

Multi Amplifier Second-order Filter Sections

8.1 Introduction

In Chapter 7, the basics of realizing second- (or third-) order filter sections using only one active device (OA) were explained. Such circuits are capable of providing any arbitrary second-order function; they are also economical from the point of view of the use of active devices. However, depending on the filter specifications and the configuration chosen, the resulting circuit may not fulfil all the requirements like small number of passive components used and specific spread, sensitivity and variability. It is for this reason that many second-order filter sections use two, three or more OAs: multi amplifier biquads (MABs). Obviously, the intention is to overcome the mentioned limitations of the single amplifier biquad (SAB). In addition, a significant feature of multi amplifier biquadratic sections is their versatility in terms of providing more than one kind of response (like LP and BP) at the output terminals leading to general biquadratic structures.

Almost all MABs use two integrators in a loop, a technique known as the state-variable approach. Based on this technique, an important practical circuit known as the KHN (Kerwin-Huelsman-Newcomb) biquad can be assembled. The scheme explained in detail in Section 8.2 realizes three types of output responses. A direct modification of the scheme, known as Tow–Thomas biquad is studied in Section 8.3. The schemes, being interesting and useful, are further studied while employing active compensation to inverting or non-inverting integrators used in the loop. Active compensation leads to another well-known biquad, the Ackerberg–Mossberg filter, which is studied in Section 8.5. Many schemes have been implemented to utilize these structures and obtain other types of responses as explained in Section 8.6. Another

scheme for obtaining a multi-response configuration using a generalized impedance convertor (GIC) is explained at the end of the chapter.

While designing a SAB, it was observed that a frequency-dependent finite gain of the OA results in a deviation in the performance parameters ω_o and pole-Q. To compensate for these deviations, biquads using *composite amplifiers* are also used, in which instead of using only a passive negative feedback, an active feedback network is used. These amplifiers increase the number of OAs used, making it a MAB, though the design itself remains a SAB type.

8.2 State Variable Multi Amplifier Biquad

There are a number of two, three or more amplifier biquad circuits. Almost all of these circuits are based on the *state variable* form of realization technique, first introduced by Kerwin, Huelsman and Newcomb (popularly known as the KHN biquad) [8.1]. The scheme, in its generality uses n integrators for an n th order transfer function, which are then appropriately connected the way integrators are connected in the analog computation method. To realize a second-order section, only two integrators are required along with a summer. Hence, in its basic form, a state variable biquad uses three amplifiers, with three outputs as shown in Figure 8.1. The configuration includes an integrator $(-a_1/s)$ with feedback k_1 making it a lossy integrator, a lossless integrator $(-a_2/s)$ with feed back factor $-k_2$ and two summers S_1 and S_2 . Here, S_2 is used to convert a lossless integrator into a lossy one by combining the lossless integrator with feedback; the summer S_1 is used to complete the feedback loop for the integrators. As there is no element in between the two summers, the summers are generally combined. It is to be noted that both integrators are in inverting mode and use negative loop feedback to ensure stability. With the transfer function of the integrators as $(-a_1/s)$ and $(-a_2/s)$, the three available transfer functions of the section in Figure 8.1 are obtained from the following equations:

$$V_{o1} = kV_{in} + (-k_2)V_{o3} + k_1V_{o2} \quad (8.1a)$$

$$V_{o2} = -\frac{a_1}{s}V_{o1} \text{ and } V_{o3} = -\frac{a_2}{s}V_{o2} \quad (8.1b)$$

The obtained transfer functions are:

$$\left(\frac{V_{o1}}{V_{in}}\right) = \frac{ks^2}{D(s)}, \left(\frac{V_{o2}}{V_{in}}\right) = -\frac{ka_1s}{D(s)}, \left(\frac{V_{o3}}{V_{in}}\right) = \frac{ka_1a_2}{D(s)} \quad (8.1c)$$

$$\text{where } D(s) = s^2 + a_1k_1s + a_1a_2k_2 \quad (8.1d)$$

The three outputs given in equations (8.1a)–(8.1c) are HP (high pass), BP (band pass), and LP (low pass), respectively, with their center frequency and pole-Q being decided by equation (8.1d).

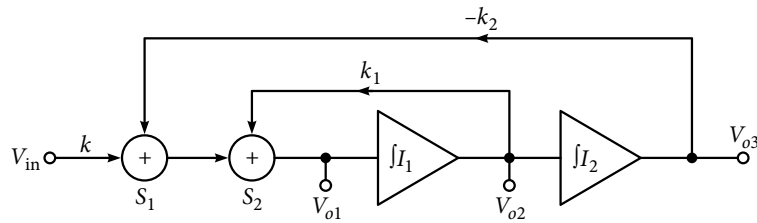


Figure 8.1 Basic two-integrator loop realizing a second-order filter in which summers S_1 and S_2 can be combined.

For the block diagram shown in Figure 8.1, three OAs are to be used, one each for the integrators and one for the combined summer. Another modification can be done by merging the summer in the lossy integrator. However, use of this modification requires the use of differential inputs for the summing of V_{in} and feedbacks for the lossy integrator and the loop. It also makes the adjustment of summing coefficients a bit difficult. To avoid this requirement, all the inputs at the summing integrator I_1 can become inverting signals. The filter will need a further modification by the addition of an inverter after I_2 as shown in Figure 8.2. It is to be noted that this configuration gives only BP and LP functions; HP is not available.

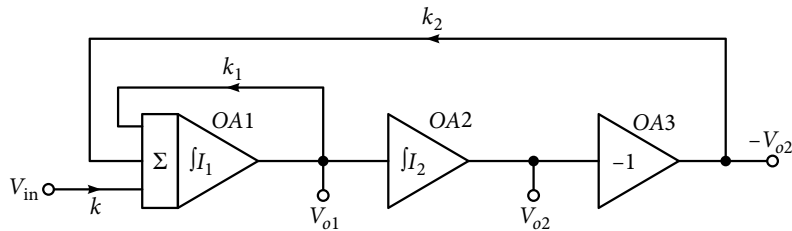


Figure 8.2 Two-integrator loop in modified form from Figure 8.1.

8.3 Tow–Thomas Biquad

A practical implementation of Figure 8.2, given in Figure 8.3, is known as the Tow and Thomas (TT) configuration [8.2,8.3]. Here, OA1 realizes a lossy integrator, OA2 realizes a lossless integrator and OA3 works as an inverter. Initially, considering OAs as ideal with infinite open-loop gain, the realized transfer function can be obtained through application of the Kirchhoff current law (KCL) at the inverting input terminals of OA1 and OA2.

$$(V_{in} - 0)/R_3 + (-V_{o2}/R_4) = (0 - V_{o1})/\{R_1/(1 + sC_1R_1)\} \quad (8.2)$$

$$(V_{o1} - 0)/R_2 = (0 - V_{o2})sC_2 \quad (8.3)$$

From equations (8.2)–(8.3), the following transfer functions are obtained:

$$\frac{V_{o1}}{V_{in}} = -\frac{(s / (C_1 R_1))(R_1 / R_3)}{s^2 + s / (C_1 R_1) + 1 / (C_1 C_2 R_2 R_4)} \rightarrow -\frac{h_{bp} (\omega_o / Q)s}{s^2 + (\omega_o / Q)s + \omega_o^2} \quad (8.4)$$

$$\frac{V_{o2}}{V_{in}} = \frac{1 / C_1 C_2 R_2 R_3}{s^2 + s / (C_1 R_1) + 1 / (C_1 C_2 R_2 R_4)} \rightarrow \frac{h_{lp} \omega_o^2}{s^2 + (\omega_o / Q)s + \omega_o^2} \quad (8.5)$$

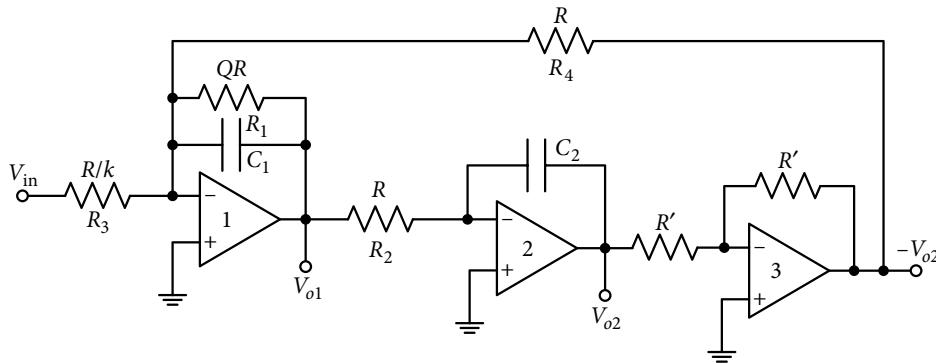


Figure 8.3 Tow-Thomas biquad. A practical implementation of Figure 8.2.

For the BP function of equation (8.4) and the LP function of equation (8.5), the important performance parameters ω_o , pole- Q , mid-band gain h_{bp} at ω_o , and gain at low frequencies or at dc for the LP case h_{lp} are as follows:

$$\omega_o = 1 / (C_1 C_2 R_2 R_4)^{1/2} \quad (8.6)$$

$$Q = \sqrt{\left(\frac{C_1}{C_2}\right)} \frac{R_1}{\sqrt{(R_2 R_4)}} \quad (8.7)$$

$$h_{bp} = (R_1 / R_3) \text{ and } h_{lp} = (R_4 / R_3) \quad (8.8)$$

Six passive components have been used in the biquad and there are only three design parameters. These multiple components give some choice in the selection of component values and flexibility in obtaining the desired parameters. As R_3 appears only in the gain factor terms of equation (8.8), it is used for fixing the dc gain of the LPF (low pass filter) and the mid-band gain of the BPF (band pass filter). Since it is always preferable to have equal valued capacitors, $C_1 = C_2 = C$. Pole- Q can be easily controlled by R_1 even when R_3 and R_4 are equalized; the component spread for resistors normally equals Q . The two resistors R' used with the inverter can be of some arbitrary value close to any one of the other resistors used.

One attractive feature of the TT configuration is its passive sensitivities, which are at their theoretical minimum. If x is a passive component, sensitivity expressions are

$$S_x^{\omega_o} = -(1/2) \text{ and } |S_x^Q| \leq 1. \quad (8.9)$$

Example 8.1: Design a BPF using a Tow–Thomas biquad having a center frequency of 3.4 kHz and pole- Q of 5. The filter's mid-band gain needs to be 20 dBs. Also discuss the obtained LP response.

Solution: With a normalized center frequency of 1 and $Q = 5$, $(\omega_o/Q) = 0.2$. Hence, we can begin with designing a normalized BP response as shown in equation (8.10).

$$H_{bpn}(S) = \frac{0.2h_{bp}S}{S^2 + 0.2S + 1} \quad (8.10)$$

The required mid-band gain being 20 dB or $h_{bp} = 10$, means $(R_1/R_3) = 10$ from equation (8.8). Selecting the normalized capacitors as $C_1 = C_2 = 1$, R_2 and R_4 will also be nominally 1 from equation (8.6). Hence, from equation (8.7), we get $R_1 = 5$ for $Q = 5$. De-normalizing the capacitors with $\omega = 2\pi \times 3.4$ krad/s and using an impedance scaling factor of 10^3 gives the following element values:

$$C_1 = C_2 = 0.0468 \mu\text{F}, R_1 = 5 \text{ k}\Omega, R_3 = 0.5 \text{ k}\Omega, R_2 = R_4 = R = 1 \text{ k}\Omega \quad (8.11)$$

Figure 8.4 shows the PSpice simulated magnitude response of the BPF with a mid-band gain of 10.828, center frequency of 3.382 kHz. Its bandwidth = 0.62 kHz resulting in a pole- Q of 5.45. The LP response is also available as V_{o2} , as shown in the figure, having a dc gain of 2 and a peak at 3.352 kHz, with peak gain being 10.915.

In another set of responses, the desired center frequency of a BPF was 300 krad/s. $Q = 10$, and mid-band gain was unity. Using the same steps as in the first set, the obtained de-normalized element values are:

$$C_1 = C_2 = 0.01 \mu\text{F}, R_1 = R_3 = 3.333 \text{ k}\Omega, R_2 = R_4 = 0.3333 \text{ k}\Omega, R' = 5 \text{ k}\Omega$$

The simulated BP response is also shown in Figure 8.4 with $\omega_o = 2\pi (44.027) = 276.74$ krad/s, bandwidth of 5.07 kHz which results in $Q = 8.67$ and a mid-band gain of 8.404. The corresponding LP response has a peak gain of 8.76 at 43.78 kHz (275.18 krad/s), and its voltage gain at dc is 1. While obtaining the transfer functions for the TT biquad in equations (8.4) and (8.5), OA were assumed to be ideal with an infinite open-loop gain. It is now well-known that the frequency-dependent gain creates deviations in performance. For example, in the first case, while using 741 type of OAs at $f_o = 3.4$ kHz and $Q = 5$, the simulated values show respective percent errors as 5.87 and 9; error in the mid-band gain was 8.28 percent. In the second case, at $f_o = 47.72$ kHz and $Q = 10$, the respective percent errors were 7.9, 13.3 and 16; a significant amount of error which increases with frequency. If suitable correction is not done,

the performance will become impracticable. Hence, passive as well as active compensations are used. In this configuration as well as in many other cases, integrators have often been used, so before moving on to other biquads, it is suggested that we find the deviations caused in ω_o and pole- Q and the methods employed in integrators for the compensation of errors.

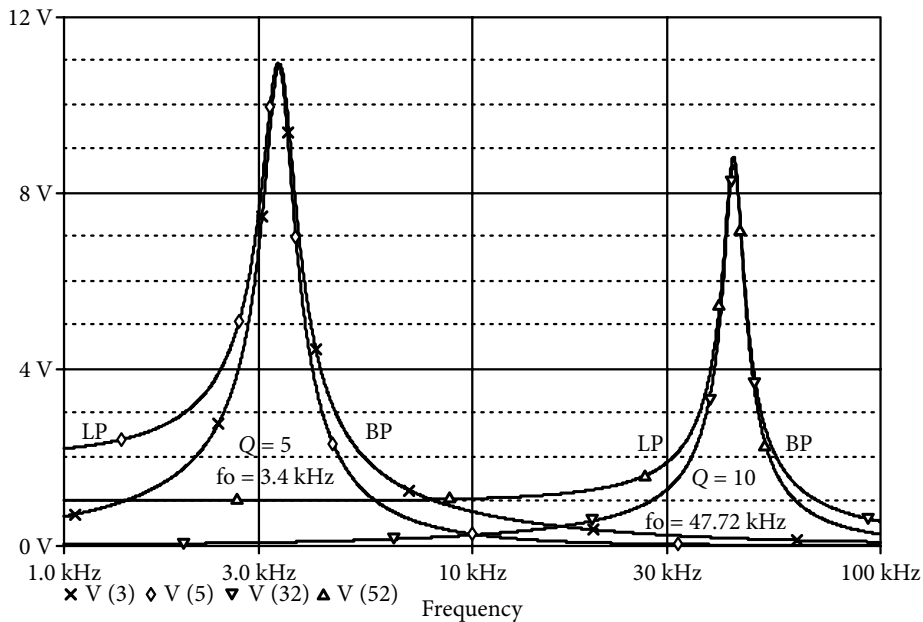


Figure 8.4 Band pass and low pass responses using a Tow-Thomas circuit at low and higher frequency levels for Example 8.1.

8.4 Active Compensation for Inverting Integrators

In Chapter 1, Section 1.8, integrators using OAs were briefly discussed. Figure 1.11(a) showed a lossy inverting integrator using an OA. The same integrator is now drawn in lossless form in Figure 8.5(a), without a feedback resistor. Using the single-pole roll-off model of equation (1.17) for the OA, the ideal integrator gets converted into a lossy integrator as expressed by equation (1.22) and rewritten as equation (8.12).

$$\frac{V_{\text{out}}}{V_{\text{in}}} = -\frac{1}{sCR} \frac{1}{1 + \frac{1}{A} \left(1 + \frac{1}{sCR} \right)} \quad (8.12)$$

With $A \cong (B/s)$, B being the gain bandwidth product, and with the condition that $(B \times CR) \gg 1$,

$$\frac{V_{\text{out}}}{V_{\text{in}}} = -\frac{1}{sCR(1 + s/B)} = -\frac{1}{j\omega CR - (\omega^2 CR/B)} \text{ for } s = j\omega \quad (8.13)$$

Equation (8.13) gives the quality factor of the inverting integrator as a ratio of imaginary parts to real parts as:

$$Q_I = -\frac{\omega CR}{(\omega^2 CR / B)} = (-B / \omega) = -|A(j\omega)| \quad (8.14)$$

The integrator quality factor Q_I is negative and depends on the magnitude of the gain. A larger value of $A(j\omega)$ is better, but as working frequency becomes large, Q_I becomes smaller. This introduces a frequency-dependent loss in the ideal integrator and therefore, error is introduced in the parameters of such filters which employ the integrator. To overcome the problem, passive or active compensation is used in integrators.

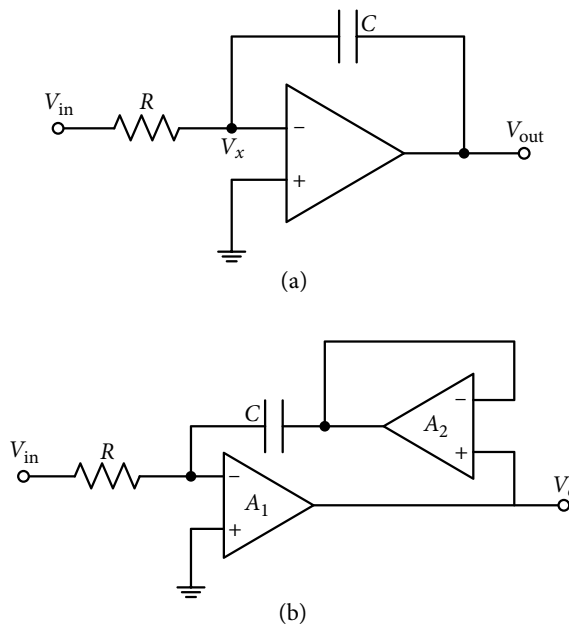


Figure 8.5 (a) Inverting integrator. (b) Active compensation for an inverting integrator.

Passive compensation has the advantage of using only one extra passive element; the compensation becomes near ideal. However, the compensation may not be accurate and will be variable with frequency.

An alternate solution is in the form of active compensation, shown in Figure 8.5(b), for which the transfer function is obtained as:

$$\frac{V_o}{V_{in}} = -\frac{1}{\frac{sCR}{(1+1/A_2)} + \frac{1}{A_1}} \quad (8.15)$$

Using simplified single-pole roll-off models for the OAs, $A_1 = (B_1/s)$ and $A_2 = (B_2/s)$. Applying truncation of Taylor's series expansion after the second-order term for $|A| \gg 1$, as we get:

$$\frac{1}{1+1/A} \cong 1 - \frac{1}{A} + \frac{1}{A^2} \quad (8.16)$$

For $s = j\omega$, equation (8.15) yields:

$$\frac{V_o}{V_{in}} = \frac{1}{\text{Re}(\omega) + j\text{Im}(\omega)} \quad (8.17)$$

$$\text{Re}(\omega) = \frac{\omega^2 CR}{B_2} \left(1 - \frac{B_2}{B_1} - \frac{\omega^2}{B_2^2} \right) \quad (8.18)$$

$$\text{Im}(\omega) = \omega CR \left(1 - \frac{1}{B_1 CR} - \frac{\omega^2}{B_1^2} \right) \quad (8.19)$$

For the matched OAs with $B_1 = B_2$, quality factor of the integrator $Q_I = \{\text{Im}(\omega)/\text{Re}(\omega)\}$ is obtained as follows:

$$Q_I = -(B_1/\omega)^3 = -|A(j\omega)|^3 \quad (8.20)$$

8.4.1 Compensation for a non-inverting integrator

In its simplest form, a non-inverting integration is obtained by cascading an inverting integrator and an inverter as shown in Figure 8.6(a). However, there are some other configurations for the non-inverting integrator as well. One such circuit is shown in Figure 8.6(b), for which the transfer function is obtained as:

$$\frac{V_o}{V_{in}} = -\frac{1}{sCR} \frac{1}{\left\{ \frac{1}{A_1(s)} + \frac{1}{sCA_1(s)} + \frac{1}{1 + \frac{2}{A_2(s)}} \right\}} \quad (8.21)$$

For matched OAs, the quality of the integrator is simplified as:

$$Q_{NI} = +(B/\omega) = +|A(j\omega)| \quad (8.22)$$

It is significant to note that here Q_{NI} is positive, whereas for the inverting integrator, Q_I was negative. This opposite nature of change in quality factor has been found to be useful while designing filters.

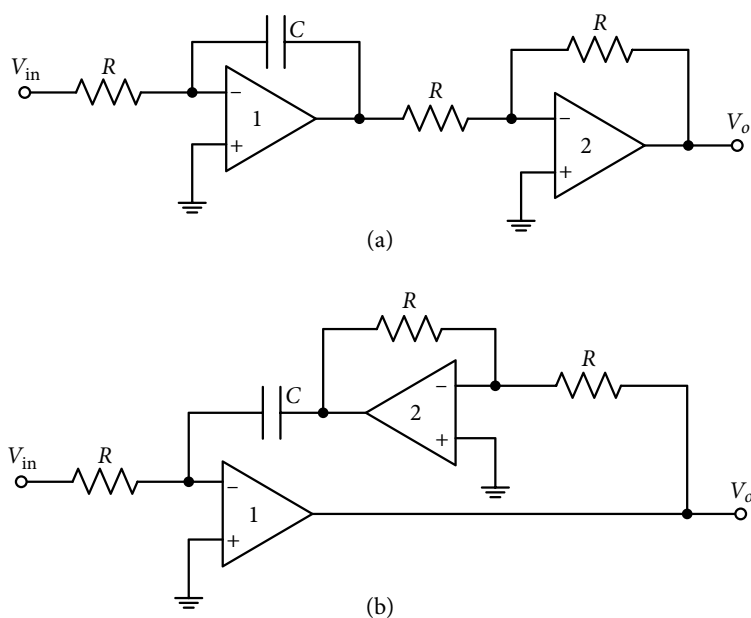


Figure 8.6 (a) A simple method to obtain a non-inverting integrator, and (b) an alternate non-inverting integrator with positive quality factor Q_{NI} .

It is not only the quality of integrators that change; the phase shift also changes. Therefore, compensating circuits of different configurations give different multi-amplifier biquadratic circuits with varied phase responses. Without going into a detailed study of the amount of affected performance due to the frequency-dependent finite open-loop gain of the OAs, let us discuss one prominent circuit which employs active compensation.

8.5 Ackerberg–Mossberg Biquad

Figure 8.7(a) shows the modification of the two-integrator loop of Figure 8.2 using the active compensation circuit of Figure 8.6(b), which was given by Ackerberg and Mossberg (AM) [8.4]. As the basic structure remains the same, the center frequency depends on the same RC product.

The circuit provides an LP response and a BP response. Assuming ideal OAs with $A \rightarrow \infty$, the obtained transfer functions are as following:

$$-\frac{V_{LP}}{V_{in}} = \frac{k / R^2 C^2}{s^2 + (s / CRQ) + 1 / R^2 C^2} \quad (8.23)$$

$$\frac{V_{BP}}{V_{in}} = -\frac{ks / CR}{s^2 + (s / CRQ) + 1 / R^2 C^2} \quad (8.24)$$

The quality factor, and the mid-band gain of the BPF = k are controlled by the resistance ratios shown in Figure 8.7(a). For the inverter OA3, equal valued resistances R^* are used; expression of the center frequency is as follows.

$$\omega_o = 1/RC \quad (8.25)$$

Using the expressions of equation (8.22) and (8.21), deviations in the quality factor and pole frequency can be obtained in the AM structure.

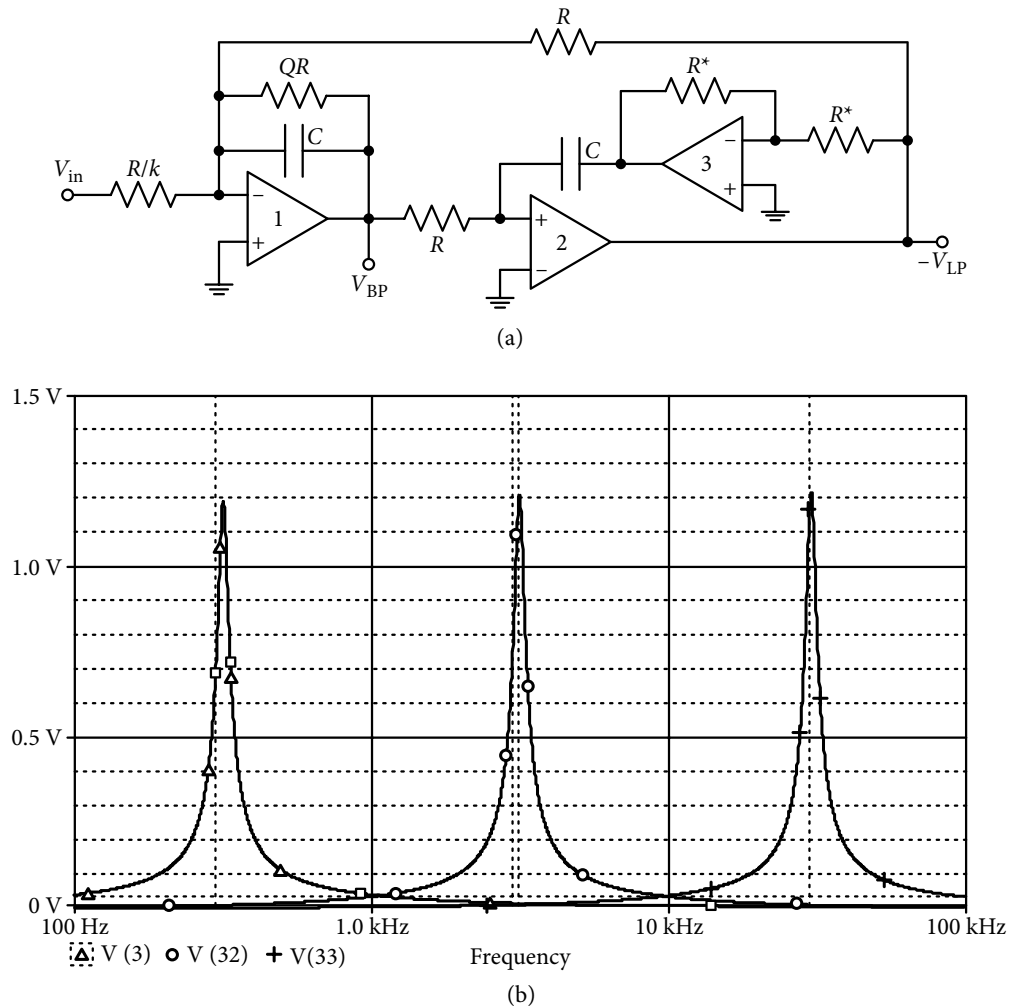


Figure 8.7 (a) Ackerberg-Mossberg biquadratic structure. (b) Band pass filter responses with $\omega_o = 2$ rad/s, 20 rad/s and 200 rad/s using the Ackerberg-Mossberg structure.

The coefficient matching technique described in Chapter 7 will be used to find element values, which will be shown in Example 8.2. Once the value of R or C is selected, the rest of the element values are evaluated from equation (8.25).

Example 8.2: Obtain a BP response using the AM structure of Figure 8.7(a) having a center frequency of 2 krad/s (3.1818 kHz), with $Q = 12$ and a mid-band gain of 12. Also obtain responses for center frequencies of 20 krad/s (3.181 kHz) and 200 krad/s (31.818 kHz) and comment on the obtained parameters.

Solution: Case (i) From equation (8.25), center frequency $\omega_o = 1/RC$, so we select $C = 0.1 \mu\text{F}$, $R = 5 \text{ k}\Omega$. Therefore, resistance $QR = 60 \text{ k}\Omega$, and for mid-band gain of 12, $k = 1$, hence, $(R/k) = 5 \text{ k}\Omega$. R^* is arbitrarily selected as $10 \text{ k}\Omega$; a value in-between values of other resistors. The circuit is simulated through PSpice and its response is shown in Figure 8.7(b). The measured center frequency is 317.75 Hz (1997.2 rad/s), mid-band gain is 11.926 and with a bandwidth of 26.61 Hz, $Q = 11.94$.

Case (ii) With all resistors remaining the same, the required value of $C = 0.01 \mu\text{F}$ for $\omega_o = 20 \text{ krad/s}$. The simulated response is also shown in Figure 8.7(b). The measured parameters are $\omega_o = 19.793 \text{ krad/s}$ (3.149 kHz), mid-band gain = 11.738 and bandwidth of 264.3 Hz resulting in $Q = 11.92$.

Case (iii) For $\omega_o = 200 \text{ krad/s}$, the required capacitor $C = 1 \text{ nF}$. The simulated response is also shown in Figure 8.7(b). The measured parameters are $\omega_o = 192.15 \text{ krad/s}$ (30.57 kHz), mid-band gain = 12.139 and bandwidth of 2.464 kHz, which results in $Q = 12.36$. Table 8.1 shows the percent error in the simulated parameters for the three cases at different frequencies.

Table 8.1 Percent error in the parameters of filters realized using AM configuration for Example 8.2

	ω_o	Q	mid-band gain
Case (i), $\omega_o = 2 \text{ krad/s}$	0.135	0.5	0.616
Case (ii), $\omega_o = 20 \text{ krad/s}$	0.375	0.66	0.516
Case (iii), $\omega_o = 200 \text{ krad/s}$	3.92	-3.0	-1.158

A comparison of percent errors in the parameters of the filters designed using Tow–Thomas and AM configurations show a marked improvement in the latter case, especially at higher frequencies. This confirms the utility of the active compensation employed in the AM circuitry.

8.6 Multi-output Biquad Using Summing Amplifier

As mentioned earlier, one of the advantages of using more than one amplifier in a circuit is its versatility in obtaining more than one kind of response simultaneously. For example, in one method, using a summing amplifier, a circuit which has already generated LP and BP responses, can also generate other kind of responses by adding the specific input. The process is illustrated by using structure of Figure 8.2 which shows a two-integrator second-order generating circuit. When the circuit's two outputs are summed with an input using an additional summing amplifier as shown in Figure 8.8, three responses are simultaneously

available as V_{o1} , V_{o2} and V_{out} . Employing the relations of equation (8.1), the obtained transfer function is as follows:

$$\frac{V_{out}}{V_{in}} = -\frac{\alpha s^2 + a_1 s(\alpha k_1 - \beta k) + a_1 a_2(\alpha k_2 - \gamma k)}{s^2 + a_1 k_1 s + a_1 a_2 k_2} \quad (8.26)$$

Selection of summing coefficients α , β and γ decides the type of available response at the output V_{out} .

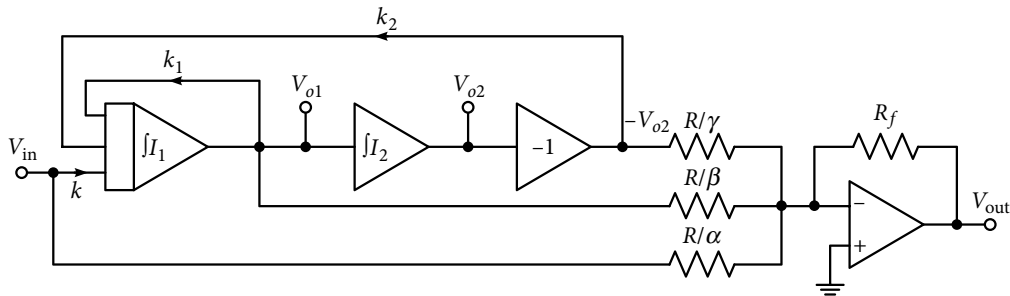


Figure 8.8 Generation of a general biquad using a two-integrator network and a summer.

If the AM circuit shown in Figure 8.7(a) is used to generate a general biquadratic circuit with a summing amplifier and inputs coming from the original input V_{in} , $+V_{LP}$ output at the output terminal of OA3, and V_{BP} , the arrangement will look as shown in Figure 8.9. The output voltage shall be as follows using equations (8.23) and (8.24); obviously any other two-integrator loop circuit can also be used.

$$V_{out} = \alpha V_{in} + \beta V_{BP} + \gamma V_{LP} = \alpha V_{in} - \beta \frac{k\omega_o}{D(s)} s V_{in} - \gamma \frac{k\omega_o^2}{D(s)} V_{in}$$

$$\frac{V_{out}}{V_{in}} = \frac{\alpha s^2 + (\alpha - kQ\beta)(\omega_o / Q)s + (\alpha - \gamma k)\omega_o^2}{s^2 + (\omega_o / Q)s + \omega_o^2} \quad (8.27)$$

From equations (8.23)–(8.24) and from Figure 8.9, the concerned relations are as follows:

$$\omega_o^2 = \frac{1}{RC}, \quad \alpha = \frac{R_f}{R_\alpha}, \quad \beta = \frac{R_f}{R_\beta} \text{ and } \gamma = \frac{R_f}{R_\gamma} \quad (8.28)$$

Selection of coefficients α , β and γ will decide the numerator terms; this will then decide the type of response at the output. The following examples will illustrate the generation of responses other than BP and LP using the coefficient matching technique.

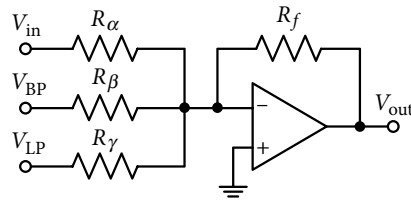


Figure 8.9 Summing amplifier used for obtaining a general biquad.

Example 8.3: Design a notch filter employing an AM circuit and a summing amplifier which should have a notch at 20 krad/s and $Q = 5$.

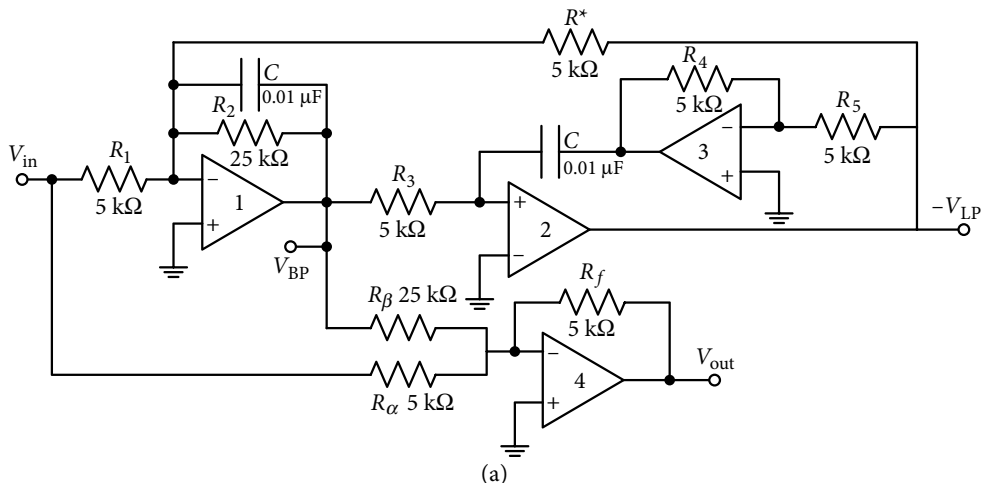
Solution: In equation (8.27), we select $\alpha = 1$. Hence, for normalized $\omega_o = 1$, $(1 - kQ\beta)$ has to be zero, which gives the coefficients as:

$$(1 - 5k\beta) = 0 \rightarrow \beta = 0.2 \text{ if } k = 1 \quad (8.29)$$

$$1 - \gamma \times 1 = -1 \rightarrow \gamma = 0 \quad (8.30)$$

The selected value of $C = 0.01 \mu\text{F}$ gives the value of R from equation (8.28) as $5 \text{ k}\Omega$ for $\omega_o = 20 \text{ krad/s}$. Hence, $R_1 = (R/k) = 5 \text{ k}\Omega$, $R_2 = QR = 25 \text{ k}\Omega$, $R_3 = R^* = R_1 = R_\alpha = R_f = R_4 = 5 \text{ k}\Omega$, $R_\beta = 25 \text{ k}\Omega$ and R_γ is open as shown in Figure 8.10(a).

Figure 8.10(b) shows the PSpice simulated response of the notch filter having a notch at 3.168 kHz (19.913 krad/s). The cut off frequencies of the filter are 3.5 kHz and 2.866 kHz, resulting a bandwidth of 634 Hz and $Q = 4.996$. The input voltage being 100 mV, the voltage level at the notch drops to 0.98 mV or an attenuation of 40.17 dBs.



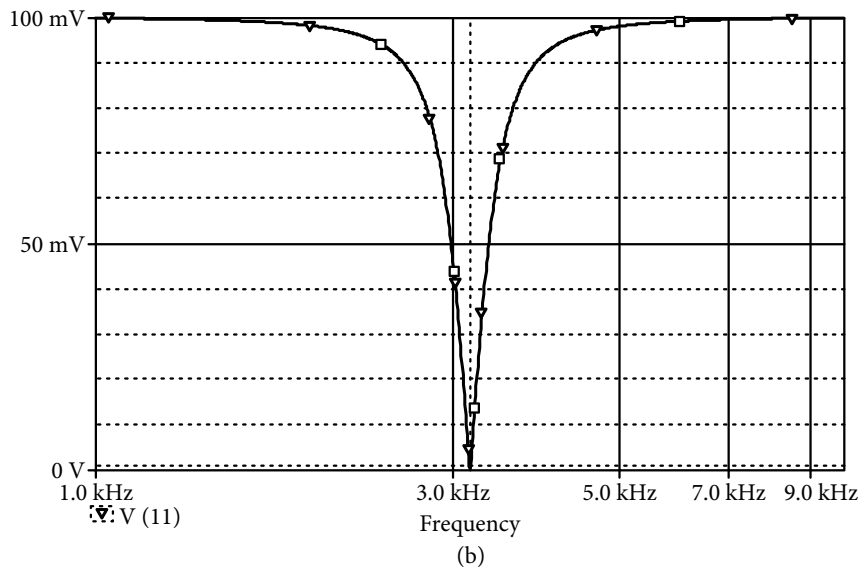


Figure 8.10 (a) Second-order notch filter circuit using an Ackerberg–Mossberg circuit and a summing amplifier. (b) Magnitude response of the notch filter of Figure 8.10(a).

Example 8.4: Design a second-order APF (all pass filter) which has a pole- $Q = 2$ and a 0° phase delay at 2 krad/s using an AM circuit and a summing amplifier.

Solution: The first consideration in getting an APF is to fix $\alpha = 1$; then, using equation (8.27):

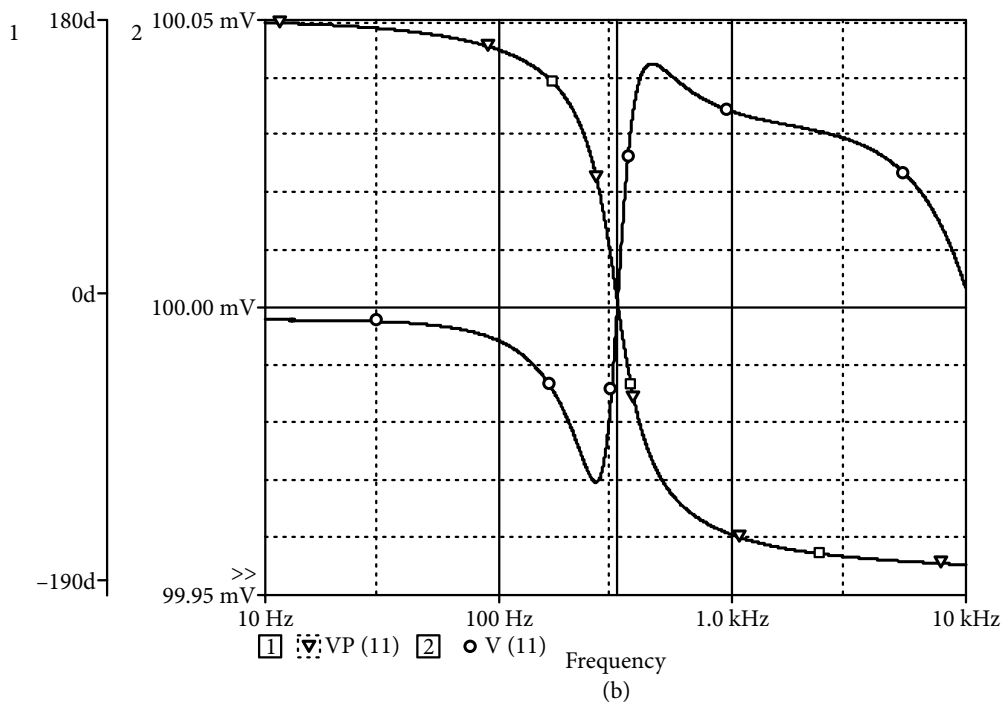
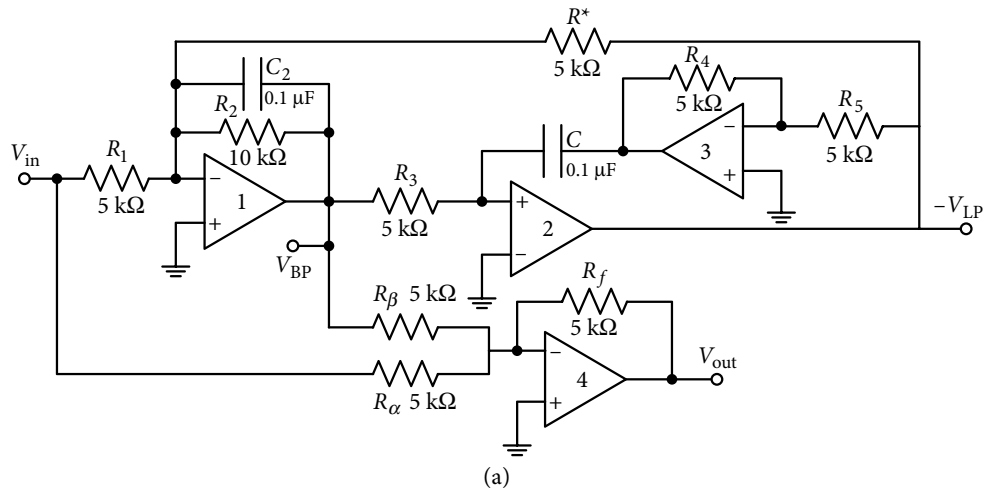
$$\omega_o^2 = (1 - \gamma k) \omega_o^2 \rightarrow \gamma k = 0 \text{ and } \gamma \text{ is taken as } 0 \quad (8.31)$$

$$\text{For } \omega_o = 1, Q = 2, \text{ and assuming } k = 1; (1 - kQ\beta\omega_o) = -1 \rightarrow \beta = 1 \quad (8.32)$$

Critical frequency being 2 krad/s, the selected value of capacitor $C = 0.1 \mu\text{F}$, which requires $R = 5 \text{ k}\Omega$. Having obtained the coefficients in equations (8.31)–(8.32), the remaining element values are:

$$C_2 = 0.1 \mu\text{F}, C_3 = 0, R_1 = 5 \text{ k}\Omega, R_2 = 10 \text{ k}\Omega, R_4 = R_5 = R^* = 5 \text{ k}\Omega \text{ and } R_f = R_\alpha = R_\beta = 5 \text{ k}\Omega.$$

The circuit diagram of the designed second-order APF with element values is shown in Figure 8.11(a). Figure 8.11(b) shows the magnitude response of the filter: there is a very small variation in magnitude; a dip of 0.13 mV and a rise of 0.042 mV from an average constant value of 100 mV. The figure also shows the phase variation in the APF from 180° to -180° with a zero-degree phase shift at 317.68 Hz (1996.8 rad/s). Figure 8.11(c) shows variation in group delay; at 310.6 Hz, peak group delay $D = 4.065 \text{ ms}$. This is a near perfect response due to the active compensation employed in the AM circuit.



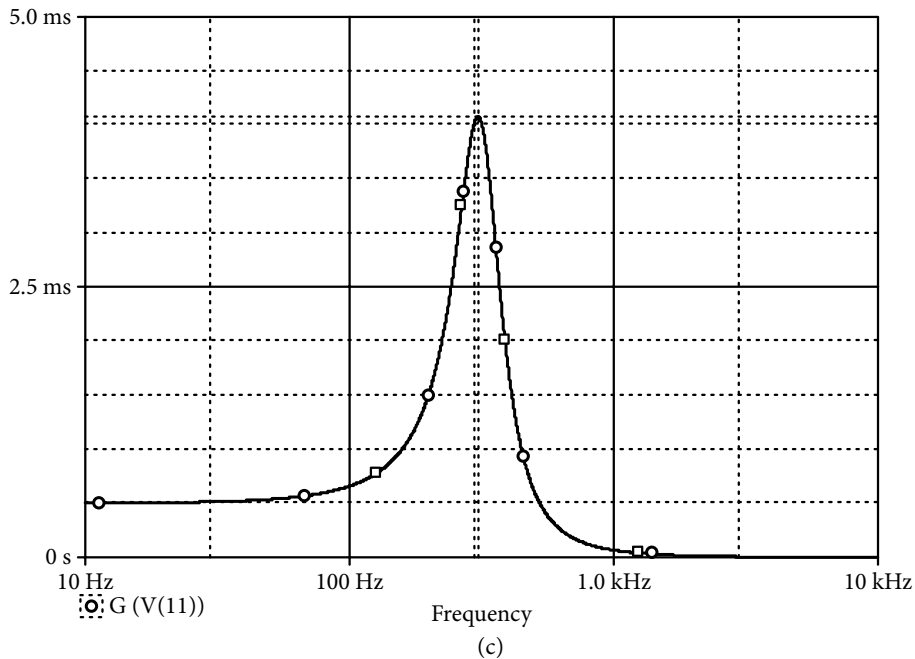


Figure 8.11 (a) Second-order all pass filter circuit using the Ackerberg–Mossberg circuit and a summing amplifier. (b) Magnitude and phase responses of the all pass filter shown in Figure 8.11(a). (c) Group delay response of the all pass filter shown in Figure 8.11(a).

8.6.1 Biquad using modified summation method

In a slightly modified but efficient approach, the additional summing amplifier can be avoided by the application of a weighted input signal at the virtual ground terminals of the two integrators. The advantage of connecting V_{in} in such a way is that the poles given by equation (8.26) are not affected. If this scheme is applied to the AM circuit shown in Figure 8.7(a), the resulting biquad becomes as shown in Figure 8.12. With the OAs considered ideal, the transfer function of the circuit is given as:

$$\frac{V_{out}}{V_{in}} = -\frac{\alpha s^2 + \{s(k - \beta)/CR\} + \gamma/C^2R^2}{s^2 + \{s/(CRQ)\} + 1/C^2R^2} \quad (8.33)$$

All types of responses are easily obtainable by selecting different weighting coefficients:

$$\text{For LP, } k = \beta = \alpha = 0, \text{ and for HP, } k = \beta = \gamma = 0 \quad (8.34a-b)$$

$$\text{For BP, } \gamma = \beta = \alpha = 0, \text{ and for notch, } k = \beta = 0, \alpha < \gamma \quad (8.34c-d)$$

For HP notch, $k = \beta = 0$, $\alpha > \gamma$, and for LP notch, $k = \beta = 0$, $\alpha < \gamma$ (8.34e-f)

Lastly for the AP, $\alpha = \gamma = k = 1$, $\beta = (1 + 1/Q)$ (8.34g)

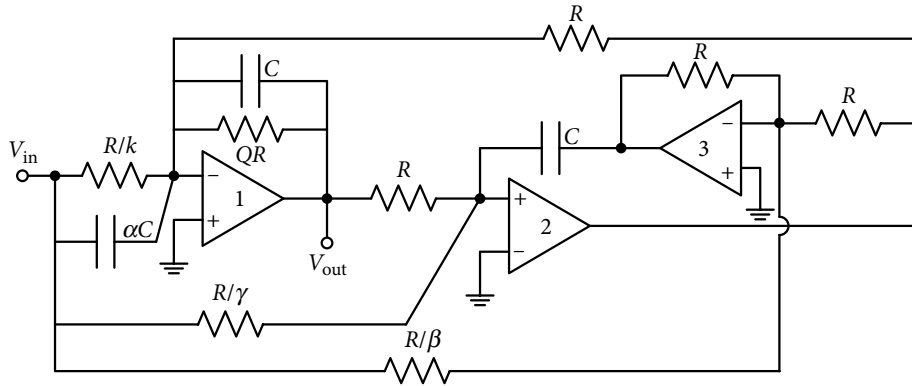


Figure 8.12 Generation of a general biquadratic circuit through the application of a weighted input at the virtual ground terminals of the operational amplifiers using an AM biquad.

The general biquad has many desirable characteristics apart from providing all types of second-order responses. Active and passive sensitivities are found to be low and since pole locations are not changed while connecting the input at the virtual ground terminals, sensitivities remain the same as that for the AM circuit. Center frequency ω_o and pole- Q are independently tuneable with the help of the input resistance connected to the second integrator and the resistor QR , respectively. Component spread is also small as will become obvious from the following examples. Additionally, when parameters are set for deciding the location of zeros or the type of response, it does not affect the pole location.

Example 8.5: Design an HPF having a 3dB cut-off frequency of 20 krad/s and $Q = 2$ using the generalized biquad shown in Figure 8.12.

Solution: From equation (8.33), as $\omega_o = (1/RC) = 20$ krad/s for the general biquadratic filter circuit shown in Figure 8.12, selection of $C = 0.01$ μ F gives $R = 5$ k Ω . To find the other component values, we use the condition $k = \beta = \gamma = 0$ from equation (8.34b); it gives, $R_1 = (R/k) = \infty = R_\beta = (R/\beta) = R_\gamma = (R/\gamma)$, $R_2 = QR = 10$ k Ω and $R_3 = R_4 = 5$ k Ω . Selection of R_3 and R_4 is intentionally done to keep as many resistances equal as possible; a good choice in integrated circuits. Capacitor $C_2 = C = 0.01$ μ F and $C_3 = \alpha C = 0.01$ μ F for $\alpha = 1$. The resulting circuit is shown in Figure 8.13(a) and the simulated response in Figure 8.13(b).

A peak occurs at 3.386 kHz where its voltage gain is 2.073 against the ideal value of 2.0. Voltage gain at high frequencies is unity. The evaluated value of the simulated $\omega_o = 3.386(1 - 1/2 \times 2^2)^{1/2} = 3.167$ kHz = 19.908 krad/s.

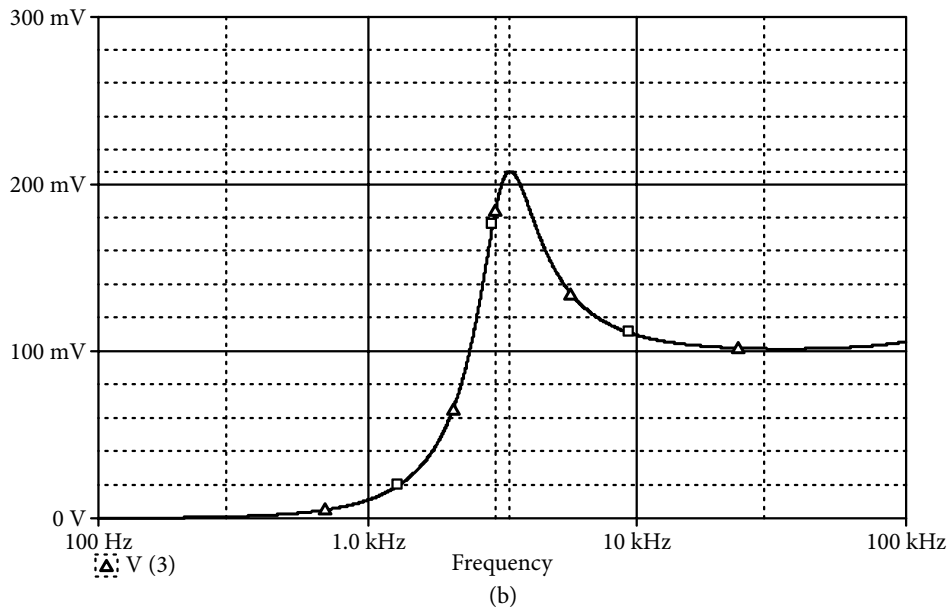
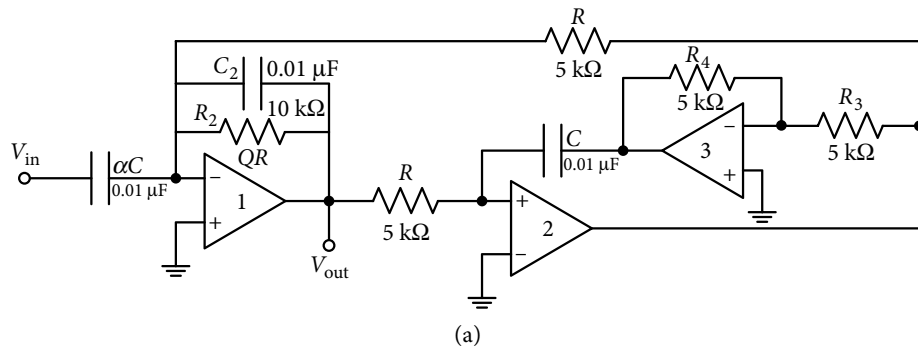


Figure 8.13 (a) High pass filter using the Ackerberg–Mossberg biquad and applying the modified summation method for Example 8.5. (b) Response of the HPF shown in Figure 8.13(a).

Example 8.6: Design a HP notch filter having a pole frequency of 2 krad/s, a zero at 1 krad/s and $Q = 2$ using the modified summation method.

Solution: From equation (8.33) for $\omega_o = 2$ krad/s, the selected capacitor $C = 0.01 \mu\text{F}$ gives $R = 5 \text{ k}\Omega$. For a zero at 1 krad/s, $\gamma^{1/2} = (1/2)$, hence, $(R/\gamma) = (R/0.25) = 20 \text{ k}\Omega$. Since α has to be more than γ for a HP notch, it is taken as 1; therefore, $\alpha C = C = 0.01 \mu\text{F}$, and $C_2 = C$. With $Q = 2$, other elements will be the same as in Example 8.5, with $k = \beta = 0$. The circuit with element values is shown in Figure 8.14(a) and its PSpice simulated response is shown in Figure 8.14(b).

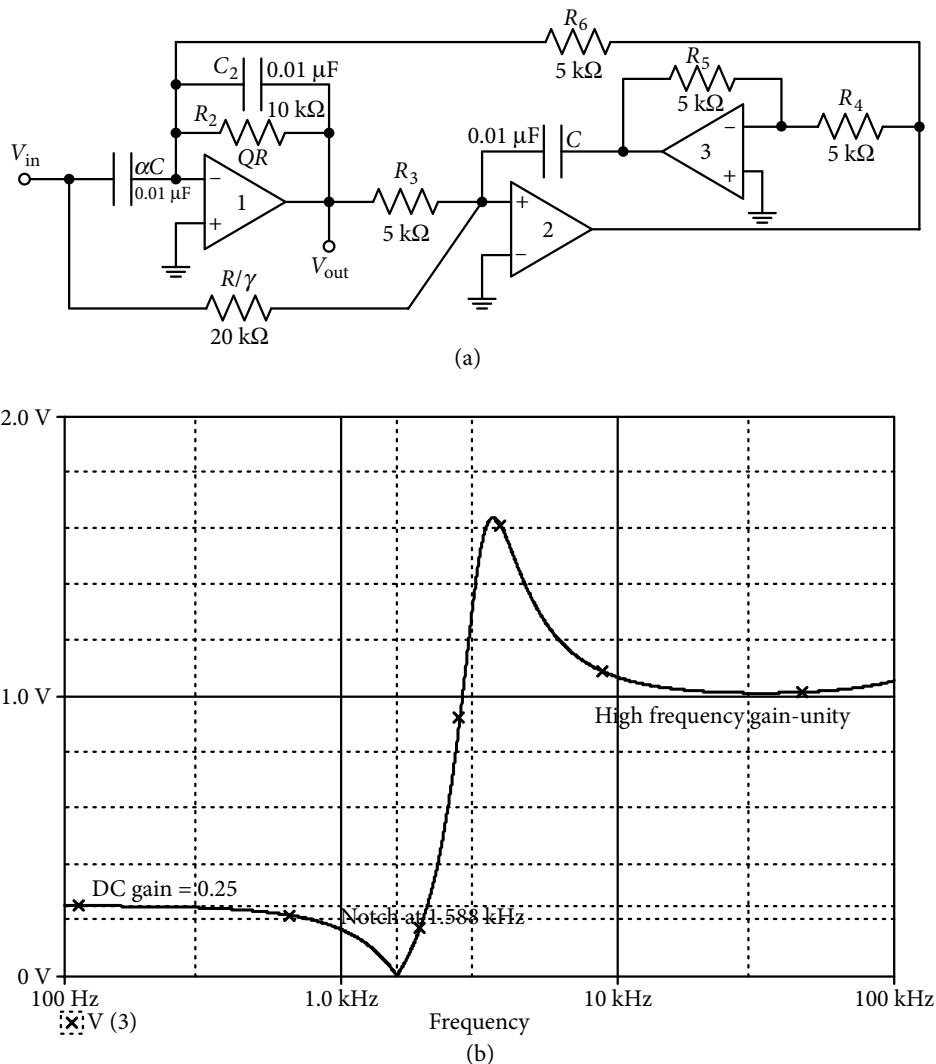


Figure 8.14 (a) Generation of a high pass notch circuit through application of the modified summation method using the Ackerberg–Mossberg biquad. (b) Magnitude response of the high pass notch filter shown in Figure 8.14(a).

The notch occurs at 1.588 kHz with a low frequency gain of 0.25; the peak occurs at 3.565 kHz where due to $Q = 2$, voltage gain is 1.636. At high frequencies, the output voltage levels at unity gain; this verifies all the specifications.

8.6.2 Active noise control: application example

Feedback control systems are used in active noise and vibration control. Such systems can be realized using either digital signal processing or analog signal methodology. Each process

has advantages and limitations. There are certain applications where analog feedback control systems are preferred, such as active control of earmuffs [8.5] and similar applications, where the process delay must be small.

In practice, analog control systems include a filter or a bank of filters [8.6]. Parameters of the analog filter are usually adjusted with variable resistance and/or changing capacitor values employing switching arrangements. The switching arrangement is normally done manually. In the application case presented here, microprocessor driven, real-time control of the parameters of the filter bank has already been developed. This circuit discussed here helps in noise control. Without going into the development process, the basic arrangement of the biquadratic filter bank system is shown in Figure 8.15 in block form.

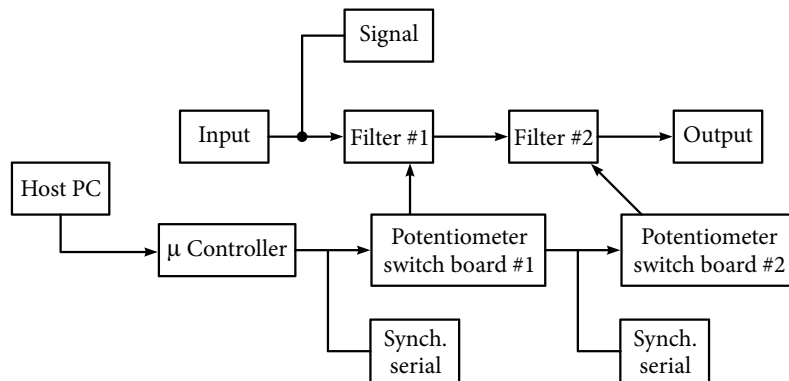


Figure 8.15 The block diagram of a system for active noise control {with permission from M. Antila et al.} [8.6].

As a specific example, equation (8.35) shows a transfer function, which was obtained as a useful transfer function for a feedback control system.

$$H(s) = \frac{1.01s^2 - 70s + 2226065}{s^2 + 272s + 319092} \quad (8.35)$$

To implement the aforementioned transfer function, a general biquadratic circuit, as shown in Figure 8.12, applying the modified summation method was used. Comparing equation (8.35) with equation (8.33) gives:

$$\omega_o^2 = 319092, \alpha = 1.01, \gamma = (2226065 / 319092) = 6.976, Q = (\omega_o / 272) = 2.076, \\ \text{and if } k \text{ is selected as } 0.5, \beta = (70 / 560.88) + 0.5 = 0.6239 \quad (8.36)$$

Applying an impedance scaling factor of 10 kΩ and a frequency scaling factor of $\omega_o = 564.88$ rad/s, element values for the circuit in Figure 8.12 are obtained as:

$$R = 10 \text{ k}\Omega, R/k = 20 \text{ k}\Omega, QR = 20.76 \text{ k}\Omega, R/\beta = 16.028 \text{ k}\Omega, R/\gamma = 1.433 \text{ k}\Omega, C = 0.177 \text{ }\mu\text{F} \\ \text{and } \alpha C = 0.1788 \text{ }\mu\text{F}$$

The simulated response of the biquadratic filter section is shown in Figure 8.16. Its low frequency gain is 16.87 dB, the high frequency gain is 0.158 dB, the notch frequency is 234.4 Hz and the peak gain of 22.3 dB occurs at 512.4 rad/s (81.5 Hz).

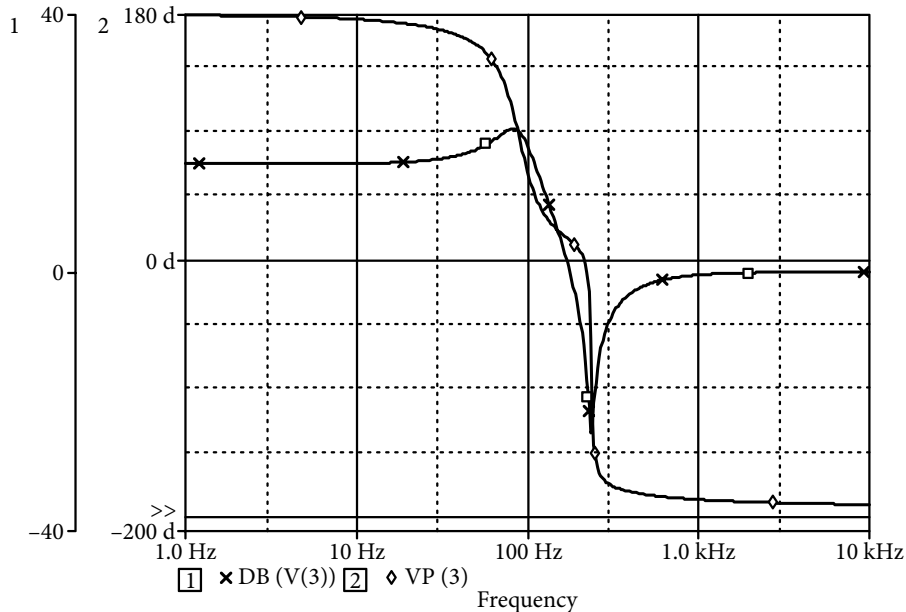


Figure 8.16 Simulated response of a biquadratic filter modeling noise control (equation (8.35)).

8.7 Generalized Impedance Converter Based Biquad

A significant alternative to obtain a multi amplifier biquad is a technique which is based on the use of a generalized impedance converter (GIC) [8.7]. In its basic form, this improvised biquad starts with a passive structure and its grounded inductor is replaced with a GIC. Presently, without going into a detailed description of a GIC, observe a second-order passive BPF structure in Figure 8.17(a) and its conversion to a second-order active filter circuit in Figure 8.17(b). GIC is shown in a dotted rectangle replacing the inductor in Figure 8.17(a). The transfer function of the passive BP filter is:

$$\frac{V_1}{V_{in}} = \frac{(1/RC)s}{s^2 + (1/RC)s + 1/(LC)} \quad (8.37)$$

Here center frequency ω_o and pole- Q (Q_o) are:

$$\omega_o = (1/LC)^{1/2} \text{ and } Q_o = R(C/L)^{1/2} \quad (8.38)$$

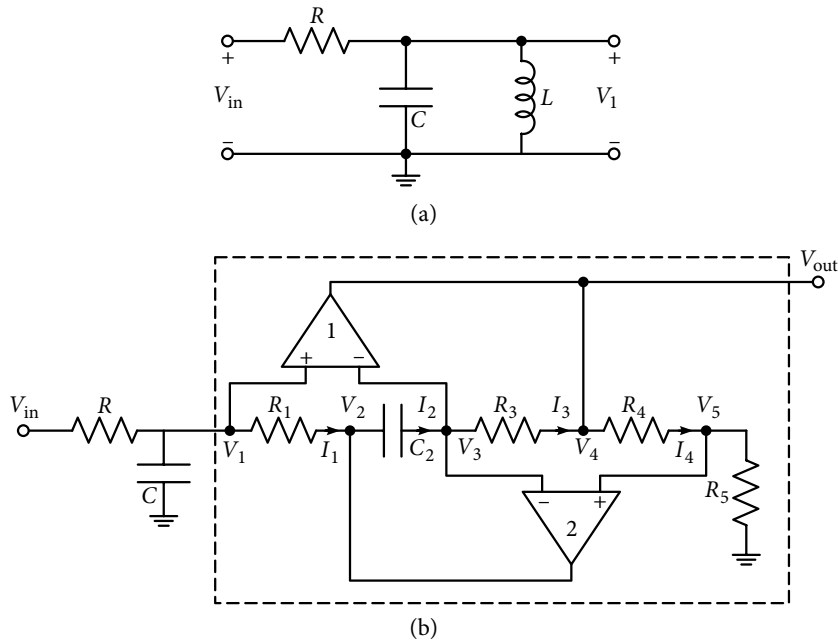


Figure 8.17 (a) A second-order passive band pass passive filter; (b) inductor in part (a) replaced with a generalized impedance converter shown inside the dotted line.

For the circuit in Figure 8.17(b), with the OAs assumed to be ideal and $A \rightarrow \infty$, $V_1 = V_3 = V_5$, the following are the current-voltage relations:

$$I_4 = (V_5/R_5), \quad V_4 = V_5 + I_4 R_4 = V_5(1 + R_4/R_5) \quad (8.39)$$

$$I_3 = I_2 \rightarrow (V_3 - V_4)/R_3 = (V_5 - V_4)/R_3 = -V_5 R_4/(R_3 R_5) = (V_2 - V_3)sC_2 \quad (8.40)$$

$$I_1 = (V_1 - V_2)/R_1 \text{ or } (V_1/I_1) = s(C_2 R_1 R_3 R_5)/R_4 \quad (8.41)$$

Equation (8.41) confirms that the circuit within the dotted rectangle in Figure 8.17(a) realizes an inductance with its expression given as:

$$L = C_2 R_1 R_3 R_5 / R_4 \quad (8.42)$$

Therefore, substituting the expression of L from equation (8.42) into equation (8.37) and V_4 from equation (8.39), the transfer function of the active second-order section of Figure 8.17(b) is obtained as:

$$\frac{V_{\text{out}}}{V_{\text{in}}} = \frac{(1/RC)(1 + R_4/R_5)s}{s^2 + (1/RC)s + (R_4/CC_2 R_1 R_3 R_5)} \quad (8.43)$$

If output voltage is V_1 , transfer function is same as in equation (8.43) but numerator will not have the term $(1 + R_4/R_5)$. So the mid-band gain will be unity in this case. Here, ω_o and Q_o are given by the following relations:

$$\omega_o = (R_4/CC_2R_1R_3R_5)^{1/2} \quad (8.44)$$

$$Q_o = R(C/C_2)^{1/2} (R_4/R_1R_3R_5)^{1/2} \quad (8.45)$$

For ω_o and Q_o , the passive sensitivities with respect to the elements x are calculated as:

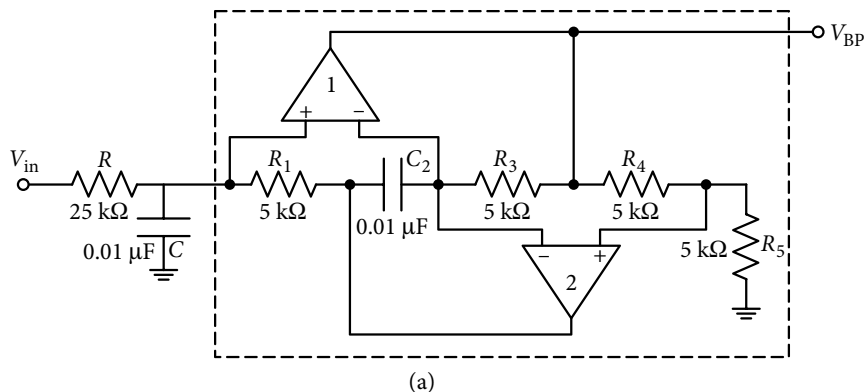
$$\left|S_x^{\omega_o}\right| = \frac{1}{2}, \left|S_x^{Q_o}\right| = \frac{1}{2} \text{ and } \left|S_R^{Q_o}\right| = 1 \quad (8.46)$$

Clearly, all passive sensitivities are very low; hence, the proposed circuit employing a GIC enjoys excellent sensitivity figures, provided its active sensitivities are also low. A detailed discussion on GIC sensitivities will be taken up later with the description of the GIC structure. In addition to the very low sensitivities, the GIC based active second-order filter structure has the following advantages, which makes the circuit very attractive.

- Component spread is small; most of the passive elements can be made equal.
- Parameters ω_o and Q_o and mid-band gain of the BP can be independently tuned.
- The circuit is suitable for cascading as it has *infinite* input impedance at the frequency ω_o .

Example 8.7: Design a BPF for a center frequency of 20 krad/s and $Q = 5$ using the GIC based configuration of inductance shown in Figure 8.17(b).

Solution: Selecting equal valued capacitors C and C_2 as $0.01 \mu\text{F}$, equation (8.44) gives $R_1 = R_3 = R_4 = R_5 = 5 \text{ k}\Omega$. For $Q = 5$, equation (8.45) gives $R = 25 \text{ k}\Omega$. Using these element values, the filter structure is shown in Figure 8.18(a). The response is simulated using PSpice and shown in Figure 8.18(b).



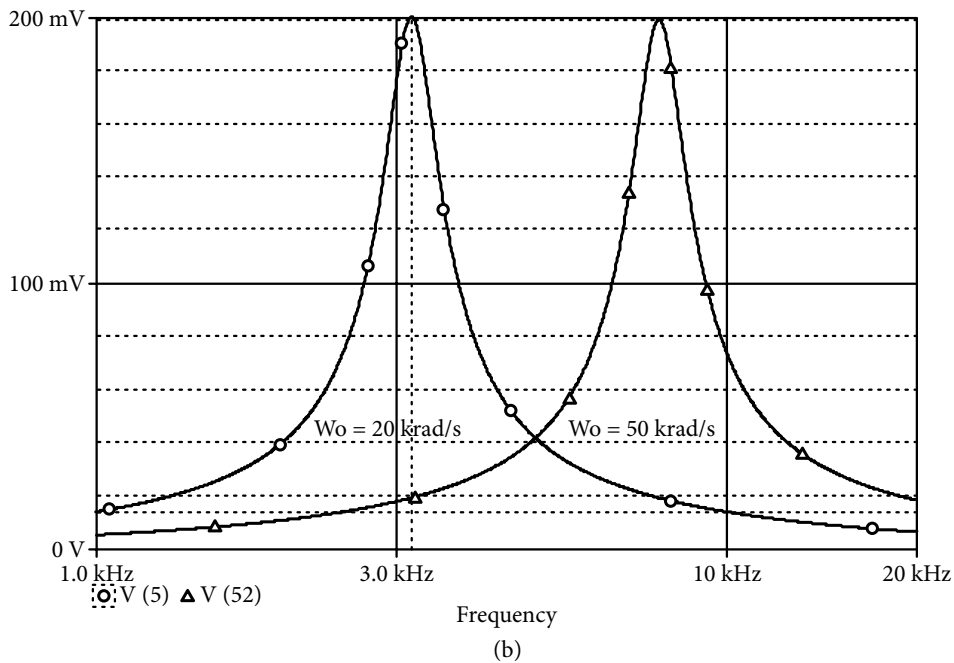


Figure 8.18 (a) Second-order active band pass filter with inductor replaced using a generalized impedance converter for Example 8.7; (b) responses of the band pass filter in part (a).

The obtained BP response realizes a center frequency as 3.177 kHz (19.969 krad/s) with a voltage gain of 1.991. The bandwidth of the filter is 635 Hz resulting in $Q = 5.003$.

Another BP response was obtained for a higher center frequency of 50 krad/s. The circuit requires capacitors C and C_2 each of 0.004 μF , with all resistors remaining the same, for same values of Q . The simulated response is also shown in Figure 8.18(b) with a center frequency of 7.821 kHz (491.6 krad/s) and a voltage gain of 1.984. Bandwidth of the filter is 1.563 kHz, resulting in the value of Q as 5.004.

8.7.1 General biquad using generalized impedance converter

In addition to the discussed BPFs in the previous section, other types of transfer functions can be realized using the well-known process of *lifting grounded elements completely or partially from the ground* while using GICs. Inclusion of resistance R_7 and splitting of input resistance R and capacitance C are used to provide feedback. Such a configuration is shown in Figure 8.19, with its transfer function obtained as:

$$\frac{V_{\text{out}}}{V_{\text{in}}} = \frac{[\alpha H - \gamma(H-1)]s^2 + [\beta H - \gamma(H-1)]s / QRC + (\gamma / R^2 C^2)}{s^2 + (1 / QRC)s + (H-1) / R^2 C^2} \quad (8.47)$$

In equation (8.47), H is the mid-band filter gain, and an appropriate choice of the weighting coefficients, α , β and γ , determines the type of obtained filter response. At this stage, it may be

noted that getting a pure LP response is very difficult in this scheme; it is advised to use some other configuration of a GIC.

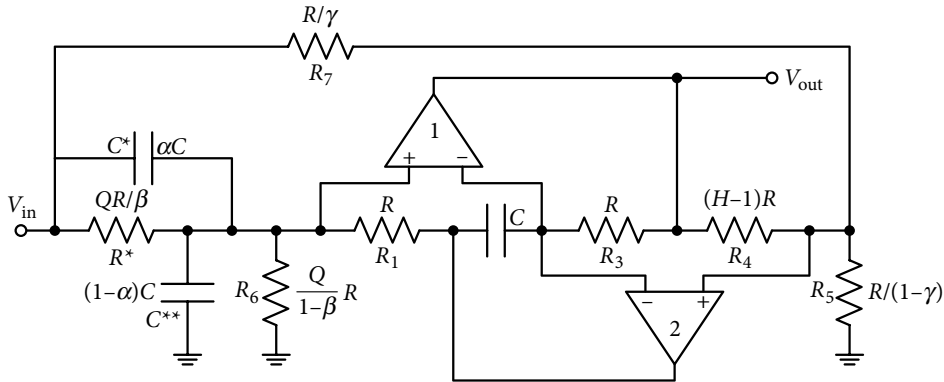


Figure 8.19 Biquadratic circuit obtained using the technique of lifting some elements from the ground partially using GIC circuit of Figure 8.17(b).

Example 8.8: Design a notch filter which should have a notch at 20 krad/s and $Q = 4$ using a GIC based general biquad.

Solution: A convenient choice of a notch filter from equation (8.47) is to assume $H = 2$ and $\gamma = 1$, with which $\omega_o = (1/RC)$. For notch frequency $\omega_o = 20$ krad/s, an easy choice of components is $C_2 = 0.01 \mu\text{F}$ and $R = 5 \text{ k}\Omega$. With $H = 2$ and $\gamma = 1$, from equation (4.49), we need to have the following:

$$\alpha H - \gamma(H - 1) = 1 \rightarrow \alpha = 1 \text{ and } \beta H - \gamma(H - 1) = 0 \rightarrow \beta = 0.5 \quad (8.48)$$

Application of equation (8.48) gives the following element values:

$$R^* = \frac{QR}{\beta} = 40 \text{ k}\Omega, R_6 = \frac{Q}{(1-\beta)}R = 40 \text{ k}\Omega, R_7 = \frac{R}{\gamma} = 5 \text{ k}\Omega, R_5 = \frac{R}{1-\gamma} = \text{open}$$

$$R_1 = R_3 = R_4 = 5 \text{ k}\Omega, C^* = \alpha C = 0.01 \mu\text{F} \text{ and } C^{**} = (1 - \alpha)C = 0$$

Figure 8.20(a) shows the circuit structure of the notch filter with element values and Figure 8.20(b) shows the PSpice simulated magnitude response. For the simulated notch which occurs at 3.169 kHz (19.91 krad/s), the output voltage level is 2.396 mV for an input voltage of 100 mV; an attenuation of 32.4 dBs. Its 3 dB bandwidth is 799.8 Hz, resulting in $Q = 3.96$.

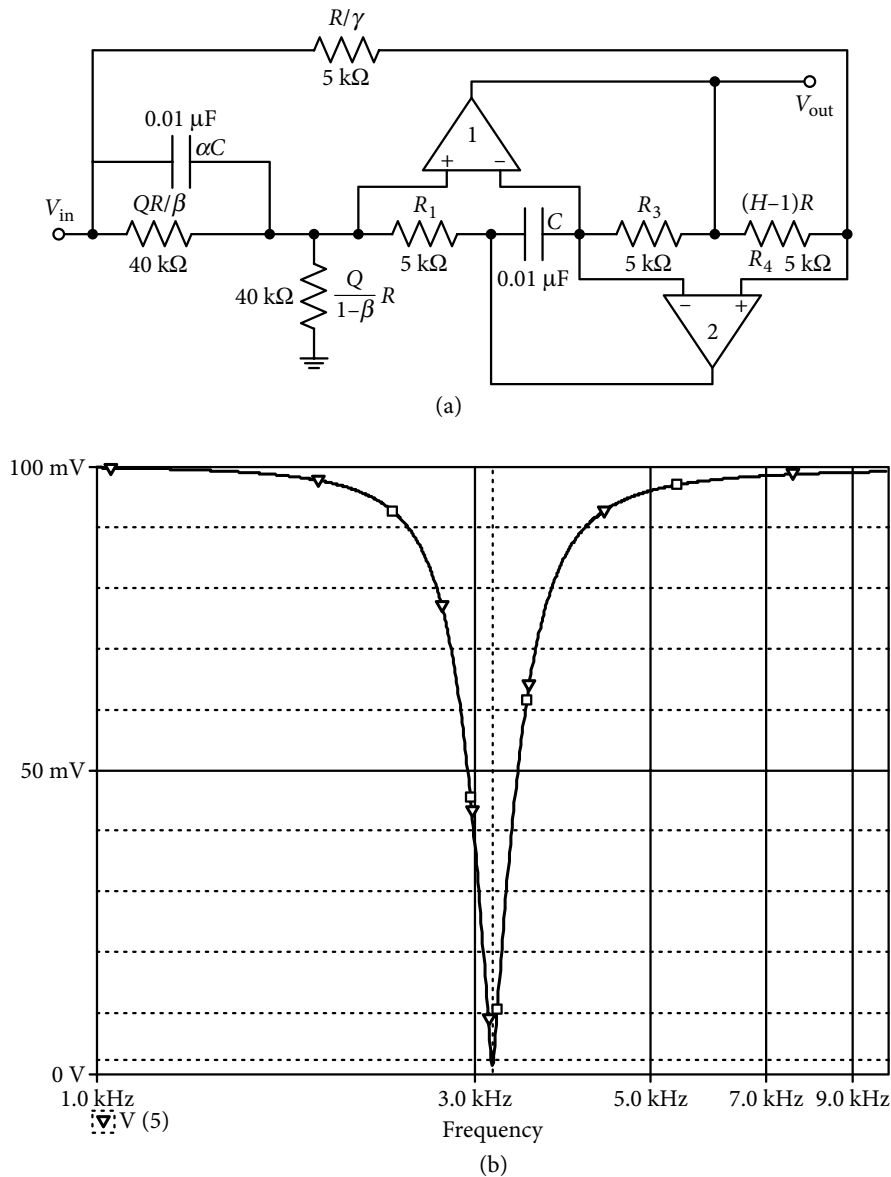


Figure 8.20 (a) Notch filter circuit obtained from the GIC circuit of Figure 8.17(b), while lifting some elements from ground partially. (b) Notch filter response for the circuit in Figure 8.20(a).

Example 8.9: Design an APF which has a phase shift of 180° at 40 krad/s and $Q = 2$ using GIC.

Solution: For the multifunctional configuration of Figure 8.19, to give an AP response, we need to have the following conditions from equation (8.47):

$$\alpha H - \gamma(H - 1) = 1, H - 1 = 1 \rightarrow H = 2 \text{ and } \beta H - \gamma(H - 1) = -1 \quad (8.49)$$

In equation (8.49), if $\gamma = 1$, $\beta = 0$ and $\alpha = 1$. For center frequency of 40 krad/s, if C_2 is selected as 5 nF, $R = 5 \text{ k}\Omega$, and with $Q = 2$:

$$R^* = \frac{QR}{\beta} = \infty, R_6 = \frac{QR}{1-\beta} = 1 \text{ k}\Omega, R_4 = (H-1)R = 5 \text{ k}\Omega, R_5 = \frac{R}{1-\gamma} = \infty, R_7 = \frac{R}{\gamma} = 5 \text{ k}\Omega,$$

$$R_1 = R_3 = 5 \text{ k}\Omega \text{ and } C^* = C_2 = 5 \text{ nF}, C^{**} = (1-\alpha)C_2 = 0 \quad (8.50)$$

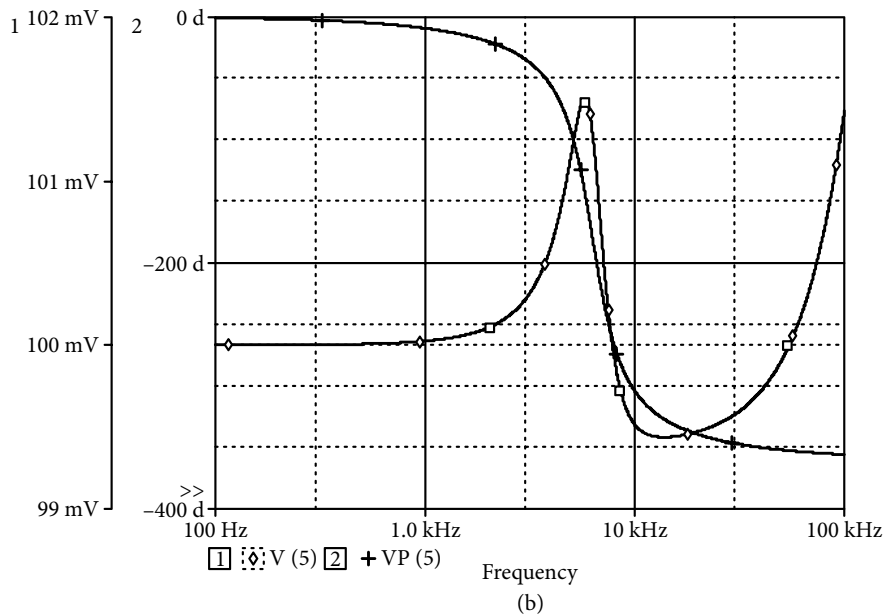
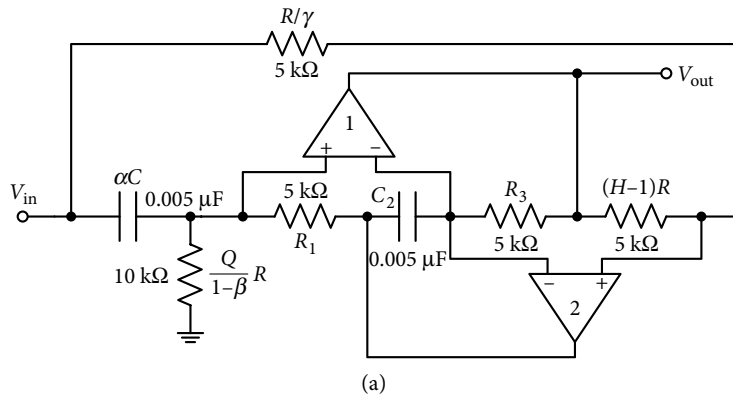


Figure 8.21 (a) All pass circuit obtained using GIC circuit of Figure 8.17(b). (b) Magnitude and phase responses of the all pass shown in Figure 8.21(a).

Figure 8.21(a) shows the circuit structure of the APF with element values and Figure 8.21(b) shows the PSpice simulated magnitude response. Gain is almost unity; there is a bit of rise having maximum gain of 1.0148 and a small drop for a minimum gain of 0.9943. Figure 8.21(b) also shows the phase response of the APF, having a phase shift of -180° at 6.3 kHz (39.6 krad/s).

References

- [8.1] Kerwin, W. J., L. P. Huelsman, and R. W. Newcomb. 1967. 'State-Variable Synthesis for Insensitive Integrated Transfer Functions,' *IEEE Journal of Solid-State Circuits* SC-2: 87-92.
- [8.2] Tow, J. 1969. 'A Step-by-Step Active Filter Design,' *IEEE Spectrum* 6: 64-8.
- [8.3] Thomas, L. C. 1971. 'The Biquad: Part I-Some Practical Design Considerations; Part II-A Multipurpose Active Filtering System,' *IEEE Transactions on. Circuit Theory* CT-18: 350-7.
- [8.4] Ackerberg, D., and K. Mossberg. 1974. 'A Versatile Active RC Building Block with Inherent Compensation for the Finite Bandwidth of the Amplifier,' *IEEE Transactions on Circuits and Systems* 21: 758.4.
- [8.5] Hall, D. L., and B. Flatua. 1998. 'On Analog Feedback Control for Magneto-strictive Transducer Linearization,' *Journal of Sound and Vibration* 211: 481-94.
- [8.6] Antila, M., K. Hakanen, and J. Kataja. 2002. 'Microcontroller-Driven Analogue Filter for Active Noise Control.' Southampton, UK: *ISVR, ACTIVE*.
- [8.7] Antoniou, A. 1967. 'Gyrators Using Operational Amplifiers,' *Electron Letters* 3: 350-2.

Practice Problems

- 8-1 Figure P8.1 shows the circuit diagram of a KHN biquad which was shown in Figure 8.1 as a block diagram form. Obtain all the three voltage ratio transfers functions available from the circuit. What kinds of responses are available?

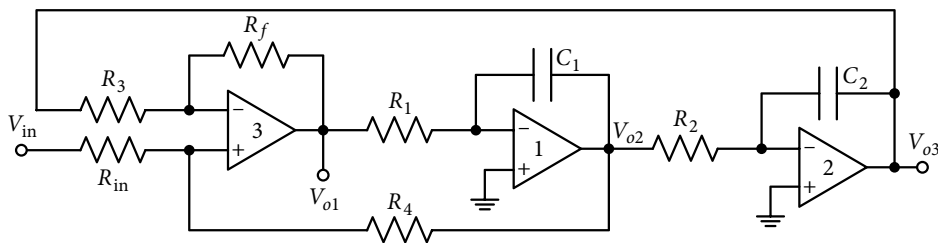


Figure P8.1

- 8-2 Design and test a second-order BP filter using a KHN circuit having center frequency of 3.4 kHz, $Q_o = 3$ and mid-band gain of 5.
- 8-3 Design and test a HP filter using KHN circuit for which attenuation falls by 2 dB at 10 krad/s.

- 8-4 Design a KHN based second-order LP filter for cut-off frequency of 10 krad/s and $Q = 5$. Use equal value capacitors C_1 and C_2 and equal valued resistors R_1 and R_2 . What is the gain of the filter at dc?
- 8-5 Design and test the second-order Tow–Thomas (TT) BP circuit of Figure 8.3 for the following specifications.
 $\omega_0 = 20$ krad/s, $Q_0 = 10$ and peak gain of 10.
- 8-6 Derive the transfer function (V_{out}/V_{in}) for the circuit shown in Figure P8.2. Design a filter, using equal value capacitors for the following transfer function:

$$\frac{V_{out}}{V_{in}} = \frac{s^2 + 0.25}{s^2 + 0.2s + 1.21}$$

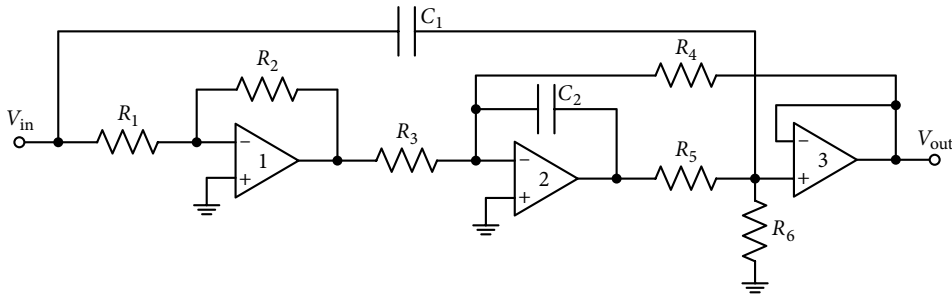


Figure P8.2

- 8-7 (a) Derive expressions for the three transfer functions for the Tow–Thomas structure shown in Figure P8.3, with OAs considered ideal.
- (b) Find the incremental sensitivity of the parameters ω_0 and Q_0 with respect to the passive elements.

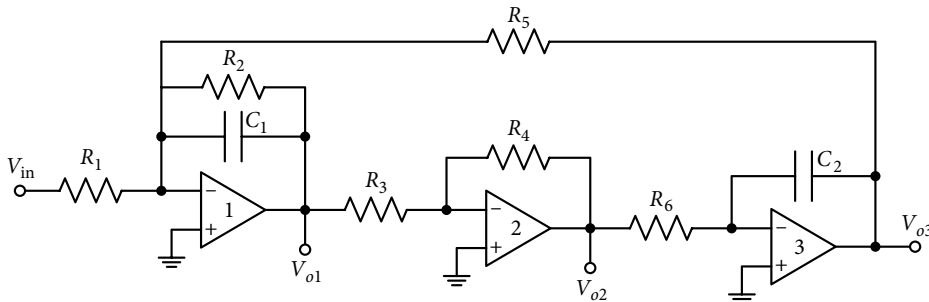


Figure P8.3

- 8-8 If the OAs are represented by the model $A(s) = B/s$, even lossless integrators become lossy. Figure P8.4 shows the representation of the two-integrator loop biquad in such a case. Show that the normalized transfer function with $\omega_0 = 1$ and $k = H(\omega_0/Q)$, where H is the mid-band gain:

$$\frac{V_{out}}{V_{in}} = \frac{\frac{H}{Q\tau_1\tau_2}(s\tau_2 + q_2)}{s^2 + \left(\frac{q_1}{\tau_1} + \frac{q_2}{\tau_2} + \frac{1}{Q\tau_1}\right)s + \left(1 + q_1q_2 + \frac{q_2}{Q}\right)\frac{1}{\tau_1\tau_2}}$$

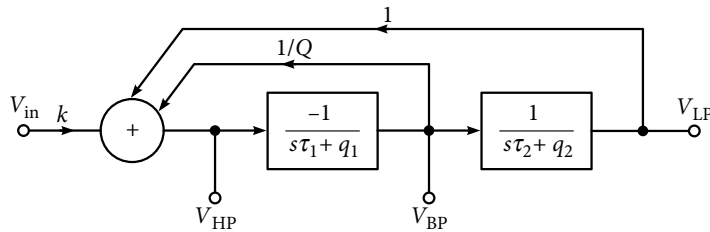


Figure P8.4

- 8-9 Compare the transfer function in Problem 8-8 with a standard form of BP transfer function and (a) show that the resultant equivalent parameters will be:

$$\omega_{oe} = \left\{ \left(1 + q_1 q_2 + \frac{q_2}{Q} \right) \frac{1}{\tau_1 \tau_2} \right\}^{1/2}$$

$$Q_{oe} = Q \frac{\omega_{oe} \tau_1}{1 + Q(q_1 + q_2 \tau_1 / \tau_2)}, \quad H_e = H \frac{1}{1 + Q(q_1 + q_2 \tau_1 / \tau_2)}$$

- (b) What is the magnitude of a parasitic zero in the transfer function?

- 8-10 Design a TT BP filter shown in Figure 8.3 for $\omega_o = 60$ krad/s and $Q_o = 15$ with mid-band gain H being unity. Estimate the deviation in the parameters of the filter if OAs have bandwidth $= 0.5 \times 10^6$ rad/s. Verify the results using PSpice simulation.
- 8-11 Repeat Problem 8-10 for $\omega_o = 120$ krad/s, $Q = 20$ and mid band gain $H = 1$.
- 8-12 Repeat Problems 8-10 and 8-11 using the Ackerberg–Mossberg (AM) circuit shown in Figure 8.7(a).
- 8-13 Design LP filters using the AM circuit shown in Figure 8.7(a) for the following specifications using 741 OAs.
- $\omega_o = 3.4 \times 2\pi$ krad/s, $Q = 1$ and gain at dc $h_{lp} = 5$.
 - $\omega_o = 60$ krad/s, $Q = 2.5$ and gain at dc $h_{lp} = 2$.
 - $\omega_o = 200$ krad/s, $Q = \sqrt{2}$ and gain at dc $h_{lp} = 3$.
- 8-14 Using 741 OAs, design BP filters using the AM circuit shown in Figure 8.7(a) for the following specifications.
- $\omega_o = 1.59 \times 2\pi$ krad/s, $Q = 10$ and mid-band gain $h_{bp} = 2$.
 - $\omega_o = 50$ krad/s, $Q = 5$ and mid-band gain $h_{bp} = 5$.
 - $\omega_o = 250$ krad/s, $Q = 12$ and mid-band gain $h_{bp} = 1$.
- 8-15 Design and test a notch filter for the following transfer function using a two-integrator network and a summer configuration shown in Figures 8.8 and 8.9. The two-integrator loop filter needs to be a KHN type. Also find the sensitivity of the parameters ω_o and Q_o , and ω_z and Q_z with respect to the passive elements used.

$$T(s) = \frac{s^2 + 10^{10}}{s^2 + 5 \times 10^4 s + 10^{10}}$$

- 8-16 Repeat Problem 8-15 employing a TT circuit in place of a AM circuit.
- 8-17 Repeat Problem 8-15 using the modified summation method shown in Figure 8.12, where AM biquad is employed.
- 8-18 Repeat Problem 8-15 using the GIC based biquadratic circuit shown in Figure 8.19.
- 8-19 Design and test a notch filter for the following transfer function using a two-integrator network and a summer configuration shown in Figures 8.8 and 8.9. The two-integrator loop filter needs to be a KHN type. Also find the sensitivity of the parameters ω_o and Q_o , and ω_z and Q_z with respect to the passive elements used.

$$H(s) = \frac{s^2 + 0.25 \times 10^8}{s^2 + 0.35 \times 10^4 s + 0.49 \times 10^8}$$

- 8-20 Repeat Problem 8-19 employing a TT circuit in place of a KHN circuit.
- 8-21 Repeat Problem 8-19 using the modified summation method shown in Figure 8.12, where AM biquad is employed.
- 8-22 Repeat Problem 8-19 using the GIC based biquadratic circuit shown in Figure 8.19.
- 8-23 Design and test a LP notch filter for the following transfer function using a two-integrator network and the summer configuration shown in Figures 8.8–8.9. The two-integrator loop filter needs to be a KHN type.

$$H(s) = \frac{s^2 + 1.44 \times 10^8}{s^2 + 0.35 \times 10^4 s + 0.49 \times 10^8}$$

- 8-24 Repeat Problem 8-23 using the modified summation method shown in Figure 8.12, where AM biquad is employed.