Chapter 10

Cascade Approach: Optimization and Tuning

10.1 Introduction

In Chapter 2, a number of first-order active filters were realized along with the basic configuration of bilinear functions. Chapters 7 and 8 were devoted to the development of second-order filter sections using one or more than one OA (operational amplifier). However, as mentioned earlier, all filter specifications are not achievable only through second-order sections; higher-order filters become necessary. For the realization of higher-order filters, ladder simulation techniques through element (inductor/frequency dependent negative resistor: FDNR) substitution was discussed in Chapter 9. Ladder simulation using signal flow graph technique, which is better known as the *operationally simulated* method was also discussed in Chapter 9. The present chapter deals with another basic method of realizing higher-order filter sections known as the *cascade design method*.

Section 10.2 will discuss the basics of cascade design and the conditions to be satisfied by first- or second-order sections in order that these could be used for cascading and obtaining higher-order filters. After taking up some examples of cascade design, the importance of *cascade optimization* will be discussed in Section 10.3 through an example section. While cascading a number of second-order sections, a proper combination of poles and zeroes (Section 10.3.1), correct assignment of gain for each section (Section 10.3.2) and their proper order (Section 10.3.2) play a very crucial role. Hence, all the three aspects will be discussed in some details.

It is well-known that due to the tolerance associated with passive elements as well as with the parameters of the active elements, and their possible variation due to the change in the biasing voltage and operating temperature, filter parameters gets deviated. Therefore, it is imperative to provide on-chip tuning of the parameters, especially the pole frequency ω_o and pole-Q of the individual second-order sections used in the cascade. This chapter introduces the idea of filter parameter tuning. In Section 10.4, we get a better insight into the cascade design method of filter design.

10.2 Cascade Design Basics

To realize higher-order filters, cascading of second-order sections finds considerable favour in the eyes of a majority of filter designers; a first-order section is also cascaded in case of an odd-order filter. It will be shown later that such a technique has few advantages like the ability to tune filter parameters easily, and that it possess controlled tunability of the filter parameters with respect to the elements used.

Lower-order building blocks are connected as shown in Figure 10.1. Over all, the transfer function is simply the product of the transfer functions of the individual building blocks as shown here.

$$H(s) = H_1(s) \times H_2(s) \times \ldots\ldots \times H_n(s) \tag{10.1}$$

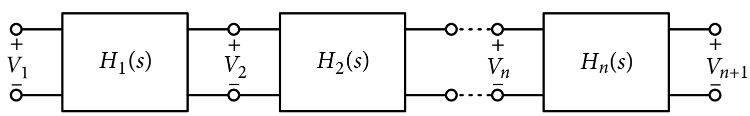

Figure 10.1 Cascading arrangement of n number of two-port networks.

Here, the transfer function $H_i(s) = V_{i+1}(s)/V_i(s)$ can be first-order; most often, second-order sections are used. However, this relation and the relation in equation (10.1) will be valid only when the individual sections are non-interactive, that is, the preceding sections do not load the previous section. From the basic knowledge of two-port networks, we know that perfect non-interactiveness will be achieved when the output impedance of each building block (Z_{oi}) is zero and their input impedances (Z_{ij}) are infinity. In practice, due to the finiteness of the input and output impedances of the individual sections, there will be some interaction between the two-port networks; hence, some deviation in the overall transfer function from the design is likely to occur. When two-port networks (second-order or first-order) are realized using OAs, it is always desirable to get its output at the output terminal of the amplifier. The output impedance of the OA being practically small, OAs are suitable for cascading. When passive two-port networks or an output terminal with higher impedance is to be cascaded, it is advisable to insert a buffer in between the networks as shown in Figure 10.2.

There are a number of advantages in cascade design method of higher-order filters. As the pole-Q and the critical frequency ω_o of each second-order section depends on one pole pair and the nature of the second-order filter depends on one zero (pair), it becomes easier to tune these parameters, which is very difficult in direct realization methods. Since individual second-order sections can be tuned easily, the overall tuning of response also becomes easy. It is to be

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noted that as only second-order sections are needed for higher-order filters, the best possible second-order section with all possible optimizations for that particular application can be used. For example, the selected section may have minimum possible sensitivity, and variability, with respect to the elements used.

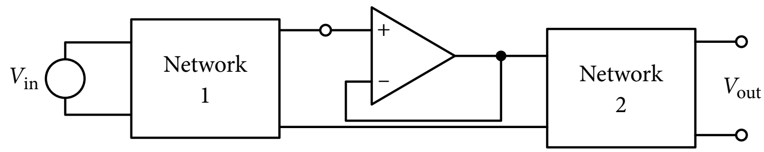

Figure 10.2 Insertion of buffer between two interactive two-port networks.

When a transfer function of order n, as given in equation (1.1) and repeated here, is to be realized, it is known that $n \geq m$. For n being even, there shall be $(n/2)$ pole pairs and $(n/2)$ second-order sections shall be connected in cascade.

$$H(s) = \frac{V_{out}}{V_{in}} = \frac{b_m s^m + b_{m-1}s^{m-1} + \ldots\ldots + b_1 s + b_0}{s^n + a_{n-1}s^{n-1} + a_{n-2}s^{n-2} + \ldots\ldots + a_1 + a_0} \tag{10.2}$$

If n is odd, $(n - 1)/2$ second-order sections and one first-order section will be connected in cascade. In general, the transfer function of each second-order function will be expressed as:

$$H_i(s) = k_i \frac{\alpha_{2i}s^2 + \alpha_{1i}s + \alpha_{0i}}{s^2 + (\omega_{oi}/Q_i)s + \omega_{oi}^2} = k_i h_i(s) \tag{10.3}$$

Obviously, all second-order functions may not have the numerator coefficients α_2, α_1 and α_0 as finite. It will depend on the number of finite zeros and the type of response of that particular section.

Let us go through the basics of the cascade approach with the help of some simple examples.

Example 10.1: Realize a Butterworth LPF (low pass filter) which will satisfy the following specifications using the cascade form of synthesis.

$$\alpha_{max} = 1 \text{ dB},\ \alpha_{min} = 40 \text{ dBs},\ \omega_1 = 2000 \text{ rad/s},\ \omega_2 = 6000 \text{ rad/s}$$

Solution: In Example 3.1, order of a filter in the form of a lossless passive ladder was found as 5 for the desired specification. For the fifth-order Butterworth filter, using the values of pole locations from Table 3.1 and with the cut-off frequency (de-normalization) given as $\omega_c = \omega_{CB} \times \omega_1 \cong 1.144 \times 2000 = 2288$ rad/s, the following forms of transfer function were obtained:

$$H(s) = \frac{1}{(s + 2288)(s^2 + 1414s + 2288^2)(s^2 + 3701.98s + 2288^2)} \tag{10.4}$$

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$$= \frac{1}{s^5 + 7.408\times10^3 s^4 + 2.741\times10^7 s^3 + 6.2714\times10^{10} s^2 + 9.11478\times10^{13} s + 6.27018\times10^{16}} \quad (10.5)$$

Obviously, the next step is to find an active network topology containing suitable active devices, and values of the passive elements used. To realize higher-order filters using the cascade form, the transfer function of equation (10.4) needs to be broken to form a product of three transfer functions. Hence, a first-order function $H_1(s)$ and two second-order functions $H_2(s)$ and $H_3(s)$, with respective dc gains of k_1, k_2 and k_3 will be used. The resulting overall transfer function will be:

$$H(s) = H_1(s) \times H_2(s) \times H_3(s) \quad (10.6)$$

where expressions of the three transfer functions obtained from equation (10.4) will be:

$$H_1(s) = \frac{2288k_1}{s + 2288} \quad (10.7)$$

$$H_2(s) = \frac{2288^2 k_2}{s^2 + 1414s + 2288^2} \quad (10.8a)$$

$$H_3(s) = \frac{2288^2 k_3}{s^2 + 3702s + 2288^2} \quad (10.8b)$$

Obviously, the dc gain of the product of the three transfer functions will become $(k_1 \times k_2 \times k_3)$, which should be equal to 1; the overall dc gain of the transfer function $H(s)$. Arbitrary values can be assigned to the individual dc gains in order to get their product as 1. However, one easy and convenient choice in the beginning is to make all three dc gains as unity (later, we shall see that the choice of dc gains for individual sections is not arbitrary for good designs).

The first-order transfer function of equation (10.6) with $k_1 = 1$ can be realized using an active section, along the lines followed in Section 2.3.1, as shown in Figure 10.3(a). Its transfer function is given as:

$$H_{11}(s) = \frac{1/CR_1}{s + 1/CR_2} \quad (10.9)$$

If the selected value of capacitor $C = 0.1$ μF, to get $k_1 = 1$, $R_1 = R_2 = 4.37$ kΩ in equation (10.7).

The remaining two second-order sections are realized using the circuit shown in Figure 2.15, redrawn in Figure 10.3(b), for which the transfer function is repeated here:

$$\frac{V_{out}}{V_{in}} = -\frac{(G_1G_3/C_1C_2)}{s^2 + s\{(G_1 + G_2 + G_3)/C_1\} + (G_2G_3/C_1C_2)} \quad (10.10)$$

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It has the expressions for ω_o, dc gain and Q as:

$$\omega_o^2 = \left(\frac{G_2 G_3}{C_1 C_2}\right), \text{ dc gain} = \frac{G_1}{G_2}, \text{ and } Q = \frac{C_1}{G_1 + G_2 + G_3}\left(\frac{G_2 G_3}{C_1 C_2}\right)^{1/2} \tag{10.11}$$

For the transfer function $H_2(s)$ of equation (10.8a), it is compared with equation (10.10). Selecting $C_1 = 0.1$ μF and with $k_2 = 1$, use of equation (10.11) gives the value of $G_1 = G_2 = G_3 = 0.1244$ mA/V, $C_2 = 0.0291$ μF; so hence, $R_1 = R_2 = R_3 = 8.101$ kΩ.

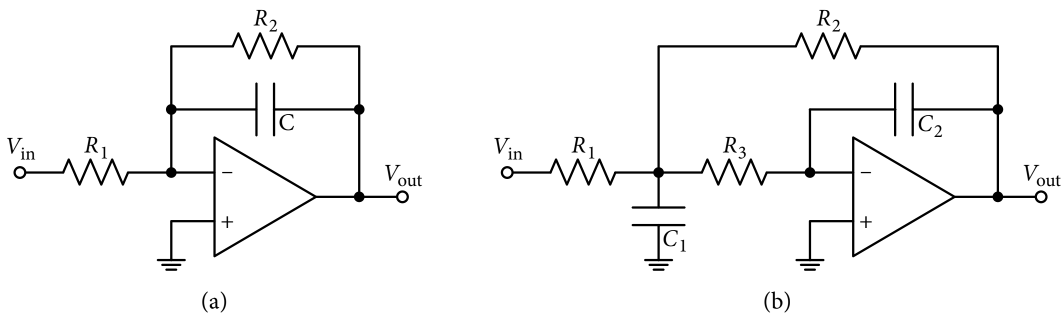
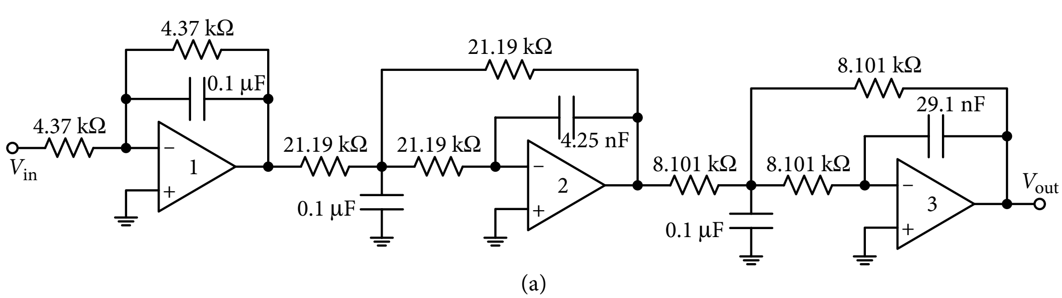


Figure 10.3 (a) An active first-order low pass circuit and (b) a second-order low pass filter circuit.

Similarly, for $H_3(s)$, element values are $C_1 = 0.1$ μF, $C_2 = 4.252$ nF, $R_1 = R_2 = R_3 = 21.196$ kΩ.

Three circuits are connected in cascade to get the overall transfer function. The complete circuit, having a sequence of sections as H_1 followed with H_3 and then H_2, along with the element values is shown in Figure 10.4(a). The simulated magnitude response is shown in Figure 10.4(b). It is observed that the output to input voltage ratio at 2000 rad/s is 0.889 or the attenuation is 1.012 dB, which is just above the design value of 1 dB. At 6000 rad/s, the output to input voltage ratio is 0.008106 which is equivalent of 41.2 dB attenuation; more than the design value of 40 dBs. However, there is a peak with a voltage of 1.28 volt at the output of the second stage. If the final output is limited to 10 volts, then the maximum input should not exceed 7.81 volts.



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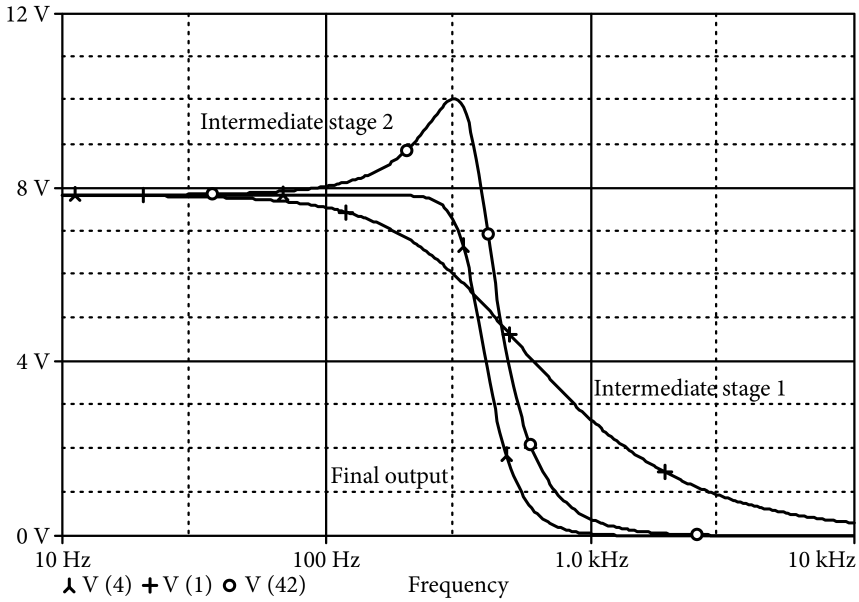
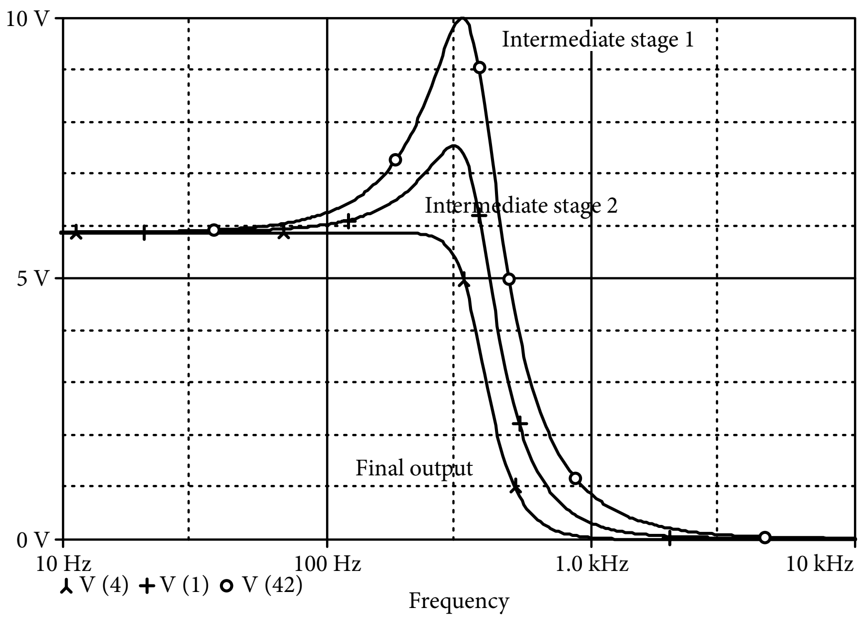


(b)



(c)

Figure 10.4 (a) Fifth-order low pass Butterworth filter using the cascade process for Example 10.1 and (b) the simulated response at intermediate and output nodes. (c) Response using cascade process with different sequence of sections for the circuit of Example 10.1.

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If the sequence of sections is changed arbitrarily so that H_3 is followed by H_1 and then H_2, the simulated response is shown in Figure 10.4(c). The final output response is unaffected, but an intermediate peak has a voltage level of 1.7 volts, which means that with this sequence of sections, the allowable input reduces to 5.88 volts for a maximum output voltage of 10 volts. The effect of selection of another sequence (correct) will be taken up later.

Example 10.2: Realize an LPF with the following specifications using Chebyshev approximation. Obtain the frequency response while realizing it through the cascade process.

$$\alpha_{max} = 0.5 \text{ dB}, \ \alpha_{min} = 40 \text{ dBs}, \ \omega_1 = 2000 \text{ rad/s}, \ \omega_2 = 6000 \text{ rad/s} \tag{10.12}$$

Solution: In Example 3.3, the required filter order was obtained as 4 for the given specifications. The following was the normalized transfer function.

$$H(s) = \frac{0.3577}{\left(s^2 + 0.3508s + 1.0636\right)(s^2 + 0.8466s + 0.3563)} \tag{10.13}$$

Since for an even-order transfer function $H(0) = \alpha_{max} = 0.5$ dB or 0.944 (normalized), the value of the numerator in $H(s)$ becomes $(0.944 \times 1.0636 \times 0.3563) = 0.3577$ for maximum pass band gain of unity.

As frequency de-normalization is to be done by 2000 rad/s, the de-normalized transfer function is obtained as:

$$H(s) = \frac{0.3577 \times 2000^2}{\left(s^2 + 0.3508 \times 2000s + 1.0636 \times 2000^2\right)(s^2 + 0.8466 \times 2000s + 0.3563 \times 2000^2)} \tag{10.14}$$

As in Example 10.1, the transfer function of equation (10.14) is broken into the following two second-order LP functions:

$$H_1(s) = \frac{1.0636 \times 2000^2}{\left(s^2 + 0.3508 * 2000s + 1.0636 \times 2000^2\right)} \tag{10.15}$$

$$H_2(s) = \frac{0.3563 \times 0.944 \times 2000^2}{(s^2 + 0.8466 \times 2000s + 0.3563 \times 2000^2)} \tag{10.16}$$

Both the transfer functions $H_1(s)$ and $H_2(s)$ are realized using the circuit shown in Figure 10.3(b) whose transfer function and expressions of parameters are given by equations (10.10) and (10.11), respectively.

For transfer function $H_1(s)$, assuming $G_{21} = 10^{-4}$ mho (second subscript 1 corresponds to the first biquad) as the dc gain is 1.0, we get:

$$(G_{11}/G_{21}) = 1; \text{ hence, } R_{11} = R_{21} = 10 \text{ k}\Omega \tag{10.17}$$

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Without losing generality, selecting R_{31} = 10 kΩ as well, with (ω_o/Q) = 0.3508 × 2000 and $\omega_o^2 = 1.0636 \times 2000^2$, application of equation (10.11) gives C_{11} = 0.4276 μF and C_{21} = 0.005498 μF.

For the transfer function $H_2(s)$ (with second subscript 2), assuming $G_{22} = G_{32} = 10^{-4}$ mho, and with dc gain being 0.944, we get:

$$(G_{12}/G_{22}) = 0.944 \text{ or } R_{22} = 10\text{ k}\Omega,\ R_{32} = 10\text{ k}\Omega,\ R_{12} = 10.593\text{ k}\Omega \tag{10.18}$$

Similar to the function $H_1(s)$, application of equation (10.11) for $H_2(s)$ gives C_{12} = 0.17387 μF and C_{22} = 0.04035 μF.

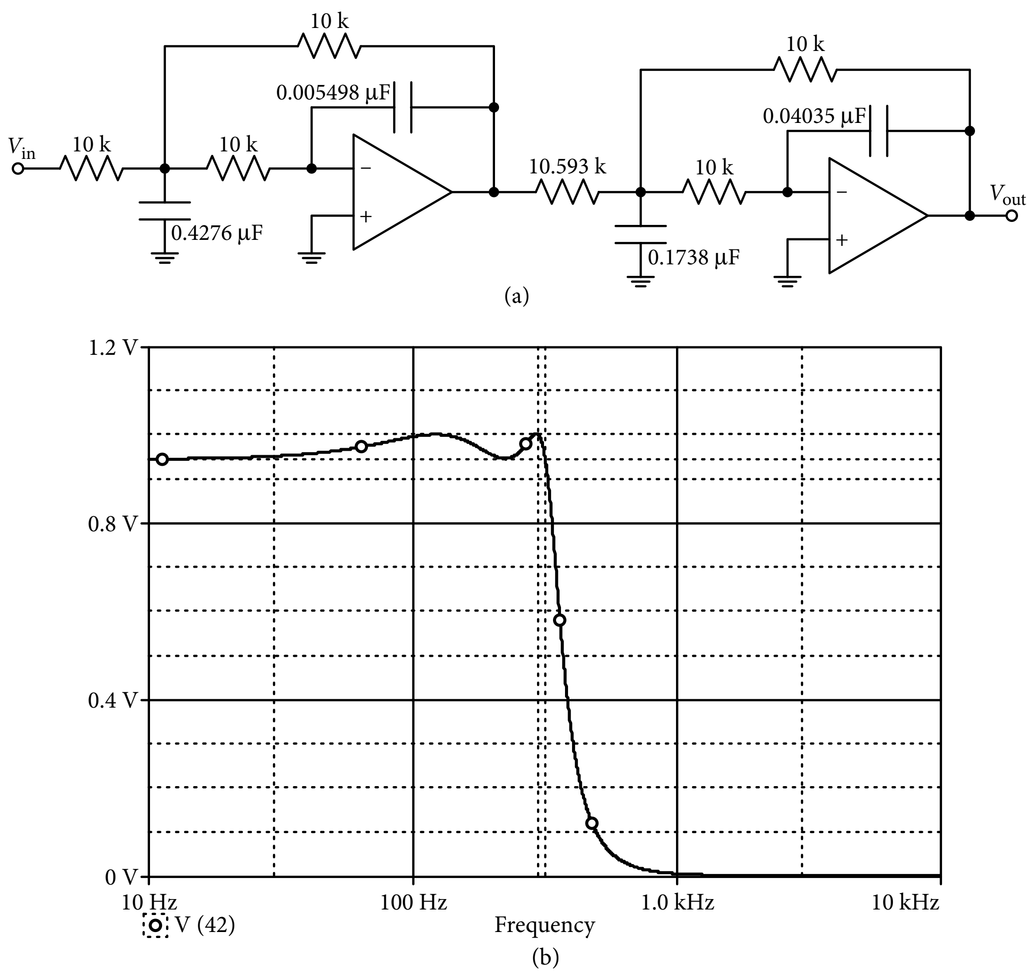

Figure 10.5 (a) Fourth-order low pass Chebyshev filter using cascade process for Example 10.2; (b) its simulated magnitude response.

Combining the circuit implementation of the transfer functions $H_1(s)$ and $H_2(s)$ in cascade, the overall circuit along with element values is shown in Figure 10.5(a). Its magnitude response

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through simulation is shown in Figure 10.5(b). The observed pass band edge frequency is 318.2 Hz or 2000.1 rad/s. Maximum output voltage at peaks being 1.002 V for input voltage of 1 volt and at the pass band edge frequency, voltage level is 944 mV, corresponding to an attenuation of 0.5 dB; the attenuation becomes 46.1 dBs at 6000 rad/s. DC gain of the filter is also 0.944, which corresponds to 0.5 dB. The circuit satisfies attenuation requirements at pass band and stop band edge frequencies with enough margin and shows maxima and minima as expected.

10.3 Optimization in Cascade Process

With the help of Examples 10.1 and 10.2, it is clear that process of cascading is simple. The main issue is to get an appropriate second-order section for each second-order function (and a first-order section, if needed). Unfortunately, actual implementation while cascading even a few second-order sections involves some serious considerations, which is important not only for the optimization of the performance, but also the correct functioning of the overall higher-order filter section.

For the simple fourth-order function of Example 10.2, two transfer functions $H_1(s)$ and $H_2(s)$ were shown to be cascaded. While forming these transfer functions, pole pairs and zeros were combined arbitrarily. Obviously, there is more than one possible combination that will result in the fourth-order filter. While cascading three sections, there are six possible combinations of pole pairs and zeros and many more for a larger number of sections to be cascaded. It will be shown in the next section that assignment of zeros with a pole pair should not be arbitrary as it affects the performance. A criterion has to be evolved for the proper combination of pole pairs and zeros.

Once the proper combination of poles and zeros is accomplished, there is more than one possibility in assigning the order in which the sections will be cascaded. Again, for the sixth-order function, there are the following six possible combinations in which the blocks may be cascaded.

$$H_1 H_2 H_3,\ H_1 H_3 H_2,\ H_2 H_1 H_3,\ H_2 H_3 H_1,\ H_3 H_1 H_2,\ H_3 H_2 H_1 \tag{10.19}$$

Unless specified, one may wonder why the order of cascading is important. In fact, along with the third issue, which will be explained soon, proper ordering of second-order sections significantly affects the working of the overall filter and may deviate its performance drastically.

The third important issue is the assignment of gain for individual two-port blocks. Since the overall gain will be the product of the gain of the individual blocks, a designer might like to assign it arbitrarily to each block. However as mentioned earlier, the assignment of gain to individual blocks along with its ordering has to be done very carefully.

The reasons behind these considerations of pole–zero pairing, ordering of the sections and assignment of gain to the sections are two-fold. The first consideration is to check that the signal level at any internal or external node does not exceed or reach the saturation level of the active device decided by the level of the supply voltage or due to the constraint of the slew rate.

If this happens, harmonics will get generated and the resulting filter parameters will deviate from the design parameters. The second consideration is to maximize the dynamic range: the ratio of the undistorted signal to the noise level present in the system at each cascading stage. The basic principle used for maximizing the dynamic range is to *maximize the minimum level of the signal in the pass band* of each second-order filter section so that the effect of noise is minimum on the signal. This action produces a flatter type of response in which the ratio of maximum signal at any node to the minimum signal comes as close to unity as possible. In addition to the aforementioned considerations, the maxima of the output voltage for individual sections should also be made equal as will be shown later.

The idea explained here can be expressed mathematically as well, which forms the starting point of developing algorithms and computer programs for the purpose. Let $V_{o,\max}$ be the level of output voltage of the OAs used, which is the upper limit of the signal level set by the power supply level or the slew rate at all frequencies including both the pass and stop band. Then the maximum magnitude of the output signal for each individual biquad $|V_{oj}(j\omega)|$ should satisfy the following condition:

$$\max |V_{oj}(j\omega)| < V_{o,\max} \text{ for } 0 \le \omega \le \infty \tag{10.20}$$

The condition imposed by equation (10.20) needs to remain valid for all signal frequencies falling in the pass and stop band; the reason being that if any signal is overdriven even in the stop band, it may generate harmonics, which may interfere with the valid output.

On the other hand, it is necessary to check that the signal does not become so small at the intermediate stage or individual block output level that it gets corrupted by the circuit noise. If the signal to noise ratio becomes small, the signal becomes indistinguishable from the noise. Hence, another condition which is required to be fulfilled is that the smaller signals at all outputs of the biquad should be enlarged in the pass band as much as practically possible.

$$\min |V_{oj}(j\omega)| \rightarrow \text{maximize in the pass band} \tag{10.21}$$

The condition imposed by equation (10.21) needs to be valid only in the pass band; it is not required in the stop band as the smallness of signal there will not do any harm.

The brief discussion in this section and the conditions given in equations (10.20) and (10.21) imply that higher magnitude signals need to be pulled down to remain below $V_{o,\max}$ and smaller magnitude signals need to be amplified. This creates a kind of flatness in signals at the output of the individual second-order section as mentioned earlier. In other words, we need to make the ratio of maximum signal to minimum signal at all the intermediate levels and the final output as small as possible. Later this point will be taken up in mathematical terms.

10.3.1 Pole–zero pairing

In order to explain the idea of *flatness* of signals at all cascaded stages, let V_{ok} be the output voltage at the kth stage. Normally, the output at any stage is taken as the output of the OA. If the biquad uses only one OA, then V_{ok} shall will be its output; however, for a multi amplifier biquad, V_{ok} will be the largest output signal amongst all the amplifiers used. This implies that for the kth stage, there will be no signal greater than V_{ok}. Therefore, as discussed in the previous section, poles and zeros should be paired in such a way that the signal maximum $M_k = \max|V_{ok}(j\omega)|$ is minimized below $V_{o,\max}$ for all the input signal frequency range of pass and stop bands. At the same time, the signal minimum $m_k = \min|V_{ok}(j\omega)|$ is maximized in the pass band. Maximization of m_k and minimization of M_k means that $|h_k(j\omega)|$ in equation (10.3) should be as flat as possible in the frequency range of interest. For a mathematical treatment, *measure of flatness* of the signal can be expressed as:

$$d_k = (M_k/m_k) \rightarrow 1 \text{ with } k = 1, 2, \ldots\ldots, n \tag{10.22}$$

Obviously, the intention is to minimize d_k and the assignment of zeros to a pole pair in equation (10.3) should be such that it minimizes the maximum value of d_k, or:

$$d_{\max} = \max[d_k] \rightarrow \text{minimum for } k = 1, 2, \ldots\ldots, n \tag{10.23}$$

For comparatively smaller order filters say (4 or 5), evaluation of d_k, though laborious, can be done in a reasonable time period and then equation (10.23) satisfied. However, for higher-order filters, manual evaluation of d_k becomes highly time consuming and requires use of computer and an appropriate software. Fortunately, for quite a few cases, instead of taking recourse to computer programs, decisions based on intuition and experience became helpful. For example, when the transfer function has more than one zero at the origin, the type of filter section can be of different types for the same final transfer function. For example, for a numerator having a term s^2, the possible combinations can be s^2 and 1, or s and s, which will mean that realization can be in the form of a combination of a HP (high pass) and a LP (low pass) or a combination of two BP (band pass) sections, respectively. However, a much more important consideration comes in the form of a thumb rule of *forming pole–zero pair combinations which are closest to each other*. This thumb rule gets its idea from the fact that when the value of the combined pole and zero is close to each other, magnitude of the section will be a minimum; which is our aim.

Example 10.3: To design a sixth-order filter in cascade form, find the optimum pole–zero pair combination for the following transfer function:

$$H(s) = \frac{s(s^2+0.25)(s^2+2.25)}{(s^2+0.09s+0.83)(s^2+0.1s+1.18)(s^2+0.2s+1.01)} \tag{10.24a}$$

Solution: Transfer function zeros and poles are:

$$z_1 = 0,\ z_{2,3} = \pm j0.5 \text{ and } z_{4,5} = \pm j1.5 \tag{10.24b}$$

$$p_{1,2} = -0.045 \pm j0.9099,\ p_{3,4} = -0.05 \pm j1.0851 \text{ and } p_{5,6} = -0.1 \pm j1.0 \tag{10.24c}$$

Poles and zeros are shown in Figure 10.6, in the second quadrant of the complex frequency s plane. Applying the thumb rule of combining the nearest poles with zeros, we get the following combination of three second-order sections:

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$$H_A(s)=\frac{(s^2+2.25)}{(s^2+0.1s+1.18)},\ H_B(s)=\frac{(s^2+0.25)}{(s^2+0.09s+0.083)},\ H_C(s)=\frac{s}{(s^2+0.2s+1.01)} \quad (10.25)$$

Each transfer function in equation (10.25) will have to be assigned a gain k_a, k_b and k_c to get the required overall gain.

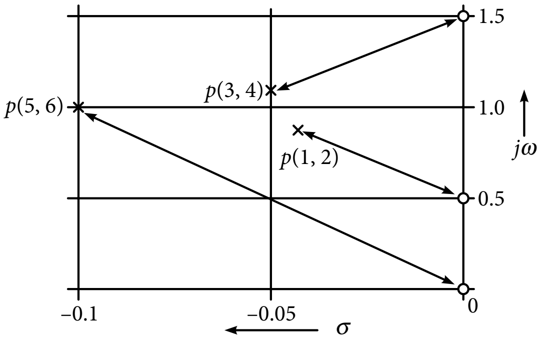

Figure 10.6 Poles and zeroes for the transfer function in Example 10.3.

It is significant to note that when performance of the fifth-order LP of Example 10.1 is to be optimized while using the cascade approach, the pole–zero pairing step is not required as all the zeros are at infinity. In fact, for all pole functions, the optimization process starts from the section ordering.

10.3.2 Section ordering

Once pole–zero assignment is made and n second-order sections are formed, it is required to decide their order in the chain of cascade. Once again, the aim is to get maximum dynamic range, and the procedure is very similar to that adopted for pole–zero pairing. We need to try and keep the variation of the signal for any individual section as flat as possible so that the ratio of the voltage at the output of the kth intermediate section to the input to the first section is as flat as possible.

As the mathematical treatment is similar for all cases, useful algorithms and computer programs have been developed. However, some simpler solutions, which do not require computer programs, are also available. It is generally desirable to keep either a LP or a BP section as the first in the cascade chain in order to suppress high frequency signal components which would have been generated due to the slew rate limitation. Similarly, a high or a BP section is put at the end of the cascade chain in order to suppress any low-frequency noise including dc offset and power supply ripples. However, a much more significant factor which decides the ordering of the sections is the value of the pole-Q of the biquads. Since a larger Q value means a larger peak gain, it is advised to keep the lowest Q section at the beginning of the cascade followed by sections with increasing values of the pole-Q. If Q_i is pole-Q of the ith section in an n section cascade, sections are selected with the following condition:

$$Q_1 < Q_2 < \ldots\ldots < Q_n \text{ for } i = 1, 2 \ldots\ldots, n \quad (10.26)$$

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Example 10.4: For the transfer function of Example 10.3, find the optimum order of the second-order sections as derived in equation (10.25).

Solution: As suggested, a good choice is to put the sections governed by the condition given in equation (10.26). Hence, the quality factor of the three sections are calculated as:

$$Q_A = \frac{\sqrt{1.18}}{0.1} = 10.86, Q_B = \frac{\sqrt{0.83}}{0.09} = 10.12, Q_C = \frac{\sqrt{1.01}}{0.2} = 5.025 \tag{10.27}$$

This means that the most appropriate order of the sections given in equation (10.25) is H_C followed by H_B and then H_A.

For illustration purposes, the transfer function $H(s)$ of equation (10.24) is realized with two different orders of second-order sections; one depending on equation (10.27) and the other a different order. Presently, the gain assigned to each section is taken as unity.

Incidentally, the transfer functions H_C and H_B have already been realized in Chapter 7 as Examples 7.4 and 7.5, respectively, with filter circuits shown in Figures 7.14a and 7.15. In order to realize the overall transfer function, H_A is also designed selecting the *general differential input single OA biquad* of Section 7.4.2, which was used for H_C and H_B as well.

For the general configuration of Figure 7.12, selecting the auxiliary polynomial $Q(s) = s + 1$, we get:

$$\frac{N(s)}{Q(s)} = \frac{s^2 + 2.25}{s+1} = (s + 2.25) - \frac{3.25s}{s+1} \tag{10.28}$$

This gives $y_a = s + 2.25,\ y_1 = \dfrac{3.25s}{s+1}$ (10.29)

$$\frac{D(s) - N(s)}{Q(s)} = \frac{0.1s - 1.07}{s+1} = -1.07 + \frac{1.17s}{s+1} \tag{10.30}$$

This gives $y_b = \dfrac{1.17s}{s+1},\ y_2 = 1.07$ (10.31)

$$\frac{D(s)}{Q(s)} = \frac{(s^2 + 0.1s + 1.18)}{s+1} = s + 1.18 - \frac{2.08s}{s+1} \tag{10.32}$$

This gives

$$y_3 = s + 1.18,\ y_c = \frac{2.08s}{s+1} \tag{10.33}$$

Here, y_1, y_b and y_c are a series combination of resistance and capacitance, y_a and y_3 are a parallel combination of resistance and capacitance, whereas y_2 is only a resistor. Normalized element values are:

R_a = 0.4444 Ω, C_a = 1 F, R_1 = 0.30769 Ω, C_1 = 3.25 F, R_3 = 0.8474 Ω, C_3 = 1 F,
R_c = 0.4807 Ω, C_c = 2.08 F, R_b = 0.8547 Ω, C_b = 1.17 F and R_2 = 0.9345 Ω

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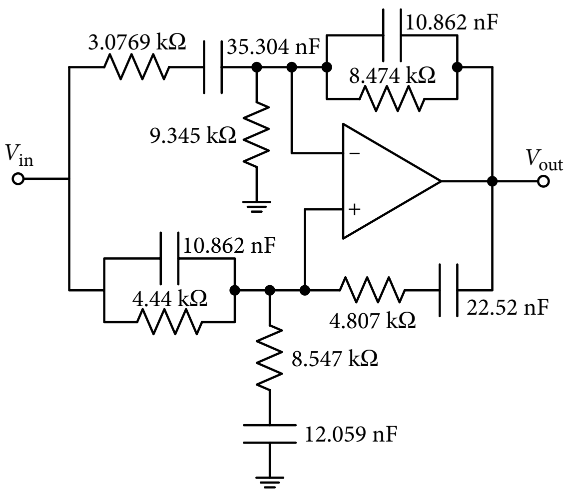
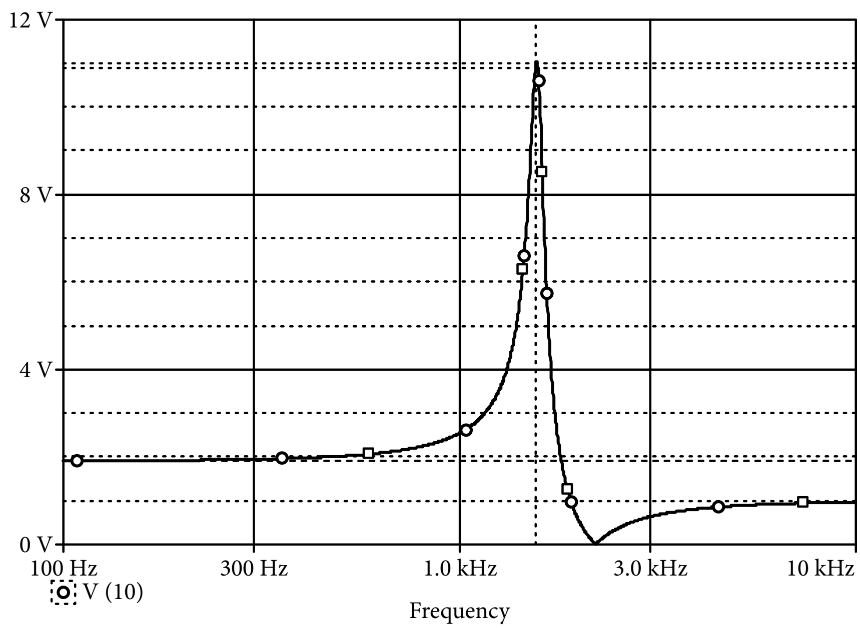
Frequency de-normalization with $1.18^{-0.5} \times 10^4$, to get its peak value at 10^4 rad/s, and impedance scaling of 10^4 gives the following element values, which are also shown in the filter circuit of Figure 10.7(a). Expected notch frequency is $1.18^{-0.5} \times 10^4 \times 2.25^{0.5}$ rad/s = (2197 Hz).



(a)



(b)

Figure 10.7 (a) Realization of the notch filter for Example 10.4. (b) Its magnitude response.

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$R_a = 4.444\ \text{k}\Omega$, $C_a = 10.862\ \text{nF}$, $R_1 = 3.0769\ \text{k}\Omega$, $C_1 = 35.304\ \text{nF}$, $R_3 = 8.474\ \text{k}\Omega$, $C_3 = 12.079\ \text{nF}$, $R_c = 4.807\ \text{k}\Omega$, $C_c = 22.52\ \text{nF}$, $R_b = 8.474\ \text{k}\Omega$, $C_b = 10.862\ \text{nF}$ and $R_2 = 9.345\ \text{k}\Omega$

Element values are shown in the filter circuit of Figure 10.7(a). Magnitude response of the BR (band reject) filter is shown in Figure 10.7(b). Its dc gain is 1.91, high frequency gain is unity, peak gain of 11 occurs at 1.569 kHz and the notch frequency is at 2.204 kHz.

Example 10.5: Determine the correct order of the sections for the fifth-order LP Butterworth filter of Example 10.1 and find the allowable input voltage for a maximum output of 10 volts.

Solution: In Example 10.1, out of the three sections, $H_1(s)$ is a first-order section which shall be the first section in the cascade. For the functions $H_2(s)$ and $H_3(s)$, the respective pole-Q are $Q = 1.618$ and $Q = 0.618$. Hence, as per equation (10.26), $H_3(s)$ will follow the first-order section and $H_2(s)$ will be the last section.

With this sequence of sections ($H_1H_3H_2$), the simulated response at all the outputs is shown in Figure 10.8. It is observed that all the three responses are monotonically decreasing with no peaking. The maximum voltage gain is unity at dc; hence, for the maximum output voltage of 10 volts, input can be 10 volts.

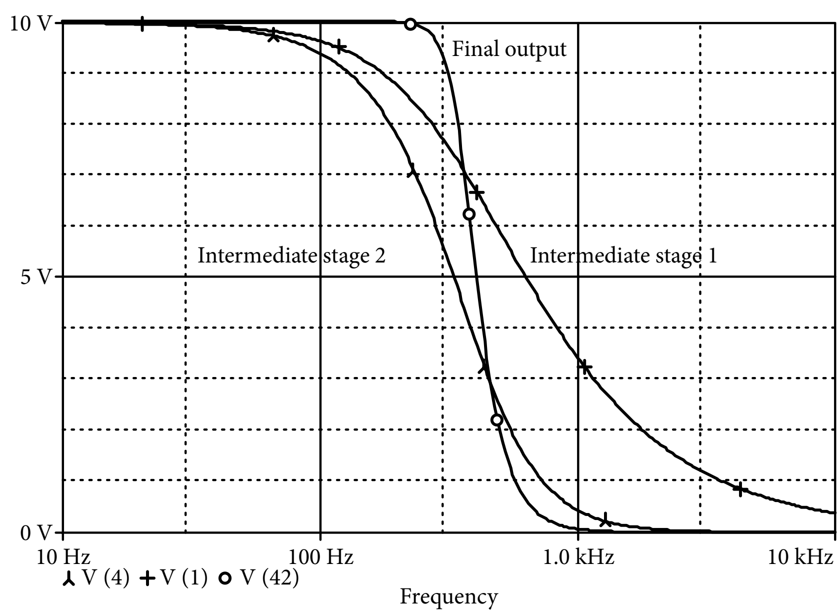

Figure 10.8 Fifth-order low pass active Butterwoth filter response using cascade process with correct sequence of sections; example 10.5.

10.3.3 Gain assignment

After completing the two steps discussed previously, we know the level of output voltage after every stage including that at the final output. Care was taken not to over-drive any output.

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The next step is to assign gain to each section. Assignment of gain follows the principle that all internal output voltages become equal (as far as possible) in magnitude corresponding to the specified final output voltage. It may be noted that the absolute value of the overall gain is not to be obtained at all costs; a simple gain amplifier in cascade will be able to adjust the overall gain if needed.

For the transfer function $H(s)$ of equation (10.2) and using the notation of equation (10.3), we can define a constant M_i as:

$$M_i = \prod_{j=1}^{i} |h_j(s)| \text{ with } i = 1, 2, \ldots, (n-1) \tag{10.34}$$

The desired condition is that $M_i = M_n$, that is, the final output after n stages. Even after the first stage, that is,

$$i = 1,\ k_1 M_1 = \max |H_1(s)| = k_1 |\max h_1(s)| = K M_n \tag{10.35}$$

$$\text{where } K = \prod_{j=1}^{n} k_j \tag{10.36}$$

It gives the relation for k_1 as:

$$k_1 = K(M_n/M_1) \tag{10.37}$$

The relation given by equation (10.37) is true for intermediate stages as well; hence, for any consecutive stages,

$$k_i = (M_{j-1}/M_j),\ j = 2, \ldots, n \tag{10.38}$$

Equation (10.38) is used for assigning gain to each stage. Assignment of gain to each second-order section ensures near equal output at the final output, as well as at intermediate stages. It ensures maximum possible input signal that can be applied without the signals going beyond saturation in OAs. The statement is correct for a single OA second-order section, where output is taken at the OA output used. For a second-order section using more than one OA, care has to be taken as mentioned before.

Example 10.6: Realize the Butterworth filter of Example 10.5 with overall gain as 6 dBs.

Solution: With the correct order as in Example 10.5, unity gain was easily achieved with each section having a gain of one. If a gain of 6 dBs is desired, it can be done without assigning gains to individual sections as discussed in Section 10.3.3 for the all pole filters which were cascaded in correct ordering. Hence, simply having a gain of 2 in the first-order section will suffice; resistance R_1 in it is changed to 2.185 kΩ, and the rest of the circuit of Example 10.5 remains the same.

Example 10.7: Assign proper values of gain to the three second-order biquadratic sections of Example 10.3, when they are in the sequence suggested in Example 10.4, such that the overall gain is 40 and center frequency is 10 krad/s. Also find:

(a) Maximum allowable input signal if the maximum allowable output is 10 volts.

(b) What happens when the second-order sections are cascaded in correct order, but each section has unity gain?

Solution: (a) Equation (10.34) relates the maximum amplitude at each output node after the first, second and third section for given example. For individual sections, it can be found mathematically (or using a computer program). However, sections H_C and H_B have already been simulated, and from Figures 7.14(b) and 7.16, max $|H_C(j\omega)|$ = 5.0 and max $|H_B(j\omega)|$ = 7.14. Section H_A has been designed and simulated in Example 10.4, and its peak value is almost 11. Hence, using the notations given in Section 10.3.3:

$$M_1 = 5,\ M_2 = 5 \times 7.14 = 35.7,\ M_3 = 35.7 \times 11 = 392.7$$

With the overall gain to be 40, KM_3 = 40, so K = 40/392.7

Gain to be assigned to the respective sections are as follows:

$$k_1 = \frac{40}{5} = 8,\ k_2 = \frac{5}{35.7} = 0.14,\ k_3 = \frac{35.7}{392.7} = 0.0909 \tag{10.39}$$

With these gain values, the overall transfer function will be:

$$H(s) = k_1\, H_C(s) \times k_2\, H_B(s) \times k_3\, H_A(s)$$

$$= \frac{8s}{(s^2+0.2s+1.01)} \times \frac{0.14\left(s^2+0.25\right)}{\left(s^2+0.09s+0.83\right)} \times \frac{0.0909(s^2+2.25)}{(s^2+0.1s+1.18)} \tag{10.40}$$

Individual second-order sections are to be designed with the new assigned gain values. The same circuit structure and methodology, which was used so far (though, any other circuit can also be used), is used now.

For the general differential input single OA biquad, the auxiliary polynomial is again assumed to be $Q(s) = (s + 1)$; this gives the following relations for $k_1 H_C(s)$:

$$\frac{N(s)}{Q(s)} = \frac{8s}{(s+1)} \to y_a = \frac{1}{\frac{1}{8}+\frac{1}{8s}},\ y_1 = 0 \tag{10.41}$$

$$\frac{D(s)}{Q(s)} = \frac{s^2+0.2s+1.01}{(s+1)} \to y_3 = \left(s+1.01\right),\ y_c = \frac{1.81s}{(s+1)} \tag{10.42}$$

$$\frac{D(s)-N(s)}{Q(s)} = \frac{s^2-7.8s+1.01}{(s+1)} \to y_b = \left(s+1.01\right),\ y_2 = \frac{9.81s}{(s+1)} \tag{10.43}$$

For $k_2H_B(s)$, the corresponding relations are:

$$\frac{N(s)}{Q(s)}=\frac{0.14s^2+0.035}{(s+1)}\to y_a=(0.14s+0.035),\ y_1=\frac{0.175s}{(s+1)} \tag{10.44}$$

$$\frac{D(s)}{Q(s)}=\frac{s^2+0.09s+0.83}{(s+1)}\to y_3=(s+0.83),\ y_c=\frac{1.74s}{(s+1)} \tag{10.45}$$

$$\frac{D(s)-N(s)}{Q(s)}=\frac{0.86s^2+0.09s+0.795}{(s+1)}\to y_b=(0.86s+0.795),\ y_2=\frac{1.565s}{(s+1)} \tag{10.46}$$

For $k_3H_A(s)$, the corresponding relations are:

$$\frac{N(s)}{Q(s)}=\frac{0.0909(s^2+0.225)}{(s+1)}\to y_a=(0.0909s+0.2045),\ y_1=\frac{0.2045s}{(s+1)} \tag{10.47}$$

$$\frac{D(s)}{Q(s)}=\frac{s^2+0.1s+1.18}{(s+1)}\to y_3=(s+1.18),\ y_c=\frac{2.08s}{(s+1)} \tag{10.48}$$

$$\frac{D(s)-N(s)}{Q(s)}=\frac{0.9191s^2+0.1s+0.8755}{(s+1)}\to y_b=(0.9191s+0.8755),\ y_2=\frac{1.6946s}{(s+1)} \tag{10.49}$$

For the three sections, with an impedance scaling factor of 10^4 and a frequency de-normalization factor of $(1.01)^{-½}\times 10^4$, $(0.83)^{-½}\times 10^4$ and $(1.18)^{-½}\times 10^4$ for the respective transfer functions, $k_1H_C(s)$, $k_2H_B(s)$ and $k_3H_A(s)$ are used and the resulting structures are cascaded in the order as decided in Example 10.4. Figure 10.9 shows the complete filter with de-normalized element values.

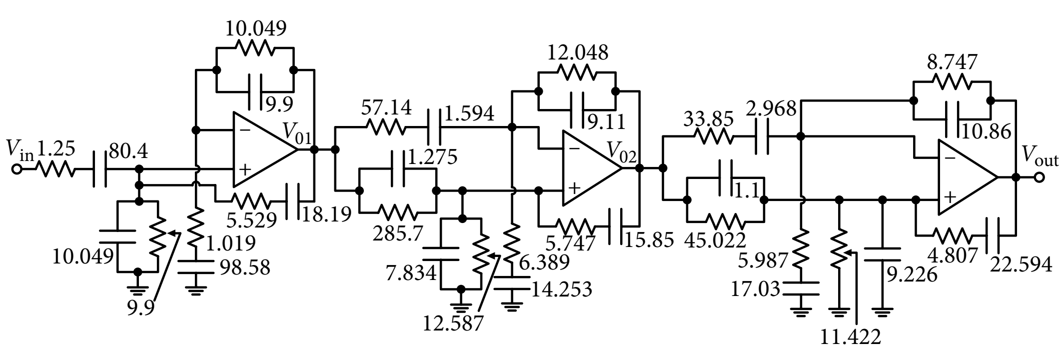

Figure 10.9 Cascade realization of the sixth-order BP filter for Example 10.7. All resistors are in kΩ and capacitors are in nF.

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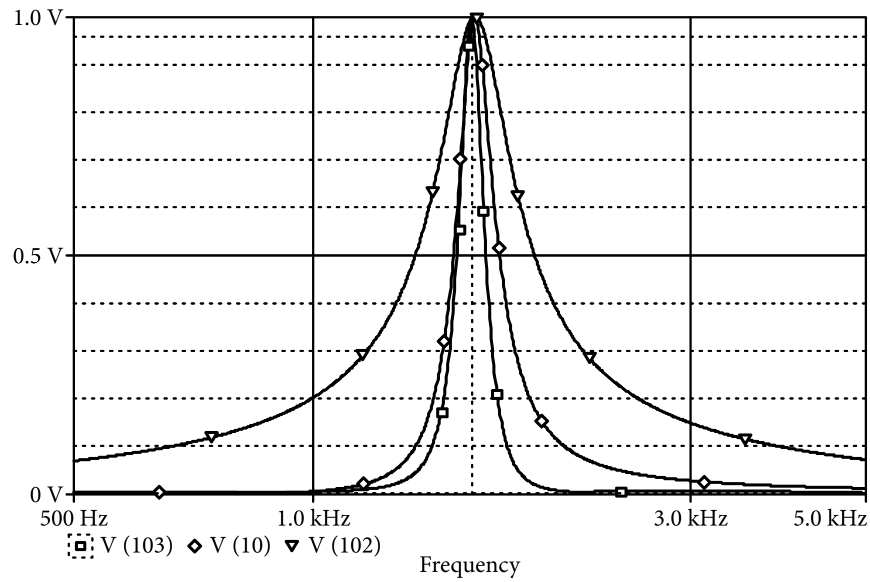
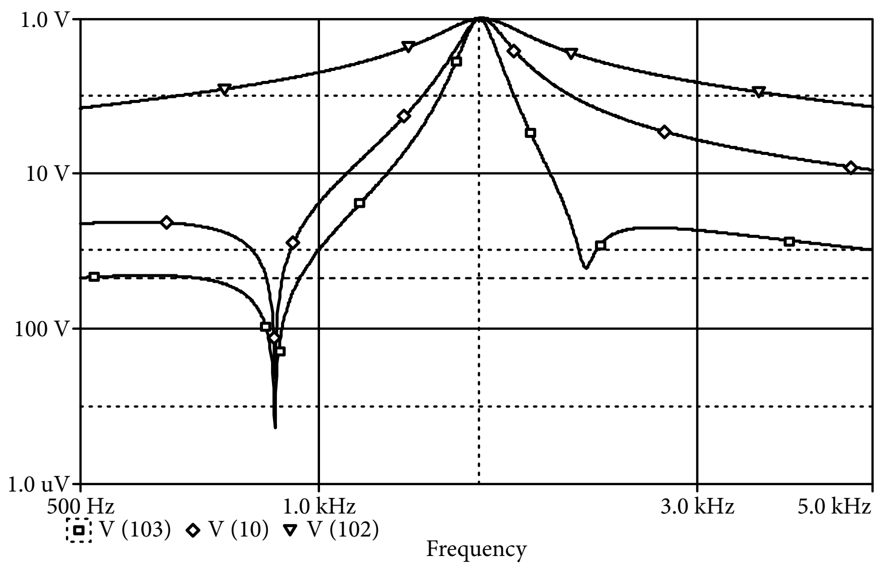
In Figure 10.10(a), the magnitude response of the sixth-order cascaded filter is shown with the output voltage on the y-axis on linear scale, while the same is shown in Figure 10.10(b) with the y-axis on log scale, depicting the notches clearly. Notch frequencies are 877.6 Hz and 2.314 kHz with 98.66 dBs and 59.3 dBs attenuation. Center frequency is 1.607 kHz (10.101 krad/s) with a gain of 40.65 against the design value of 40.



(a)



(b)

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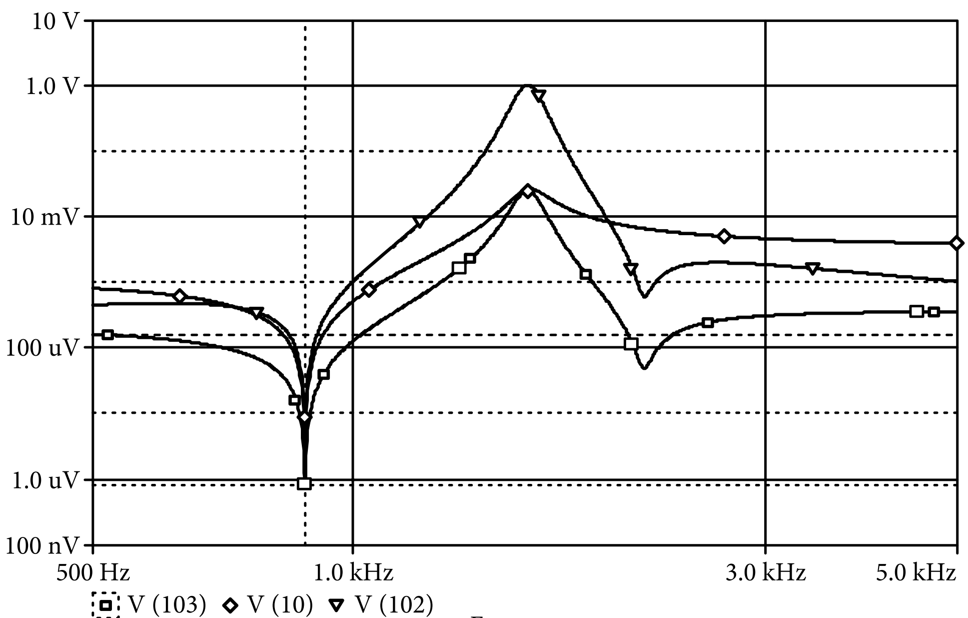

(c)

Figure 10.10 (a) Magnitude response of the sixth-order BP filter for Example 10.7. (b) Magnitude response of the sixth-order BP filter with the ordinate on log scale. (c) Magnitude response with order of the sections different from what is recommended.

(a) In order to make the level of the output voltage as 10 V, the input level can be as high as 250 mV (theoretically), and practically, 246 mV as simulated. If the order of sequence is changed with $k_2H_B(s)$ at first, followed by $k_3H_A(s)$ and then $k_1H_C(s)$, the responses at the three outputs are shown in Figure 10.10(c). The final output level and the gain remains the same but the intermediate levels have gains of approximately unity. Notches do appear at 872.7 Hz and 2.31 kHz in the final output.

(b) If the section is formed with correct pole–zero combination and cascaded in correct sequence, but is assigned unity gain for each individual sections, instead of assigning gains as calculated, the simulated response is as shown in Figure 10.11. The final output amplitude becomes 10 V for an input voltage of 28.4 mV, compared to 250 mV for the same design but with correct gain assignment, as shown earlier. For this input level, the intermediate voltages are 142.53 mV and 1.015 V; obviously, all three voltage levels are not nearly the same and the overall gain becomes 350.14.

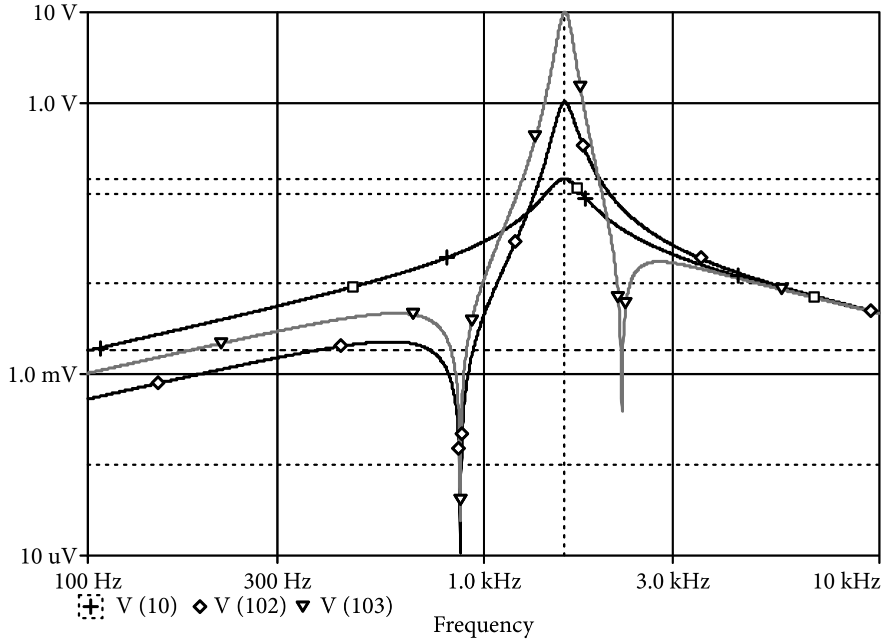

Figure 10.11 Magnitude response of the sixth-order BP filter for Example 10.7 with correct order of the sections but with unity gain assignment to the three sections.

Example 10.8: Discuss what happens with the response of the transfer function of equation (10.24) if the closest pole and zeros are not paired.

Solution: Let a different pole–zero pairing be done; the resulting three sections obtained by combining $z_{4,5}$ with $p_{3,4}$, $z_{2,3}$ with $p_{1,2}$ and z_1 with $p_{5,6}$ from the zero and pole locations given in equation (10.24b,c) are as follows:

$$H_1(s)=\frac{s}{(s^2+0.1s+1.18)},\ H_2(s)=\frac{\left(s^2+2.25\right)}{\left(s^2+0.09s+0.083\right)},\ H_3(s)=\frac{s^2+0.25}{(s^2+0.2s+1.01)} \quad (10.50)$$

For the general differential input single OA biquad, the auxiliary polynomial is again assumed to be $Q(s) = (s + 1)$; this gives the following relations for $H_1(s)$:

$$\frac{N(s)}{Q(s)}=\frac{s}{(s+1)}\rightarrow y_a=s/(s+1)\quad y_1=0 \quad (10.51)$$

$$\frac{D(s)}{Q(s)}=\frac{s^2+0.1s+1.18}{(s+1)}\rightarrow y_3=(s+1.18),\ y_c=\frac{2.08s}{(s+1)} \quad (10.52)$$

$$\frac{D(s)-N(s)}{Q(s)}=\frac{s^2-0.9s+1.18}{(s+1)}\rightarrow y_b=(s+1.18),\ y_2=\frac{3.08s}{(s+1)} \quad (10.53)$$

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For $H_2(s)$, the corresponding relations are:

$$\frac{N(s)}{Q(s)}=\frac{s^2+2.25}{(s+1)}\rightarrow y_a=(s+2.25), y_1=\frac{3.25s}{(s+1)} \tag{10.54}$$

$$\frac{D(s)}{Q(s)}=\frac{s^2+0.09s+0.83}{(s+1)}\rightarrow y_3=(s+0.83), y_c=\frac{1.74s}{(s+1)} \tag{10.55}$$

$$\frac{D(s)-N(s)}{Q(s)}=\frac{0.09s-1.42}{(s+1)}\rightarrow, y_b=\frac{1.33s}{(s+1)}, y_2=1.42 \tag{10.56}$$

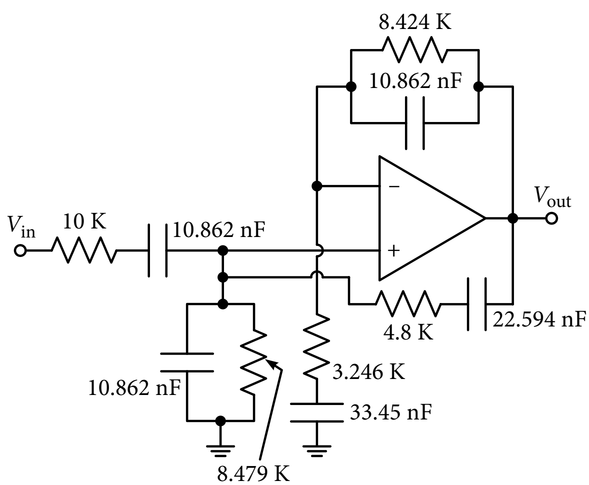
For $H_3(s)$, the corresponding relations are:

$$\frac{N(s)}{Q(s)}=\frac{s^2+0.25}{(s+1)}\rightarrow y_a=(s+0.25), y_1=\frac{1.25s}{(s+1)} \tag{10.57}$$

$$\frac{D(s)}{Q(s)}=\frac{s^2+0.2s+1.01}{(s+1)}\rightarrow y_3=(s+1.01), y_c=\frac{1.81s}{(s+1)} \tag{10.58}$$

$$\frac{D(s)-N(s)}{Q(s)}=\frac{0.2s+0.76}{(s+1)}\rightarrow y_b=0.76, y_2=\frac{0.56s}{(s+1)} \tag{10.59}$$

For the three sections, an impedance scaling factor of 10^4 and a frequency de-normalization factor of $(1.18)^{-½}\times 10^4$, $(0.83)^{-½}\times 10^4$ and $(1.01)^{-½}\times 10^4$ are used for the transfer functions, $H_1(s)$, $H_2(s)$ and $H_3(s)$. The resulting structures, with element values calculated from the respective equations are shown in Figures 10.12(a), 10.13(a), and 10.14(a), respectively. Figure 10.12(b)



(a)

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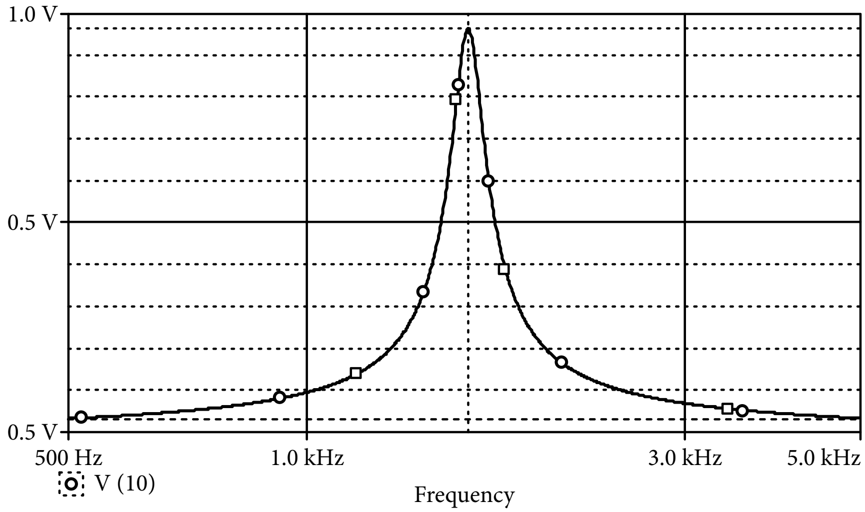
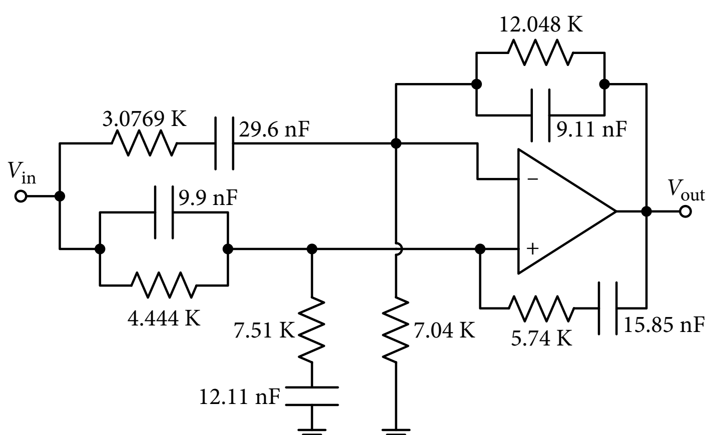


(b)

Figure 10.12 (a) Realization of the transfer function $H_1(s)$ for Example 10.8. (b) Magnitude response of the band pass function with unity gain.

shows the simulated response of $H_1(s)$, a BP response with center frequency of 1.6035 kHz, pole-Q of 10.46 and mid-band gain of 9.61. Figure 10.13(b) shows the simulated response of the function $H_2(s)$, BR characteristic with a notch at 2.639 kHz, dc gain being 2.71, high frequency gain being unity and peak gain of 30.79 at 1.607 kHz. Figure 10.14(b) shows a BR response for function $H_3(s)$ with a notch at 791.7 Hz, dc gain of 0.247, high frequency gain of unity and peak gain of 3.85 at 1.622 kHz.



(a)

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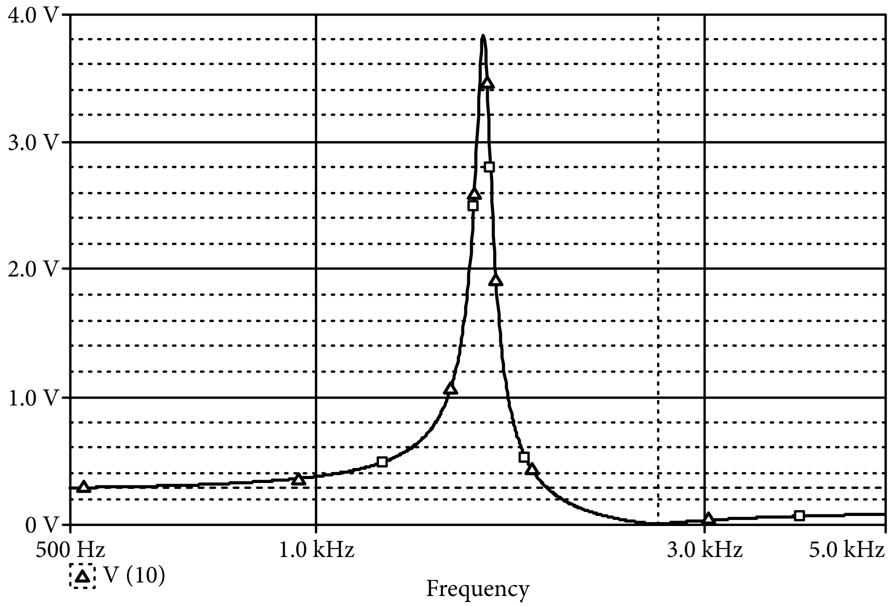
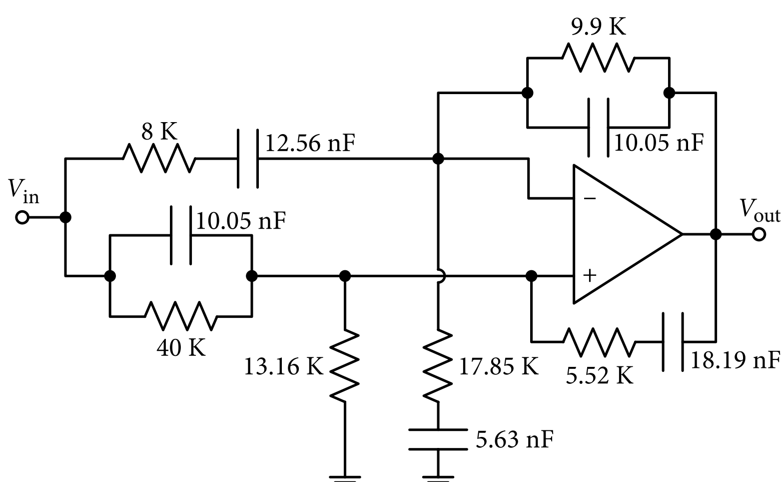


(b)

Figure 10.13 (a) Circuit realization of the transfer function $H_2(s)$ for Example 10.8 with unity gain. (b) Magnitude response of the transfer function $H_2(s)$ with unity gain.



(a)

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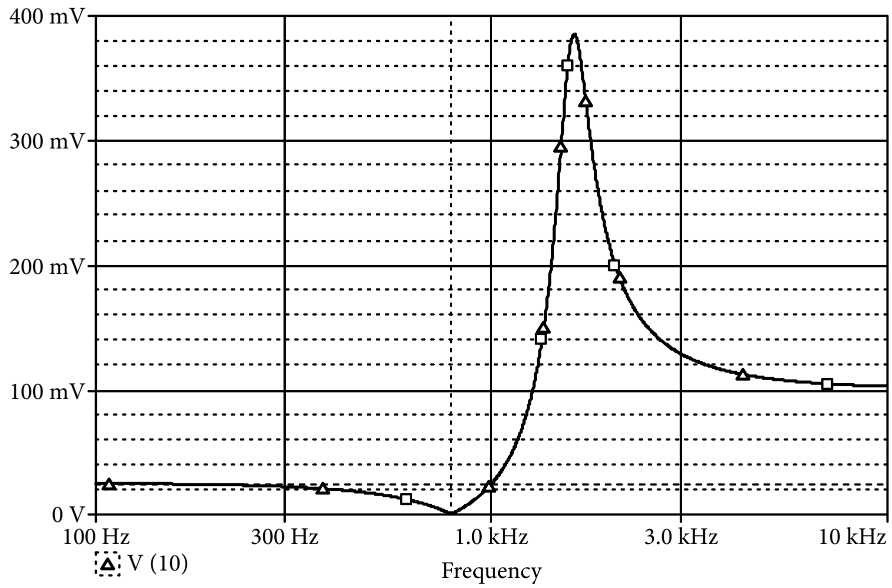
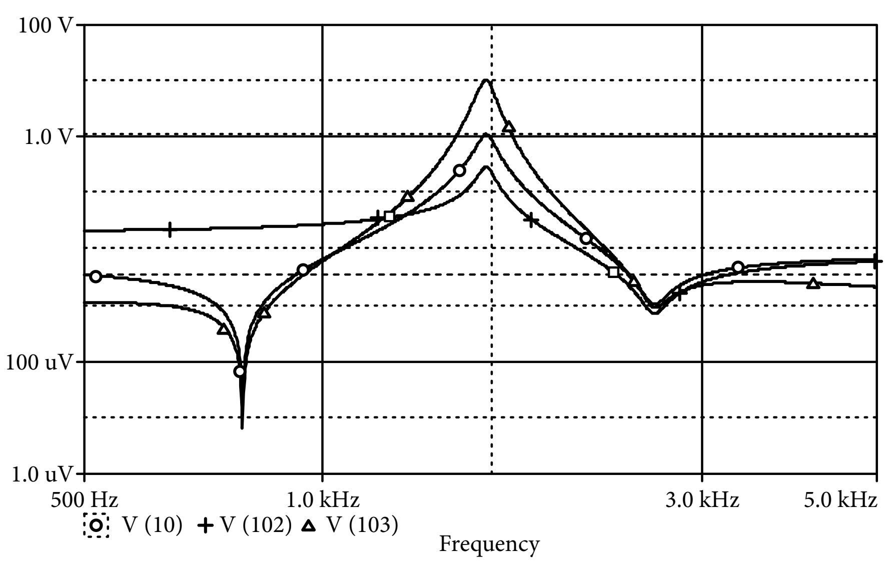


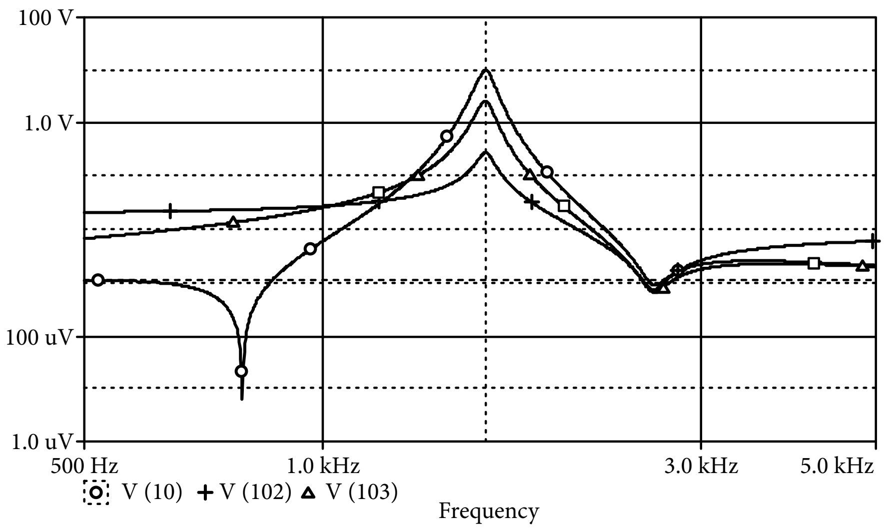
(b)

Figure 10.14 (a) Circuit realization of the transfer function $H_3(s)$ for Example 10.8. (b) Magnitude response of the transfer function $H_3(s)$ for Example 10.8 with unity gain.

The three sections are cascaded and Figures 10.15(a)–(b) show the response of the complete filter. Overall gain is 139.4; hence, to get 10 V output, the input has to be less than 71.7 mV.



(a)



(b)

Figure 10.15 (a) Response of the sixth-order filter for Example 10.8 with a pole–zero combination other than the case when the nearest poles and zeros are combined. (b) Response of the sixth-order filter while the sequence order is also different.

Example 10.9: Employing the cascade approach, realize an active RC filter for the following specifications, with dc gain as unity:

$$\alpha_{max} = 1 \text{ dB}, \alpha_{min} = 40 \text{ dBs}, \omega_1 = 2000 \text{ rad/s}, \omega_2 = 6000 \text{ rad/s}$$

Solution: In Example 3.4, it was found that a fourth-order inverse Chebyshev filter will be required for these specifications, for which the obtained transfer function is as given here:

$$H(s) = \frac{\{s^2 + 1.1757\}\{s^2 + 6.8283\}}{(s^2 + 0.3423s + 0.2559)(s^2 + 1.0091s + 0.31255)} \quad (10.60)$$

For the transfer function in equation (10.60), poles and zeros are:

$$z_{1,2} = \pm j1.0843, z_{3,4} = \pm j2.6131, p_{1,2} = -0.17117 \pm j0.47611, p_{3,4} = -0.50455 \pm j0.2408$$

Cascade optimization suggests a combination of $z_{1,2}$ with $p_{1,2}$ and $z_{3,4}$ with $p_{3,4}$ which results in the following two second-order functions from equation (10.60).

$$H_1(s) = \frac{\{s^2 + 1.1757\}}{(s^2 + 0.3423s + 0.2559)} \quad (10.61)$$

$$H_2(s) = \frac{\{s^2 + 6.8283\}}{(s^2 + 1.0091s + 0.31255)} \quad (10.62)$$

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Parameters of the two second-order normalized functions are:

$\omega_{o1} = 0.50586$, $\omega_{o2} = 0.55906$, $Q_1 = 1.4778$ and $Q_2 = 0.554$

Both the transfer functions are LP notch, for which a number of circuits and approaches are available; we select the *modified summation method* of Section 8.6.1.

With the required overall dc gain being unity, use of equations (10.36) to (10.38) gives the value of gain coefficients as:

$k_1 = 0.2178$ and $k_2 = 0.04577$

With these values of gain coefficients, the transfer functions modify as:

$$H_1(s) = \frac{0.2178s^2 + 0.2559}{(s^2 + 0.3423s + 0.2559)} \tag{10.63}$$

$$H_2(s) = \frac{0.04577s^2 + 0.31255}{(s^2 + 1.0091s + 0.31255)} \tag{10.64}$$

To realize the aforementioned transfer functions, the Ackerberg–Mossberg biquadratic circuit is used in the modified summation approach; the respective design values of elements after denormalization by 6000 rad/s and impedance scaling of 10 kΩ, are as follows:

$QR_1 = 14.778$ kΩ, $R_{31} = R_{41} = R_{51} = R_{61} = 10$ kΩ, $R_{\gamma 1} = 10$ kΩ, $C_{11} = C_{21} = 32.94$ nF and $C_{\alpha 1} = 7.1743$ nF

$QR_2 = 5.54$ kΩ, $R_{32} = R_{42} = R_{52} = R_{62} = 10$ kΩ, $R_{\gamma 2} = 10$ kΩ, $C_{12} = C_{22} = 29.811$ nF and $C_{\alpha 2} = 1.3644$ nF

The cascaded fourth-order filter is shown in Figure 10.16(a) and its simulated magnitude response is shown in Figure 10.16(b). The pass band is maximally flat, having unity gain at dc with an attenuation of 0.422 dB at 2000 rad/s and attenuation of 40.1 dBs at 6000 rad/s; this satisfies the specifications easily.

Example 10.10: Realize a maximally flat LPF in which it is desired that its dc gain remains unity and its gain drops by 1 dB at 20 krad/s by introducing transmission zeroes at 40 krad/s and 50 krad/s to increase rate of fall of attenuation.

Solution: For the given specifications, the desired transfer function was obtained in Example 3.7 of Chapter 3. The required sixth-order transfer function as obtained in Example 3.7 is as follows:

$$H_6(S) = \frac{3.119\left\{\left(\frac{S}{2}\right)^2 + 1\right\}\left\{\left(\frac{S}{2.5}\right)^2 + 1\right\}}{\left(S^2 + 2.511S + 1.744\right)\left(S^2 + 1.524S + 1.449\right)\left(S^2 + 0.475S + 1.234\right)} \tag{10.65}$$

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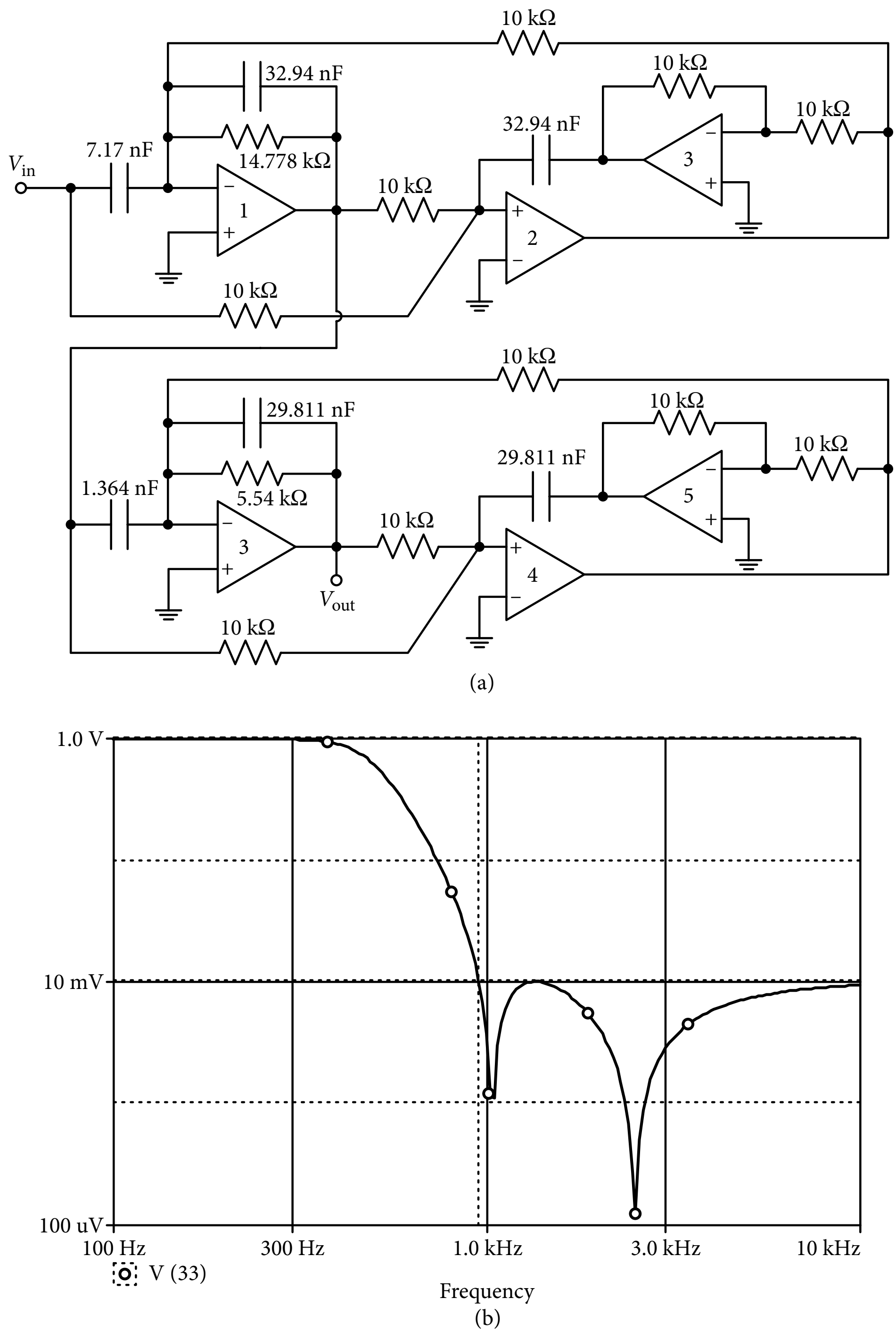

Figure 10.16 (a) Fourth-order inverse Chebyshev filter circuit for Example 10.9. (b) Its magnitude response.

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Transfer function in equation (10.65) is to be broken into second-order sections. As a first step, the nearest poles and zeros are to be combined. Hence, the values of the poles and zeros are:

$$z_{1,2} = \pm j2,\ z_{3,4} = \pm j2.5,\ p_{1,2} = -1.256 \pm j0.41,\ p_{3,4} = -0.762 \pm j0.932, \text{ and } p_{4,5} = -0.238 \pm j1.085$$

Based on the closeness between poles and zeros, the following second-order sections are formed.

$$H_1(s) = \frac{\left\{(s/2)^2 + 1\right\}}{\left(s^2 + 2.511s + 1.744\right)} \tag{10.66}$$

$$H_2(s) = \frac{3.119}{\left(s^2 + 1.524s + 1.449\right)} \tag{10.67}$$

$$H_3(s) = \frac{\left\{(s/2.5)^2 + 1\right\}}{\left(s^2 + 0.475s + 1.234\right)} \tag{10.68}$$

The next step is the selection of the sequence of the sections for which the mentioned thumb rule in terms of pole-Q values can be applied. The respective values of the pole-Q for the transfer functions H_1, H_2 and H_3 are:

$Q_1 = 0.5259$, $Q_2 = 0.7898$ and $Q_3 = 2.3386$

Hence, in the proposed cascade H_1 will be followed with H_2 and H_3 will be at the end. Otherwise, it will be preferred to have the LP section in the beginning.

Next, the assignment of gain is to be done for the individual sections so that dynamic range is maximized and the responses at all intermediate section nodes become as flat as possible. The voltage maxima at the output of each section are to be evaluated either by calculation or by inspection. As the function H_2 is an LP section with $Q_2 = 0.7899$, its maxima will occur at dc, which means $h_2 = 3.119/1.449 = 2.1525$; similarly, H_1 which functions as an LP notch, with $Q_1 = 0.5259$, has its maxima also at dc, which results in $h_1 = 1/1.744 = 0.5733$. As we are designing an LPF with gain unity at dc, for H_3, the value of h_3 was also taken at dc, which is $h_3 = 1/1.234 = 0.81$. From these values, we can write:

$M_1 = |h_1|_{max} = 0.5733$, $M_2 = |h_1\ h_2|_{max} = 2.1525 \times 0.5733 = 1.234$ and $M_3 = |h_1\ h_2\ h_3|_{max} = 1.234 \times 0.81 = 1.0$

As the final gain is to be unity, $KM_3 = 1$, and value of the gain coefficients of the three sections are evaluated as:

$k_1 = 1/0.5733 = 1.744$, $k_2 = 0.5733/1.234 = 0.4646$, $k_3 = 1.234/1.0 = 1.234$

Having obtained the gain coefficients, the biquadratic arrangement known as *modified summation method* of Section 8.6.1 was selected for the realization of the three transfer

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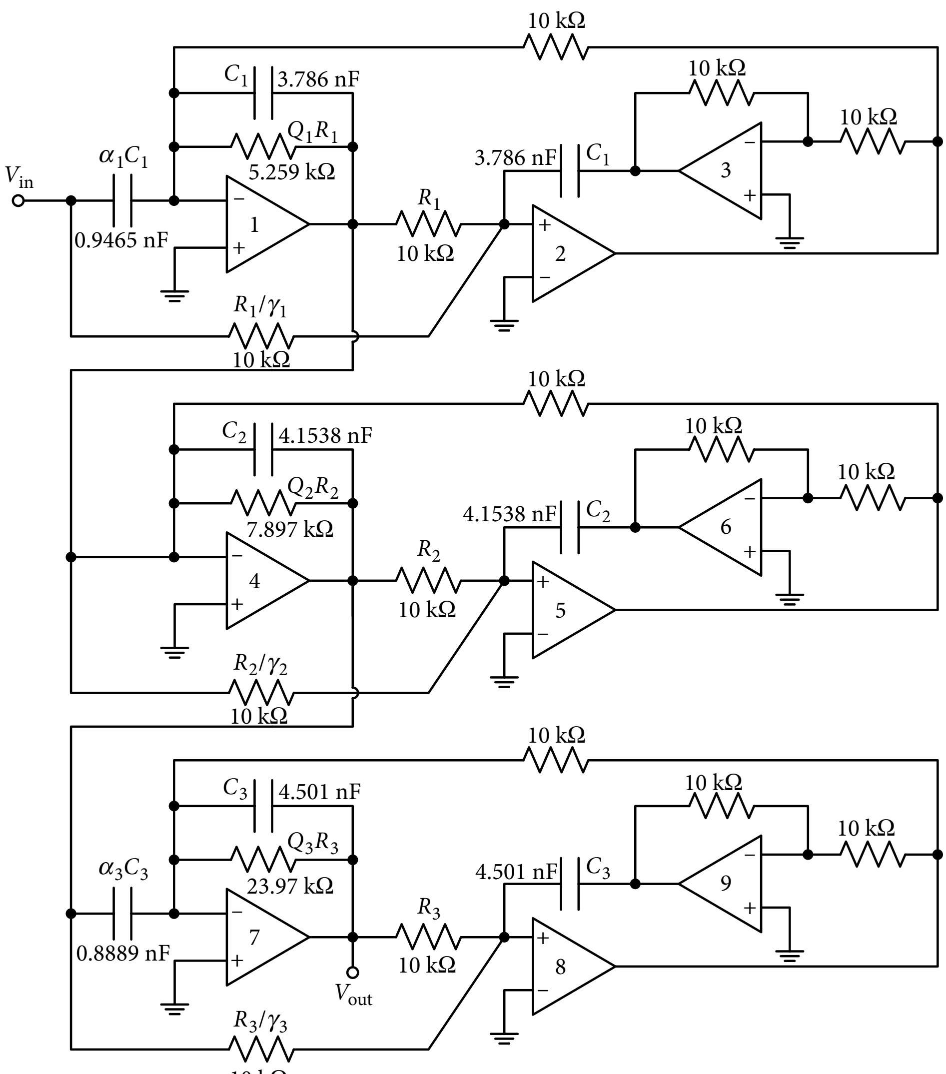
functions. For the sake of reference, the expression of the biquad is repeated here as equation (10.69).

$$\frac{V_{\text{out}}}{V_{\text{in}}} = -\frac{\alpha s^2 + s(k-\beta)/CR + \gamma / C^2R^2}{s^2 + \{s/(CRQ)\} + 1/C^2R^2} \tag{10.69}$$

Comparing equation (10.66) with equation (10.69), $k = \beta = 0$, $\alpha = 0.25$ and $\gamma = \frac{1.744}{1.744} = 1$. If normalized $R = 1$, we get $C = 1/1.744^{0.5} = 0.7572$. As the frequency normalization was done by a factor of 20 krad/s in Example 3.7, the same is used for de-normalization, and with an impedance scaling factor of 10 kΩ, the values of the elements for $H_1(s)$ are obtained as:

$$C_1 = 3.7861\text{ nF},\ \alpha_1 C_1 = 0.9465\text{ nF},\ R_1 = 10\text{ k}\Omega,\ Q_1 R_1 = 5.259\text{ k}\Omega,\ R_1/\gamma_1 = 10\text{ k}\Omega$$



(a)

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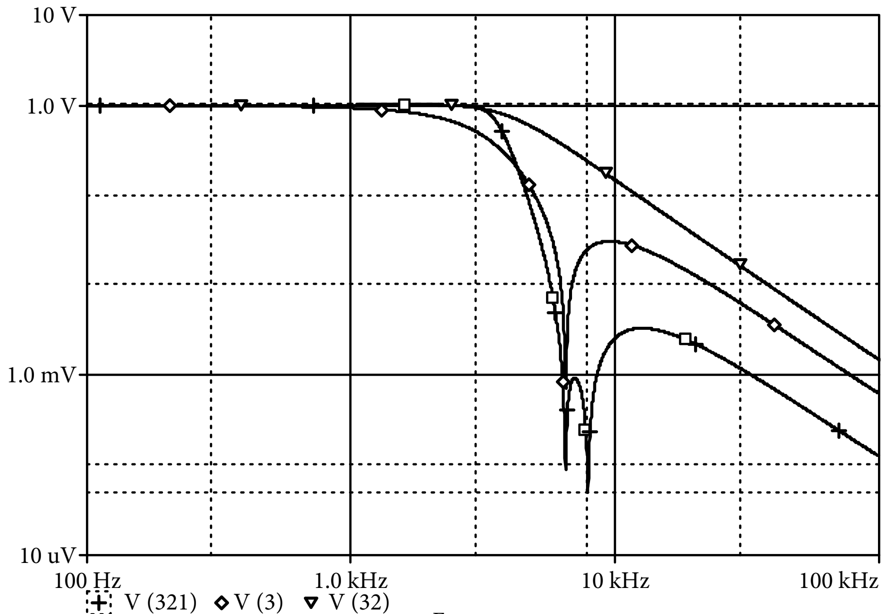

(b)

Figure 10.17 (a) Sixth-order cascaded low pass filter structure for Example 10.10. (b) Its magnitude response.

With the same circuit configuration and frequency and impedance scaling factors, element values for the remaining transfer functions H_2 and H_3, respectively, are as follows:

$$C_2 = 4.1538 \text{ nF}, \alpha_2 C_2 = 0, R_2 = 10 \text{ k}\Omega, Q_2 R_2 = 7.898 \text{ k}\Omega, R_2/\gamma_2 = 10 \text{ k}\Omega$$
$$C_3 = 4.501 \text{ nF}, \alpha_3 C_3 = 0.8889 \text{ nF}, R_3 = 10 \text{ k}\Omega, Q_3 R_3 = 23.386 \text{ k}\Omega, R_3/\gamma_3 = 10 \text{ k}\Omega$$

Figure 10.17(a) shows the complete sixth-order cascaded filter with element values and Figure 10.17(b) shows its simulated magnitude response. The filter's dc gain is unity, attenuation at 20 krad/s is 1.09 dB and zeros occur at 40.14 krad/s and 49.82 krad/s; this is very close to the design specifications.

10.4 Tuning of Filters

There has been a significant improvement in the fabrication processes and manufacturing methods of electronic components and devices and it has resulted in considerable advances in the performance capability of active filters. However, increasing complexities in various application fields such as communication, instrumentation and control has necessitated in more miniaturization. Fabrication processes of integrated circuits (ICs) have advanced from the thin and thick film technology to the hybrid and to the monolithic form. Unfortunately, even with tremendous advancements, both passive and active components suffer from manufacturing tolerances of varying degrees, affecting the performance parameters of the filters. Deviation

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in the filter parameters depend on the amount of tolerance of the components. Therefore, it becomes necessary to adopt correction methods such that the practically obtained performance is within the prescribed limits of the design, and the circuit and the system works as desired.

It has been mentioned a number of times that for OA based continuous-time filter circuits, parameters are set by RC products; this is barring OA-R and OA-C circuits where parameters depend on gain bandwidth product as well. In OTA based circuits, the parameters depend on the capacitance to the trans-conductance ratios. This means that not only the passive components but also the active components should be realized with maximum accuracy and their parameters should remain stable.

The presence of production tolerance in components requires that post-design adjustment is an essential step in meeting tight specifications. While pre-distortion is also an important step for compensating the effects of component imperfections, it is the post-fabrication parameter tuning which is almost essential. Of course, the specific tuning employed depends upon the function to be realized, the network configuration used and the technology of implementation. Over the years, *functional tuning* [10.1], *deterministic tuning*[10.2] and *automatic tunings* have been developed and used; these are briefly discussed here.

Functional and Deterministic Tuning Functional tuning is performed by adjusting the parameters. This is done by changing the components at a frequency of known phase shift while the circuit is in operational mode. Selection of phase, instead of magnitude is done because it was observed that change in phase is more pronounced than change in magnitude of filter near the region of the critical frequency, ω_o.

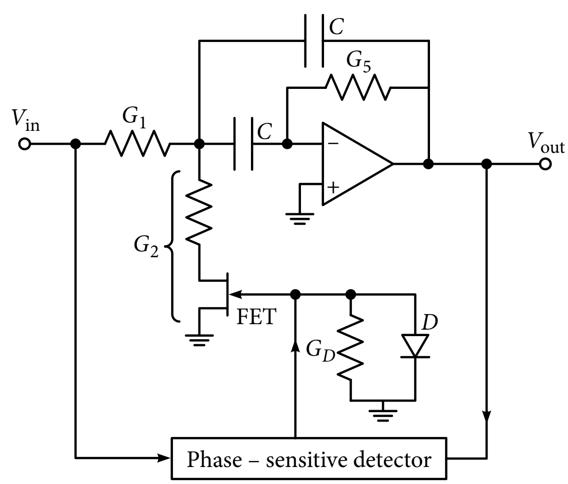

Figure 10.18 Self-tuned signal-tracking multi-loop feedback filter configuration.

Ideally, the process of tuning proceeds by controlling the parameters of interest in a non-interactive fashion through change in a single circuit element; which in almost all the cases, is a resistor. Obviously, complete non-interactiveness is practically difficult to achieve, which necessitates a tuning sequence. Without going into the details of the functional tuning, it can be said that the trial and error process for precision is slow and suitable only for simpler

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circuits. Moreover, the process of change in the controlling resistor through laser trimming is irreversible. Finally, this kind of tuning is possible in hybrid circuits and not in monolithic ICs.

In deterministic tuning, the time-consuming iterations of functional tuning are avoided by introducing a predictive step of performing initial circuit analysis. Exact design equations are formulated considering all known imperfections, like frequency dependent amplifier gain and capacitor losses. However, once the analysis is done without making the circuit interconnections, final design equations become a set of non-linear expressions and demand a computer solution. At the end, laser trimming is performed, making this approach unsuitable for monolithic ICs.

10.4.1 Automatic tuning

In applications where the input signal frequency varies over a wide range, a wide bandwidth filter is not suitable for effective rejection of noise. One solution for the problem is to use a high Q BPF with its center frequency ω_o being continuously adjusted to a desired value. In such a case, it is also required that the bandwidth BW = ω_o/Q and mid-band gain H_o remain constant and unaffected by the change in ω_o. These conditions are achievable in OA-RC circuits in which tuning of ω_o is possible by a single resistor, and this resistor does not affect BW or H_o. There are quite a few multi-OA circuits like state variable and GIC based configurations satisfying the required conditions. The single OA multiple feedback circuit discussed in Section 7.3 comes in that category, for which equal capacitance C:

$$\omega_o = \sqrt{G_5(G_1+G_2)/C}, \text{ BW} = 2G_5/C \text{ and } H_o = -G_1/2G_5 \tag{10.70}$$

It is clear from equation (10.70) that G_2 is present in the expression of ω_o only but not in the expressions of BW and H_o. Automatic tuning is implemented by means of a phase detector as shown in Figure 10.18 [10.3]. The phase detector differentiates the input and output filter waveforms before their comparison and necessary steps, such as gating, smoothing and feeding to the FET acting as a voltage-variable resistor.

In practice, the arrangement of automatic tuning comprises the phase-locked loop (PLL), which tracks a given signal while passing signals only in a small bandwidth [10.4]. Such a system can be integrated in monolithic form. The method will be discussed later in connection with the realization of active R and active C filters in Chapter 17.

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Practice Problems

10-1 A LP filter has the following specifications:

ω_1 = 10 krad/s, ω_2 = 36 krad/s, A_{max} = 1 dB ripple and A_{min} = 50 dBs

Design an active RC filter to satisfy the specifications using single amplifier second-order sections in cascade approach and test it.

10-2 A HP filter has the pass band from 10^4 rad/s to infinity. The peak to peak ripple in the pass band has to be less than 2 dBs. For $\omega \leq 2$ krad/s, the loss must be greater than 50 dBs. Design and test an active RC filter to satisfy the specifications using the cascade approach.

10-3 Design and test an equal-ripple BP filter to satisfy the specifications: (a) the pass band extends from 2000 to 8000 rad/s. The ripple width in the pass band does not exceed 0.5 dB. (b) The magnitude is at least 30 dB down at 24 krad/s from its peak value in the pass band.

10-4 Realize a fifth-order LP Chebyshev filter using cascade approach. Optimize its dynamic range. Ripple width in the pass band is 2 dBs, and frequency is normalized by 25 krad/s. Use practical element values and test the filter.

10-5 Realize the following transfer functions by cascading second-order sections using single amplifier biquads while frequency de-normalization will be with 10 krad/s.

(a) $$H(s)=\frac{4}{\left(s^2+0.77s+1\right)\left(s^2+1.85s+1\right)}$$

(b) $$H(s)=\frac{2}{\left(s^2+3s+3\right)\left(s^2+1.414s+1\right)}$$

(c) $$H(s)=\frac{s^2(s^2+1)}{\left(s^2+3s+3\right)\left(s^2+1.414s+1\right)}$$

(d) $$H(s)=\frac{1}{\left(s^2+0.77s+1\right)\left(s^2+1.85s+1\right)\left(s^2+1.414s+1\right)}$$

10-6 Repeat Problem 10-1 using multi amplifier biquads.

10-7 A fifth-order normalized LP filter can be realized by a cascade of first- and second-order functions as shown here:

$$H(s)=\frac{k_1\times k_2\times k_3}{(s+1)\left(s^2+1.61803s+1\right)\left(s^2+0.61803s+1\right)}$$

A possible design for the system is suggested in Figure P10.1. Prove its adequacy or provide an alternate design.

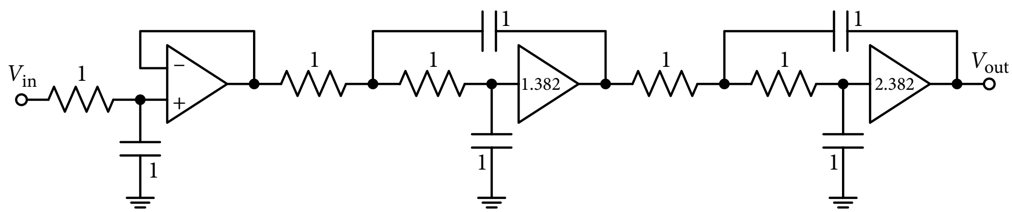

Figure P10.1

10-8 Specifications of an inverse Chebyshev function as shown in figure are:

α_{max} = 0.5dB, α_{min} = 20 dBs, ω_1 = 36 krad/s and ω_2 = 80 krad/s.

Determine the order of the filter and find transfer-function satisfying the specifications in terms of the product of second-order (a first-order also if needed) sections. Optimize dynamic range of cascaded filter.

10-9 Repeat problem 10-8 for the following specifications:

α_{max} = 0.5 dB, α_{min} = 30 dBs, ω_1 = 1 krad/s, and ω_2 = 3.45 krad/s.