basic structure shown in Figure 13.1 employing the lossy integrator shown in Figure 13.4(a) as the basic block. Care is taken to assign positive or negative sign for the coefficients k_i in the feedback in order to have all feedbacks negative. The process is illustrated here with an example.

Example 13.2: Design a third-order Chebyshev filter having a ripple width of 2 dBs and a pass band edge frequency of 10 krad/s using the lossy inverting integrator shown in Figure 13.4(a).

Solution: From Table 3.4, location of the poles for the desired filter is as follows:

$$s_{1-2} = -0.1845 \pm j0.9231,\ s_3 = -0.3689$$

Hence, the normalized transfer function of the all-pole LP filter having unity dc gain will be:

$$H(s)=\frac{V_{\text{out}}}{V_{\text{in}}}=\frac{0.3269}{s^3+0.7379s^2+1.02218s+0.3269} \tag{13.17}$$

Putting $n = 3$ in equation (13.16), we get expressions for b_i and k_i as:

$$b_2 = 3a + k_1 \rightarrow k_1 = b_2 - 3a$$

$$b_1 = 3a^2 + 2ak_1 + k_2 \rightarrow k_2 = b_1 - 3a^2 - 2ak_1$$

$$b_0 = a^3 + a^2k_1 + ak_2 + k_3 \rightarrow k_3 = b_0 - a^3 - a^2k_1 - ak_2 \tag{13.18a}$$

For a selected value of the parameter, $a = 1$, and using values of the coefficients b_i from equation (13.17), equation (13.18a) gives the values of coefficients k_i as:

$$k_1 = -2.2621,\ k_2 = 2.5464,\ k_3 = -0.9574 \tag{13.18b}$$

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With these values of coefficients k_i, for $n = 3$, equation (13.17) can be realized using the structure shown in Figure 13.4(b). For the design value of the pass band edge frequency, de-normalization is done by a factor of 10 krad/s and impedance scaling is done by a factor of 10^4, which makes the capacitance value in each integrator as 10 nF and the feedback resistor of 10 kΩ. The transfer function of equation (13.17) will become as follows.

$$\frac{V_{\text{out}}}{V_{\text{in}}} = -\frac{0.34727}{\{-(s+1)\}^3 + 2.2621\{-(s+1)\}^2 + (2.5464)\{-(s+1)\} + 0.9574} \tag{13.19}$$

$$\frac{V_{\text{in}}}{1/0.34727} + \left\{-(s+1)^3 + \frac{(s+1)^2}{1/2.2621} - \frac{(s+1)}{1/2.5464} + \frac{1}{1/0.9574}\right\} V_{\text{out}} = 0 \tag{13.20}$$

Figure 13.5 shows the circuit diagram of the filter with de-normalized element values. Figure 13.6(a) shows the phase response at the output of each OA, showing whether the outputs are in phase or out of phase with the input signal. Due to the negative sign with coefficients k_1 and k_3, output from integrators 1 and 3 are also applied directly to the summer OA0, with respective feedback resistance values of (10^4/2.2621) = 4.4206 kΩ and (10^4/0.9574) = 10.444 kΩ. The feedback resistance value from the output at integrator 2 is (10^4/2.5464) = 3.927 kΩ and input resistance is (10^4/0.3269) = 30.59 kΩ. Figure 13.6(b) shows the simulated magnitude response of the filter. Initially, dc gain is 0.945, with a ripple width of 1.96 dB. The ripple near the pass band does not reach its dc level, and the corner frequency is 1.532 kHz (962.9 rad/s). However, with small adjustments in the coefficient of $(s+1)^3$ from 1.0 to 0.98 by making the feedback resistor in the summing inverter-0 as 10.2 kΩ, the filter shows improvement in the shape of the response with ripple at the pass band reaching the dc level; corner frequency is now 1.55 kHz (974.3 rad/s), dc gain is unity and the ripple width is 1.85dBs.

13.3 Feed-forward Path Based FLF Structure

So far, the techniques used for the realization of FLF structure could give only all-pole filters. Researchers have shown that the required building blocks needs to be biquadratic sections for the realization of finite transmission zeros. However, some alternate methods which do not need biquadratic building blocks are also available. One such method which employs feed-forward paths will be shown here.

The structure shown in Figure 13.1 and implemented in the form of Figure 13.2(a), which employs lossless integrators as basic building blocks, realizes all-pole filters that have the transfer function as given in equation (13.5). In this basic structure, outputs are available in terms of $s^n V_{\text{out}}$, $s^{n-1} V_{\text{out}}$, ……, $s\,V_{\text{out}}$ and V_{out}. These outputs can be used further in a feed-forward form to get a general nth order filter with arbitrary transmission zeros as shown in Figure 13.7. In

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the scheme, the original outputs from the lossless integrators are multiplied by coefficients a_i and then added together to obtain the final output.

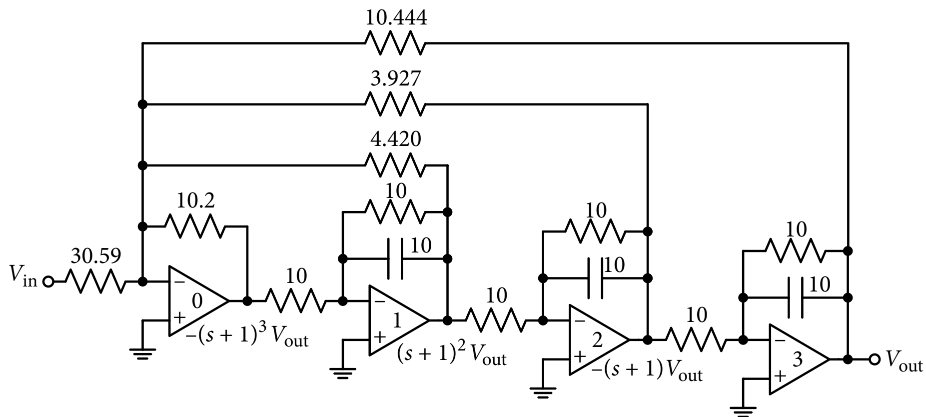
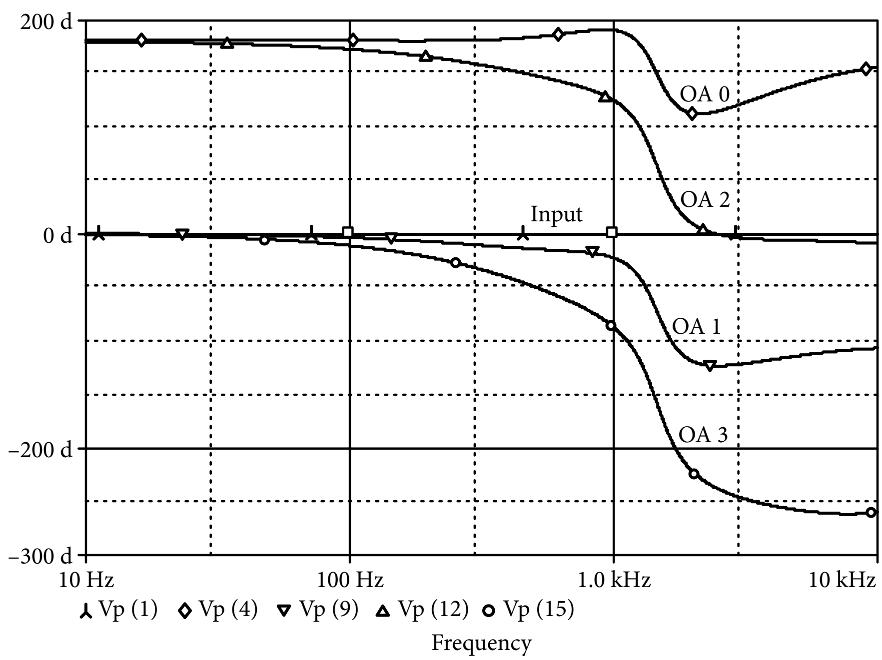


Figure 13.5 Circuit structure of the third-order filter for Example 13.2. All resistances are in kΩ and capacitors are in nF.



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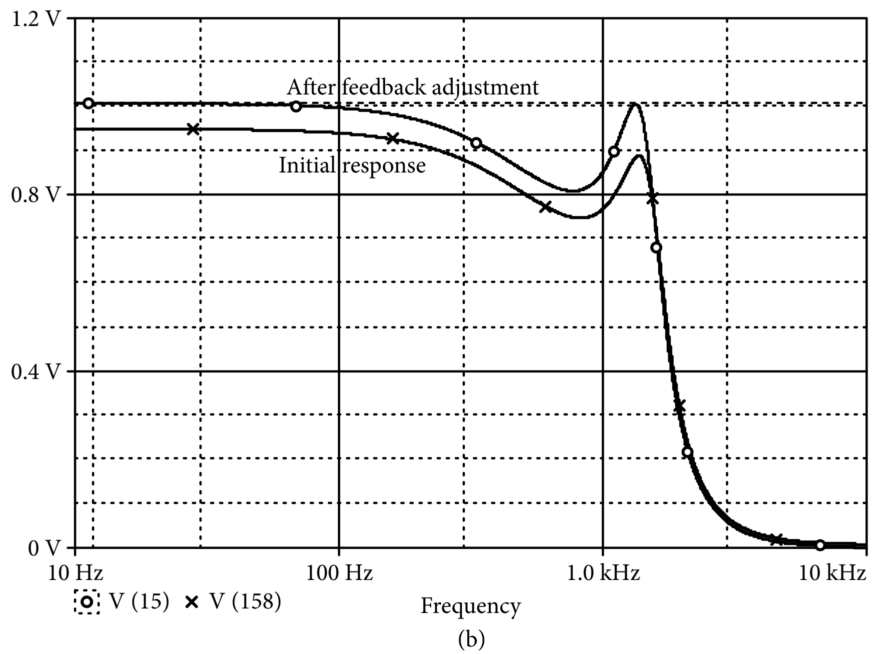

Figure 13.6 (a) Phase responses at the input, and outputs of the OAs in the third-order filter for Example 13.2. (b) Magnitude response of the third-order filter for Example 13.2.

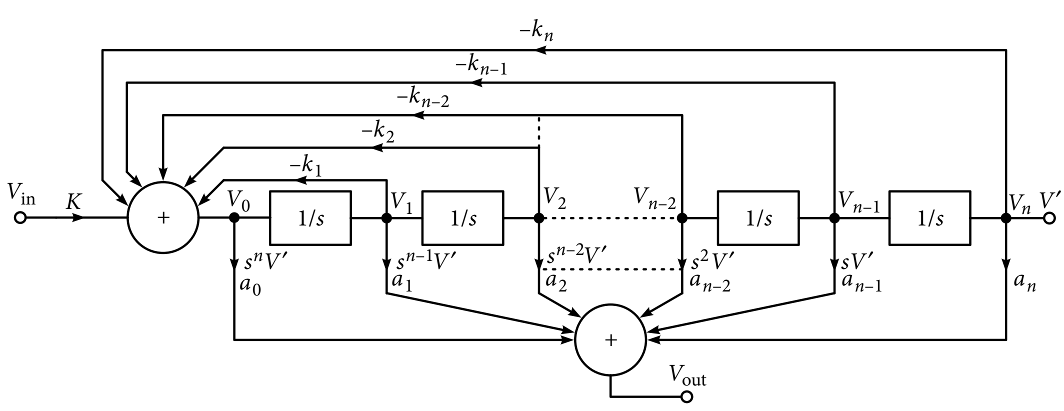

Figure 13.7 Realization of an n^{th} order filter with finite zeros in the FLF structure employing lossless integrators and feed-forward paths.

As before, the ratio of the intermediate output V' to V_{in} in Figure 13.7 will be given as:

$$\frac{V'}{V_{in}} = H_1(s) = \frac{1}{s^n + k_1 s^{n-1} + k_2 s^{n-2} + \ldots\ldots + k_{n-1}s + k_n} = \frac{1}{D(s)} \tag{13.21}$$

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The ratio of the final output (V_{out}) with intermediate voltages V' will be

$$(V_{out}/V') = H_2(s) = a_0 s^n + a_1 s^{n-1} + \ldots + a_{n-2}\, s^2 + a_{n-1}\, s + a_n = \mathrm{N}(s) \tag{13.22}$$

Combining equations (13.21) and (13.22)

$$\frac{V'}{V_{in}} \times \frac{V_{out}}{V'} = H_1(s) H_2(s) \tag{13.23}$$

$$\frac{V_{out}}{V_{in}} = H(s) = \frac{N(s)}{D(s)} = \frac{a_0 s^n + a_1 s^{n-1} + \ldots + a_{n-2} s^2 + a_{n-1} s + a_n}{s^n + k_1 s^{n-1} + k_2 s^{n-2} + \ldots\ldots + k_{n-1} s + k_n} \tag{13.24}$$

Hence, a general transfer function of equation (13.24) can be realized using only lossless integrators. Once again, as inverting integrators are easy to use, appropriate positive or negative multipliers will be assigned to the coefficients a_j and k_i.

While doing final summation using the inverting summer, some of the received signals may have negative sign and some may have positive sign. Some coefficients may be zero as well; in these cases, feed-forward path will be simply open. The following example will illustrate the process.

Example 13.3: Realize an active LP filter in FLF mode with the feed-forward technique. Its pass band extends up to 20 krad/s with a maximum ripple width of 1.0 dB; the stop band is beyond 40 krad/s with a minimum attenuation of 34 dBs. The filter has unity gain in the pass band.

Solution: The design obtains the following normalized transfer function for the given specifications.

$$H(s) = \frac{V_{out}}{V_{in}} = K \frac{\left(s^2 + 5.1532\right)}{(s + 0.54)\left(s^2 + 0.434s + 1.01\right)} \tag{13.25}$$

For the unity dc gain, $K = 0.10589$. Transfer function of equation (13.25) is expanded, and functions $H_1(s)$ and $H_2(s)$ corresponding to equations (13.21) and (13.22) are obtained from it as:

$$H_1(s) = \frac{V'}{V_{in}} = \frac{0.10589}{s^3 + 0.974s^2 + 1.2443s + 0.5454} \tag{13.26}$$

$$H_2(s) = (V_{out}/V') = s^2 + 5.1532 \tag{13.27}$$

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Equation (13.26) can be written as:

$$\frac{V_{\text{in}}}{1/0.10589} - \left(\frac{s^2}{1/0.974} + \frac{1}{1/0.5454}\right)V' + \left(-\frac{s}{1/1.2443} - s^3\right)V' = 0 \tag{13.28}$$

Equation (13.28) is realized as the top portion of Figure 13.8(a) using inverting lossless integrators with the value of coefficient multipliers $k_1 = (1/0.974) = 1.0267$, $k_2 = (1/1.2443) = 0.8038$, and $k_3 = (1/0.5454) = 1.8335$. As $K = 0.10589$, V_{in} is to be divided by 9.4437.

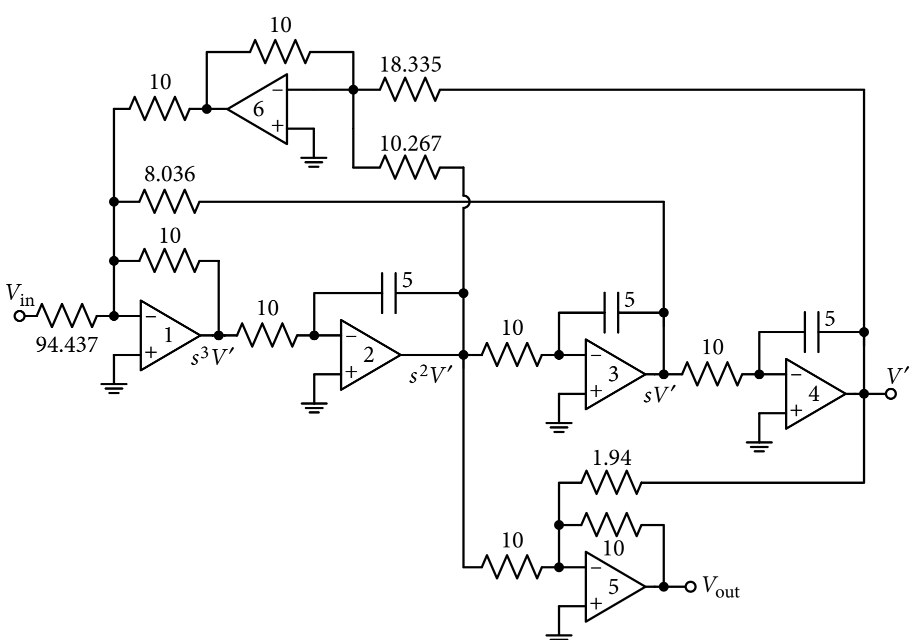
While realizing $H_2(s)$ of equation (13.27), $a_0 = a_2 = 0$. Hence, it is written as:

$$V_{\text{out}} + \left(-s^2 - \frac{1}{1/5.1532}\right)V' = 0 \tag{13.29}$$

For the realization of the part played by equation (13.29), the lower half part of Figure 13.8(a) needs only one summing inverter with multipliers from V' being (1/5.1532) = 0.194 and unity from s^2V'.

Frequency de-normalization by a factor of 20 krad/s is done, and an impedance scaling factor of 10^4 is used to find suitable element values, which are shown in Figure 13.8(a).

Figure 13.8(b) shows the PSpice simulated magnitude response of the filter. Peak gain is 1.005 but the dc gain is 0.9862 and ripple width is 10.49 mV for an input of 100 mV, which corresponds to 0.962 dB. The corner frequency is 3.2168 kHz (20.219 krad/s) and gain at 40 krad/s (6.38 kHz) is 0.017427 (–35.17 dBs). A zero occurs at 7.24 kHz (from the numerator of equation (13.25)). All the simulated parameters are very close to the design parameters.



(a)

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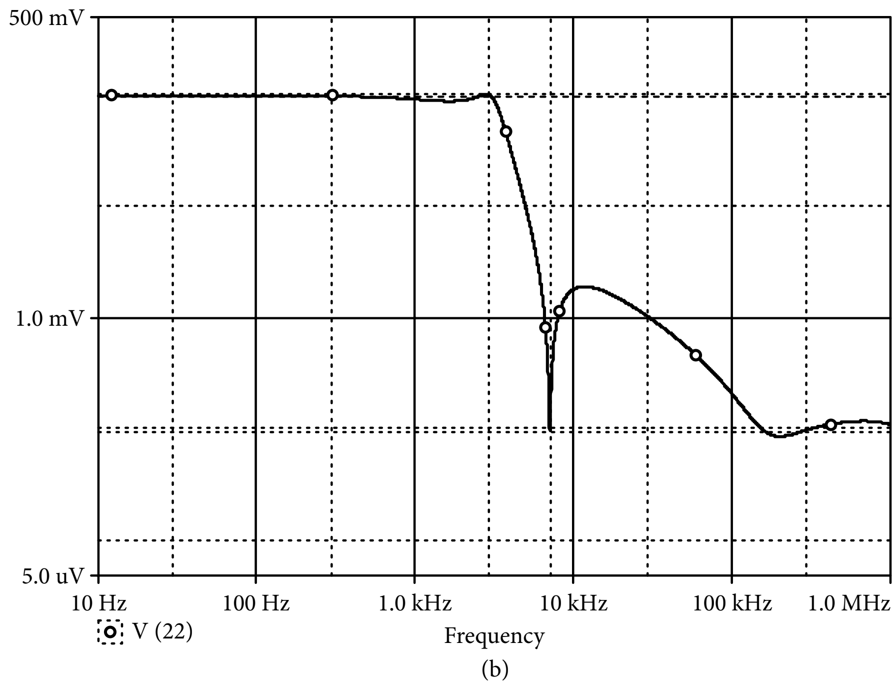

Figure 13.8 (a) Third-order filter section for Example 13.3, using feed-forward methodology. All resistances are in kΩ and capacitors are in nF. (b) Magnitude response of the third-order elliptic filter for Example 13.3.

13.4 Shifted Companion FLF Structure

In Section 13.2.2, lossy inverting integrators are used to realize all-pole filters where the first feedback coefficient, k_1 was derivable as $(b_{n-1} - na)$. The common practice is to select $a = 1$ for the sake of simplicity in calculations. However, if a is selected as $\left(\frac{b_{n-1}}{n}\right)$, k_1 will reduce to zero, which means that there will be no feedback after the first integrator (or any other building block used in place of the integrator). Such an arrangement is known as the *shifted companion feedback* (SCF) structure as shown in Figure 13.9(a)

Figure 13.9(b) shows the normalized inverting integrator when $a \neq 1$, which is to be used as the basic building block in the SCF scheme.

The SCF structure can also use feed-forward paths like those in Section 13.3 in order to realize the general transfer function having arbitrary zeros. The suggested process is also shown in Figure 13.9(a) with dotted lines joining a summer. An example will illustrate the procedure.

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Example 13.4: Realize the third-order Chebyshev filter of Example 13.2 to have a pass band edge frequency of 5 krad/s using the SCF structure.

Solution: The normalized transfer function from Example 13.2 is repeated here for unity dc gain from equation (13.17):

$$H(s)=\frac{V_{\text{out}}}{V_{\text{in}}}=\frac{0.3269}{s^3+0.7379s^2+1.02218s+0.3269} \tag{13.30}$$

Equation (13.30) can be written as:

$$\frac{V_{\text{in}}}{1/0.3269}-\left(\frac{1}{1/0.7379}s^2+\frac{1}{1/0.3269}\right)V_{\text{out}}+\left(-\frac{s}{1/1.02218}-s^3\right)V_{\text{out}}=0 \tag{13.31}$$

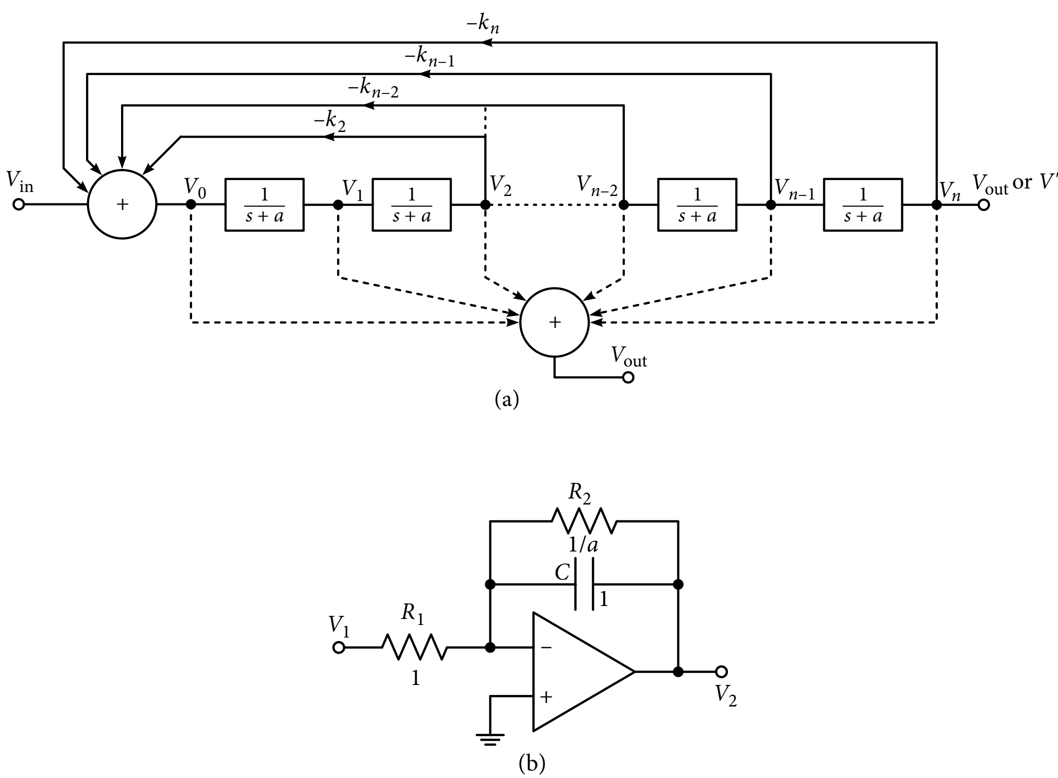

Figure 13.9 (a) Shifted companion feedback FLF structure (and feed-forward as well) (b) normalized inverting integrator to be used in part (a).

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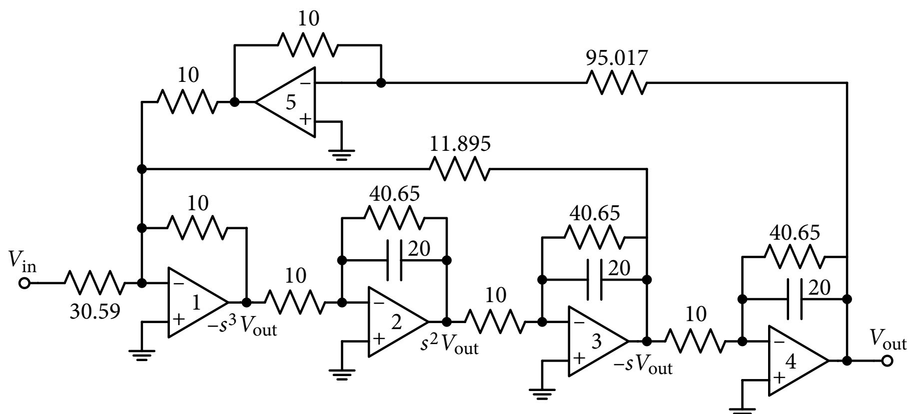
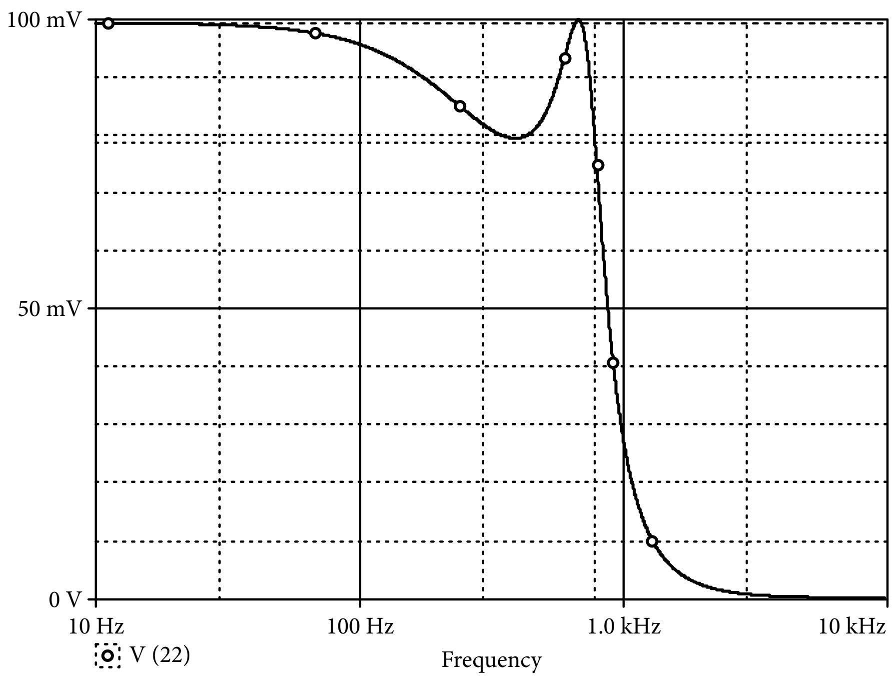
Equation (13.31) may be realized using the circuit of Figure 13.10(a). Calculation of the element values of the basic block, as well as values of the resistances providing feedback and feed-forward, are given here.



(a)



(b)

Figure 13.10 (a) Circuit realization of third-order Chebyshev filter for Example 13.4 using shifted companion feedback process. All resistors are in kΩ and capacitors in nF. (b) Response of the third-order filter.

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Order of the filter n being three, the coefficient $k_1 = b_2 - 3a$. While making $k_1 = 0$:

$$a = (b_2/3) = 0.7379/3 = 0.24596$$

$$k_2 = b_1 - n(n-1) \times a^2/2 = 0.84069$$

$$k_3 = b_0 - ak_2 - a^3 = 0.10524 \tag{13.32}$$

A frequency de-normalizing factor of 5 krad/s and impedance scaling factor of 10 kΩ is used. For $a = 0.24596$, the selected capacitor in each inverting integrator shown in Figure 13.9(b) will be 20 nF; R_1 becomes 10 kΩ and R_2 = 40.65 kΩ. The rest of the resistors realizing the coefficients k_2 and k_3 are obtained after impedance scaling as ($1/0.84069 = 1.1895 \times 10^4\ \Omega$) and ($1/0.10524 = 9.5017 \times 10^4\ \Omega$), which are shown in Figure 13.10(a). To get the dc gain as unity, the input resistance after impedance scaling will be ($1/0.3269 = 3.059 \times 10^4\ \Omega$).

The simulated magnitude response of the circuit is shown in Figure 13.10(b). DC gain is found to be 0.995 and a ripple width of 19.96 mV for an input voltage of 100 mV is equivalent to 1.96 dB. Corner frequency is 791.5 Hz (4975 rad/s).

13.5 General FLF Structure

In Section 13.2, a basic structure of the FLF feedback filter was discussed. The structure was shown in Figure 13.1. Building blocks were initially lossless, but later on lossy integrators were used to realize all-pole filters. The idea was extended to include feed-forward paths to realize arbitrary transmission zeros while using lossless or lossy integrators. The shifted companion approach was introduced in Section 13.4.

All the aforementioned schemes can be shown to be special cases of a generalized FLF process. In this section, the FLF structure will be studied from a generalized view, which will not only help in revising the simpler versions studied so far in the chapter, but also leads to another useful special class of FLF filters, known as the *primary resonator block* (PRB) based filters.

For Figure 13.7, if the transfer functions of each block is taken as $H_i(s)$, we can write:

$$-V_0 = KV_{\text{in}} + \sum_{i=1}^{n} k_i V_i \tag{13.33a}$$

And the final output is written as:

$$V_{\text{out}} = a_0 V_0 + \sum_{i=1}^{n} a_i V_i \tag{13.33b}$$

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Previous examples have shown that other than the basic blocks, the filter realization requires two summers using OAs; the realization of multipliers k_i and a_i is through resistors. A generalized form of FLF filters is easily derivable from Figure 13.7, which is shown in Figure 13.11. Here as:

$$k_i = \frac{R_{f0}}{R_{fi}}\Big|_{i=1\,to\,n} \text{ and } a_i = \frac{R_0'}{R_i}\Big|_{i=0\,to\,n} \tag{13.34}$$

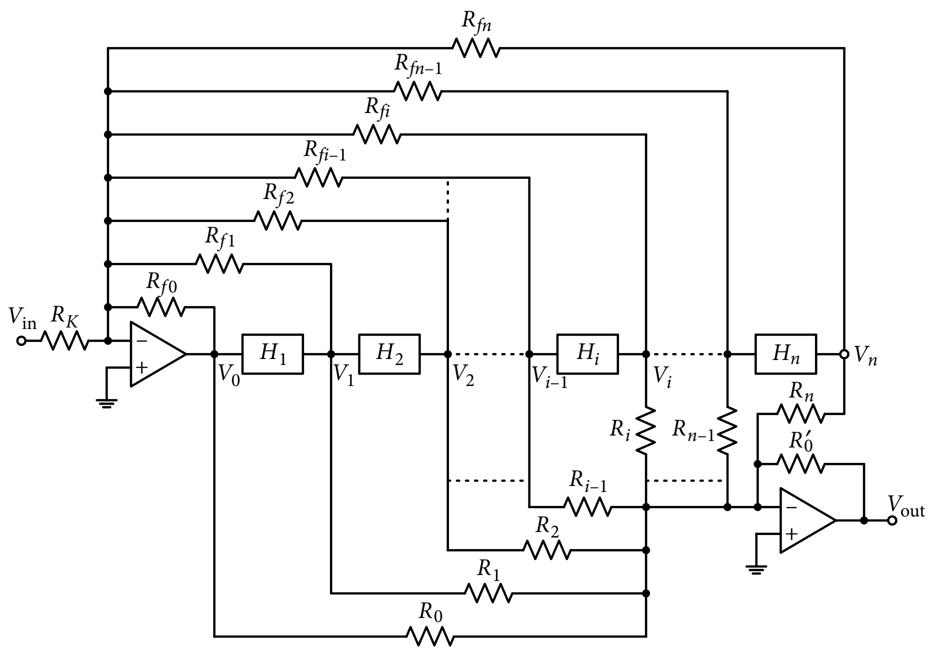

Figure 13.11 A general FLF structure where H_i could be first- or second-order sections, and summing amplifiers are assumed to be ideal.

Equations (13.33a) and (13.33b) can be written respectively as:

$$-V_0 = \frac{R_{f0}}{R_K} V_{in} + \sum_{i=1}^{n} \frac{R_{f0}}{R_{fi}} V_i \tag{13.35}$$

$$V_{out} = -\sum_{i=0}^{n} \frac{R_0'}{R_i} V_i \tag{13.36}$$

Internal voltages V_i are easily calculated from the following expression:

$$V_i = V_0 \prod_{j=1}^{i} H_j(s) \quad i = 1, 2, \ldots\ldots, n \tag{13.37}$$

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If V_i from equation (13.37) is substituted in equation (13.33a), then the expression of output voltage will be:

$$-V_0 = KV_{\text{in}} + \sum_{i=1}^{n} k_i \left\{ V_0 \prod_{j=1}^{i} H_j(s) \right\} \tag{13.38a}$$

$$= KV_{\text{in}} + k_1 V_0 H_1(s) + k_2 V_0 H_1(s) H_2(s) + \ldots + k_n V_0 H_1(s) H_2(s) \ldots H_n(s) \tag{13.38b}$$

Hence, $$\frac{V_0}{V_{\text{in}}} = -\frac{K}{1 + \sum_{m=1}^{n} \left\{ k_m \prod_{j=1}^{m} H_j(s) \right\}} \tag{13.39}$$

In the expanded form, it can be modified as:

$$\frac{V_0}{V_{\text{in}}} = -\frac{K}{1 + k_1 H_1(s) + k_2 H_1(s) H_2(s) + \ldots + k_n H_1(s) H_2(s) \ldots H_n(s)} \tag{13.40}$$

Substituting V_0 from equation (13.39) in equation (13.37), transfer function at the ith stage is obtained as:

From $$h_i = \left(\frac{V_i}{V_0}\right)\left(\frac{V_0}{V_{\text{in}}}\right)$$

$$h_i(s) = \frac{V_i}{V_{\text{in}}} = -\frac{K \prod_{j=1}^{i} H_j(s)}{1 + \sum_{m=1}^{n} \left\{ k_m \prod_{j=1}^{m} H_j(s) \right\}} \tag{13.41}$$

Since $$h_i(s) = h_n(s) \prod_{j=i+1}^{n} 1/H_j(s) \tag{13.42}$$

Here, $h_n(s)$ is the transfer function for feedback network only and its expression is obtained from equations (13.42) and (13.41) as:

$$h_n(s) = \frac{h_i(s)}{\prod_{j=i+1}^{n} 1/H_j(s)} = \frac{K \prod_{j=1}^{n} H_j(s)}{1 + \sum_{m=1}^{n} \left\{ k_m \prod_{j=1}^{m} H_j(s) \right\}} \tag{13.43}$$

As discussed before, it is clear from equation (13.43) that for this transfer function, the zeros are decided by the zeros of $H_j(s)$. For realizing $h_n(s)$ with arbitrary transmission zeros, $H_j(s)$ should be a second-order function with finite transmission zeros, treatment of which is quite involved. Instead, if the basic blocks are second-order band pass (BP) functions, calculations become a little easier. Let the normalized second-order BP function for the building block in standard format be:

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$$H_i(s) = h_{0i}\frac{(1/Q_i)s}{s^2 + (1/Q_i)s + 1} = h_{0i}h_i(s) \tag{13.44}$$

In equation (13.44), h_{0i} is the mid-band gain and Q_i is the quality factor of the ith stage.

Finally substituting V_i from equation (13.41) and V_0 from equation (13.39) in equation (13.33):

$$H(s) = \frac{V_{\text{out}}}{V_{\text{in}}} = -K\frac{a_0 + \sum_{m=1}^{n}\{a_m \prod_{j=1}^{m} H_j(s)\}}{1 + \sum_{m=1}^{n}\left\{k_m \prod_{j=1}^{m} H_j(s)\right\}} \tag{13.45}$$

Implementation of using a second-order BP section as $H_i(s)$ can be explained better with the help of a transfer function having a smaller value of n. Hence, for $n = 4$, $H(s)$ will be expanded as shown in equation (13.46) and will be studied further.

$$H(s) = \frac{V_{\text{out}}}{V_{\text{in}}} = K\frac{a_0 + a_1H_1 + a_2H_1H_2 + a_3H_1H_2H_3 + a_4H_1H_2H_3H_4}{1 + k_1H_1 + k_2H_1H_2 + k_3H_1H_2H_3 + k_4H_1H_2H_3H_4} \tag{13.46}$$

Calculations involving equation (13.46) become easier if the $H_i(s)$ are arithmetically asymmetric BPFs obtained from the low pass (LP) section using the transformation already discussed in Chapter 5, and repeated here.

$$S = \frac{\omega_o}{B}\frac{s^2 + \omega_o^2}{s} \rightarrow S = Q\frac{s^2 + 1}{s} \text{ for normalized } \omega_o \tag{13.47}$$

In equation (13.47), S is the normalized complex frequency variable of the LP prototype and $Q = (\omega_o/B)$ is the quality factor of the BP filter. The numerator and denominator of the BP transfer function of equation (13.44) is multiplied by (Q/s) to get the transfer function of the prototype LP.

$$H_i(s) = h_{0i}\frac{Q/Q_i}{Q\frac{s^2+1}{s} + \frac{Q}{Q_i}} \rightarrow h_{0i}\frac{Q/Q_i}{S + \frac{Q}{Q_i}} \tag{13.48}$$

Equation (13.48) is the first-order prototype LP transfer function, where h_{0i} becomes the dc gain of the LP filter $-H_{LP}(S)$ and if we write $(Q/Q_i) = q_i$, then

$$H_{LP}(S) = h_{LPi}\frac{q_i}{S + q_i} \tag{13.49}$$

To find the poles of the general FLF transfer function, if the BP to LP transformation is used, its order becomes $(n/2)$ and the poles are decided by the values of q_i and k_i. For the nth order

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FLF, the transfer function expression of the (n/2)th order LP has been derived for further processing. However, as suggested earlier, we will consider the case of the eighth-order FLF filter for the sake of easier understanding.

Substituting the relation in equation (13.49) in the denominator of equation (13.46), we get:

$$\begin{aligned}&=(S+q_1)(S+q_2)(S+q_3)(S+q_4)+\\&k_1\{h_{LP1}q_1(S+q_2)(S+q_3)(S+q_4)\}+\\&k_2\{h_{LP1}q_1h_{LP2}q_2(S+q_3)(S+q_4)\}+\\&k_3\{h_{LP1}q_1h_{LP2}q_2h_{LP3}q_3(S+q_4)\}+\\&k_4h_{LP1}h_{LP2}h_{LP3}h_{LP4}q_1q_2q_3q_4\end{aligned} \tag{13.50}$$

When the relation in equation (13.49) is substituted in the numerator of equation (13.46), we get the following relation for finding the zeros of the LP filter.

$$\begin{aligned}&=a_0(S+q_1)(S+q_2)(S+q_3)(S+q_4)+\\&a_1\{h_{LP1}q_1(S+q_2)(S+q_3)(S+q_4)\}+\\&a_2\{h_{LP1}q_1h_{LP2}q_2(S+q_3)(S+q_4)\}+\\&a_3\{h_{LP1}q_1h_{LP2}q_2h_{LP3}q_3(S+q_4)\}+\\&a_4h_{LP1}h_{LP2}h_{LP3}h_{LP4}q_1q_2q_3q_4\end{aligned} \tag{13.51}$$

Coefficients of the denominator in equation (13.46) are found by comparing its expanded form in equation (13.50). However, we get four equations with $b_4 = 1$ (as in equation (13.9)), in terms of eight unknowns, k_1, k_2, k_3, k_4, q_1, q_2, q_3, q_4. One of the solutions which also simplifies the procedure is the one which assumes equal quality factors for all the second-order BP sections, that is, $q_1 = q_2 = q_3 = q_4 = q$. This means we need to use identical BP sections. Such a scheme is known as the *primary resonator blocks* (PRB) technique. Hence, we will continue the study of FLF filters in PRB form, which also includes the shifted companion approach.

13.5.1 Primary resonator block technique

For identical BP sections with the same value of quality factor q, if we compare equation (13.50) with the denominator of equation (13.46) and simplify, we get the following relations.

$$k_1 = b_3 - 4qb_4 \tag{13.52a}$$

$$k_2 = b_2 - 6q^2b_4 - 3qk_1 \tag{13.52b}$$

$$k_3 = b_1 - 4q^3b_4 - 3q^2k_1 - 2qk_2 \tag{13.52c}$$

$$k_4 = b_0 - q^4b_4 - q^3k_1 - q^2k_2 - qk_3 \tag{13.52d}$$

Here, there are four equations with five unknowns. It is used to advantage by making $k_1 = 0$, through selecting q given by the following relation from equation (13.52a):

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$$k_1 = 0 = b_3 - 4qb_4 \rightarrow q = (b_3/4b_4) \tag{13.53}$$

It needs to be mentioned that for the nth order filter, equation (13.53) will become:

$$q = (b_{n-1}/nb_n) \tag{13.54}$$

The remaining feedback coefficients in equations (13.52) can be found recursively as has been shown in previous examples.

The process of finding coefficients while realizing transmission zeros is exactly the same as that for the case of poles; this has been explained in Section 13.3.

So far, the process of finding the feedback and feed-forward coefficients has been discussed when PRBs are used in FLF design. Obviously, finding the actual resistors realizing these coefficients should not be difficult. However, gain constants have not yet been specifically decided. In fact, these are free parameters and in a good design, they are selected to get maximum dynamic range without over-shooting of signals at any intermediate terminal, which would have distorted signals and thus might have given erroneous outputs. A process similar to the one discussed in the cascade process is to be followed for optimum design.

In case, one is interested in simulating a function without finite zeroes, no feed forward circuitry is required. In such case, numerator of the prototype LPF can be found out by substituting the relation of equation (13.49) in equation (13.43). Evaluation would then be replaced as shown below for a fourth order LP prototype:

$$N(S) = Ka_4\, h_{\mathrm{LP}1}\, h_{\mathrm{LP}2}\, h_{\mathrm{LP}3}\, h_{\mathrm{LP}4}\, q_1\, q_2\, q_3\, q_4 \tag{13.55}$$

Obviously, for identical prototyped expression will simplify as:

$$N(S) = Ka_4\, (h_{\mathrm{LP}}\, q)^4 \tag{13.56}$$

$K = 1$ in equation (13.56) for the dc gain of unity for the LP prototype.

Example 13.5: Design an eighth-order Butterworth BPF using the PRB structure having a center frequency of 2.5 kHz and quality factor of 20. Mid-band gain needs to be unity.

Solution: To realize the eighth-order Butterworth filter, we need to start with a fourth-order LP Butterworth filter, for which the normalized transfer function is given as:

$$H_{LP}(s) = \frac{V_{\mathrm{out}}}{V_{in}} = \frac{1}{s^4 + 2.6131s^3 + 3.414s^2 + 2.6131s + 1} \tag{13.57}$$

To employ the shifted companion scheme and simplicity in FLF structure, equation (13.54) gives:

$$q = \frac{2.6131}{4} = 0.65328 \tag{13.58}$$

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Applying equation (13.52), we get the coefficient values of equation (13.34) as:

$$k_2 = 0.8536,\ k_3 = 0.3838 \text{ and } k_4 = 0.2036 \tag{13.59}$$

As $q = Q_{BP}/Q_{LP} = 0.6532$ (from equation 13.48), for the given value of Q_{BP} as 20, $Q_{\mathrm{LP}} = 30.616$. Therefore, expression of the normalized transfer function of the PRB is written as:

$$h_{\mathrm{PRB}}(s) = \frac{h_0\left(1/Q_{LP}\right)s}{s^2 + \left(1/Q_{LP}\right)s + 1} \tag{13.60}$$

In equation (13.60), $h_0 = (1/q) = 1.5308$ in order to get unity mid-band gain for the PRB.

Since this example is for an all-pole filter, no feed-forward path is required.

The de-normalized transfer function of the PRB will be

$$\frac{V_2}{V_1} = \frac{1.5308\left(5000\pi/30.616\right)s}{s^2 + \left(5000\pi/30.616\right)s + \left(5000\pi\right)^2} \tag{13.61}$$

It is realized by a GIC based circuit shown in Figure 13.12(a), for which the expression of the transfer function, with $R_1 = R_3 = R$ and $C_2 = C$ is:

$$\frac{V_2}{V_1} = \frac{\left(1/CR_Q\right)\left\{1 + \left(R_4/R_5\right)\right\}s}{s^2 + \left(1/CR_Q\right)s + \left(R_4/R_5\right)\left\{1/(RC)^2\right\}} \tag{13.62}$$

Comparison between equations (13.61) and (13.62) give the following component values.

With $R_4 = R_5 = R$, and $\omega_o = 2500(2\pi) = 1/RC$, for the selected value of $C = 20$ nF, we get $R = 3.1818$ kΩ and $(1/CR_Q) = 5000\pi/30.616 \rightarrow R_Q = 97.412$ kΩ. With $R_4 = R_5$, the realized mid-band gain becomes 2. Hence, to bring it down to 1.5308, a potential divider at the input is to be used. For the potential divider, the resistor ratio will be (R_Q/α) and $(R_Q/1 - \alpha)$, where $\alpha = 1.5308/2 = 0.7654$; it results in the input resistors of 127.27 kΩ and 415.22 kΩ.

The calculated values of the components are shown in Figure 13.12(a). PSpice simulated response of the PRB section is shown in Figure 13.12(b). Its mid-band gain is 1.5296, center frequency is 2.488 kHz and with a bandwidth of 80.98 Hz, realized $Q_{LP} = 30.73$.

PRB of Figure 13.12(a) is used in the realization of the eighth-order Butterworth BPF shown in Figure 13.13(a). Feedback resistances are calculated to realize the coefficients k_3, k_2 and k_4 given in equation (13.59).

$$\text{Selecting } R_{f0} = 10 \text{ k}\Omega, \text{ we get } R_{f2} = 11.715 \text{ k}\Omega,\ R_{f3} = 26.155 \text{ k}\Omega,\ R_{f4} = 49.115 \text{ k}\Omega, \text{ and for } K=1,\ R_{\mathrm{in}} = 10 \text{ k}\Omega. \tag{13.63}$$

The simulated response of the Butterworth BPF is shown in Figure 13.13(b). Cumulative effect of the gain (1.529) of the PRB results in the mid-band gain of unity. Center frequency

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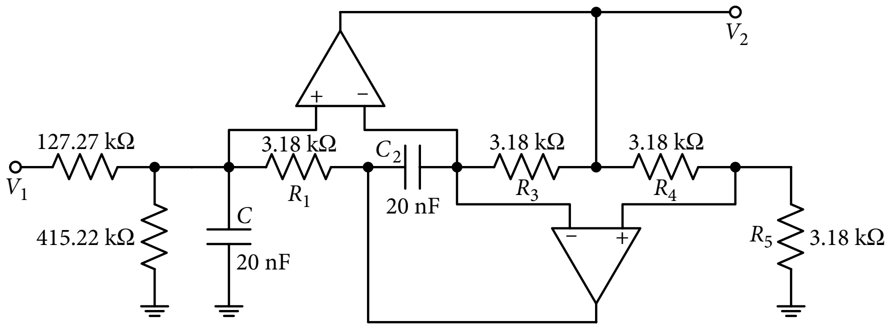
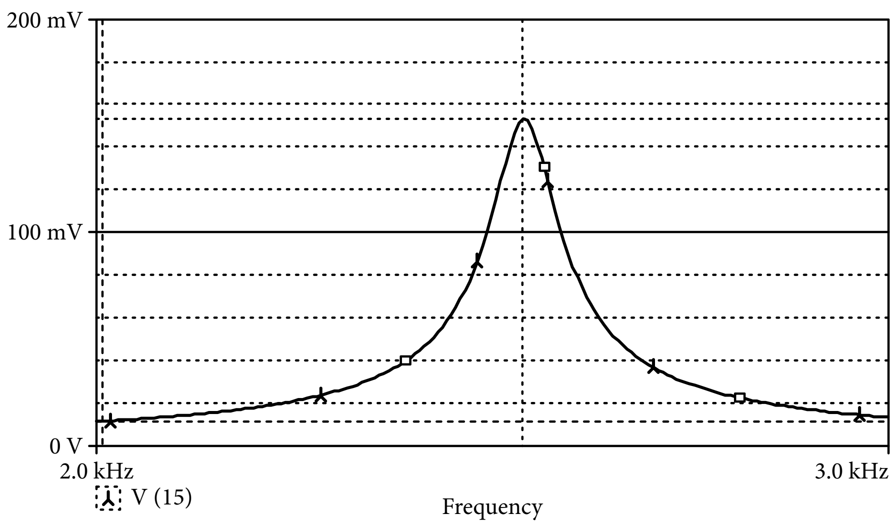
is 2.492 kHz and obtained with a bandwidth of 123.44 kHz, Q_{BP} = 20.19 against the design value of 20. Rate of fall of the output corresponds to an eighth-order filter. Outputs at the other PRBs are also shown in Figure 13.13(b).

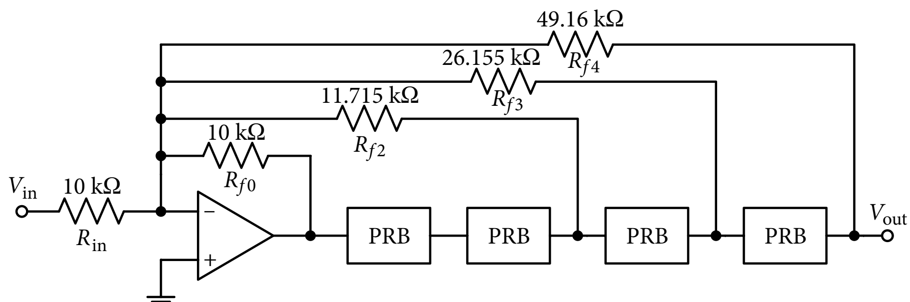


(a)



(b)

Figure 13.12 (a) Generalized impedance convertor based second-order PRB section for Example 13.5. (b) Its simulated magnitude response.



(a)

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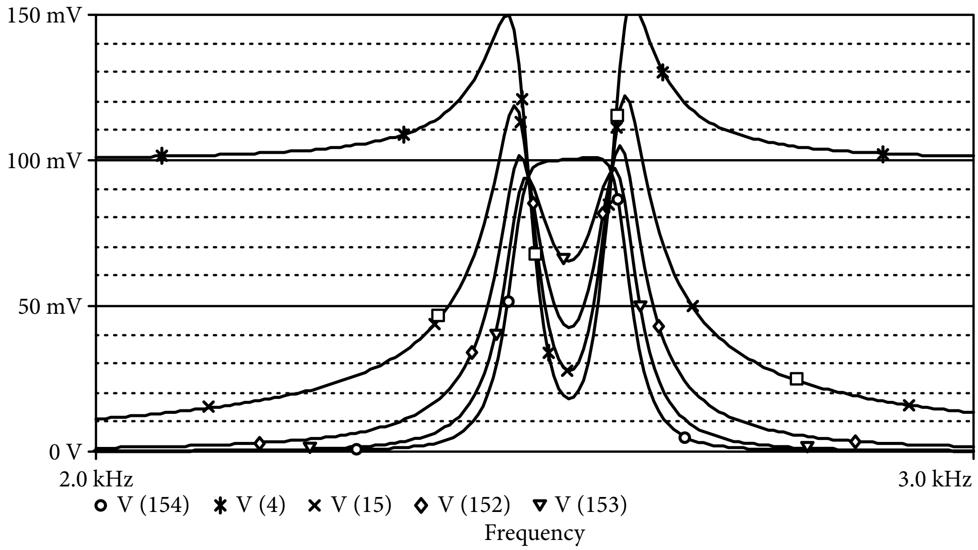

(b)

Figure 13.13 (a) Eighth-order band pass maximally flat filter using PRB of Figure 13.12(a) as the basic block for Example 13.5. (b) Simulated magnitude response.

Practice Problems

13-1 Design and test a fourth-order low pass filter with maximally flat characteristics having 1 dB attenuation at 1 kHz. Use the follow the leader feedback approach employing lossless integrators.

13-2 Repeat Problem 13-1 using integrators with loss. All capacitors should be below 10 nF.

13-3 Realize a fifth-order LP filter having Butterworth characteristics using the follow the leader feedback technique using lossless integrators. The filter is to have a cut-off frequency of 5 kHz.

13-4 Design the filter in Problem 13-3 using lossy integrators.

13-5 Realize a fifth-order LP filter having Chebyshev characteristics with ripple width of 2 dBs using the follow the leader feedback technique while employing lossless integrators. The filter is to have a cut-off frequency of 5 kHz.

13-6 Use shifted companion FLF structure to design a fourth order LP filter with maximally flat characteristic having 1 dB attenuation at 1 kHz.

13-7 Realize a notch filter in FLF mode with feed-forward technique for the transfer function:

$$H(s) = \frac{(s^2 + 0.25)}{(s^2 + 0.09s + 0.083)}$$

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It is preferred that notch appears at 5 kHz. What is the peak value for the function and at what frequency it occurs?

13-8 Design a fourth-order Butterworth BPF using PRB technique. Its center frequency is 5 kHz and quality factor is 10. Use single amplifier biquad as the basic building block,

13-9 Repeat problem 13-8 using multi amplifier biquad as building block.

13-10 Apply a frequency de-normalization factor of 3.4 kHz for the following transfer function:

$$H(s)=\frac{0.274(s^2+2.41)}{(s+0.635)(s^2+0.361s+1.042)}$$

Design and test the realization using lossless or lossy integrators and feed-forward FLF structure approach.

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