Chapter 1

Analog Filter: Concepts

1.1 Introduction

The study of electric network theory has two related but distinct spheres, namely *network analysis* and *network synthesis*. The two terms are easily identified in terms of the *network*, the *excitation* or input, and the *response* or output as shown in Figure 1.1. Here *network* means some combination of passive elements, such as the resistor, the capacitor and the inductor, and (not necessarily) dependent energy sources. The *excitation* or input is an electrical energy source connected to the network. The *response* or output is observed in different forms such as voltage across certain element(s), current through certain element(s), or energy dissipated in a resistor. This response is observed either in the *time domain* or in the *frequency domain*. In the time domain, the output voltage/current variation is observed/measured with respect to time, whereas in the frequency domain, the same observation is taken in terms of frequency.

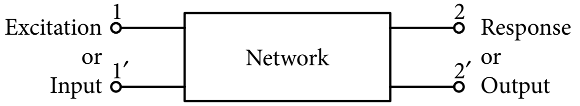

Figure 1.1 Network: a physical entity comprising passive (and active) components.

When the response of a given network to a certain excitation is observed/evaluated, it is known as *network analysis*; whereas, the process of finding a network for a given excitation and response is known as *network synthesis*. A significant difference between analysis and synthesis is that while analyzing the network, for a certain input or excitation, the response solution is unique; however, during the network synthesis process, it may be possible to obtain a number of solutions and even no solution for a given combination of excitation and response. It is this

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possibility of obtaining many solutions which has led to the development of a large number of synthesis procedures. Classification of the various synthesis methods shall be discussed in Section 1.5. Sensitivity concept, being an important issue in the practical utilization of filters, is introduced in Section 1.6 and discussed in more detail in a separate chapter, Chapter 6

The basic principles behind the classification of various types of network functions that filter signals on the basis of their frequency, and the terminologies used, are explained in Sections 1.2 and 1.3. An important stage of conversion/evolution from passive filters (networks which use only passive elements) to active filtering (using at least one active device) is discussed in Section 1.4.

One very important reason for changing from passive filters to active filters in the integrated circuit (IC) form was the inability of realizing practically feasible inductors in passive filters. For a long time, active RC (resistance–capacitance) filter structures using resistance, capacitance and operational amplifiers (OAs) were synonymous to active filters. Though the usage of OA is still prevalent, other active devices are also being used now in a big way.

It is assumed that the reader is familiar with OAs; hence, only a brief discussion about these amplifiers is included in Section 1.7. The application of OA as an integrator is reviewed in Section 1.8. A brief discussion about the characteristics of resistors and capacitors fabricated in the IC form is also included in Section 1.9 with the assumption that a detailed study of the realization of these passive components is undertaken in the separate subject 'microelectronics' or an allied subject.

1.2 Network Functions

External connections from a network to another network or excitation and response observations are made through terminals or ports: terminals are commonly paired to form a *port*. For example, in Figure 1.2, there are four terminals in the network forming two ports.

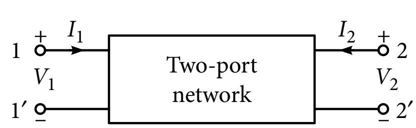

Figure 1.2 A two-port network showing conventional direction of current and voltage references.

For a single-port network, only one voltage and one current are identified and only one network function (and its reciprocal) is defined, which is given as:

$$Z(s) = V(s)/I(s) \tag{1.1}$$

This function is known as the *driving-point impedance function* because the terminals are connected to the driving force or energy source. Generally, it is simply known as the impedance

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of a network. Reciprocal of the impedance function is known as the *driving-point admittance function* or simply admittance, $Y(s)$.

For the two-port network in Figure 1.2, two currents and two voltages must be defined as shown. The four quantities $V_1(s)$, $I_1(s)$, $V_2(s)$ and $I_2(s)$ can be related to each other in six different forms (and their six reciprocals), when only two quantities are taken at a time. For example, in Figure 1.2, the driving point impedance at input terminal port 1 for a passive termination at port 2 is given as:

$$Z_{11}(s) = V_1(s)\ /I_1(s) \tag{1.2}$$

Similarly, the driving point impedance at port 2 for a passive termination at port 1 is given as:

$$Z_{22}(s) = V_2(s)\ /I_2(s) \tag{1.3}$$

Example 1.1: Find the driving point impedance function for the network shown in Figure 1.3.

Solution: As input voltage V_1 is connected across the two impedances Z_1 and Z_2 in parallel, total impedance shall be the parallel combination of these two impedances.

$$Z_1 = \frac{s}{2} + \frac{3}{2s} = \frac{s^2+3}{2s} \text{ and } Z_2 = \frac{s}{8} + \frac{5}{8s} = \frac{s^2+5}{8s}$$

$$\frac{1}{Z} = \frac{1}{z_1} + \frac{1}{Z_2} \rightarrow Z(s) = \frac{s^4+8s^2+15}{10s^3+34s} \tag{1.4}$$

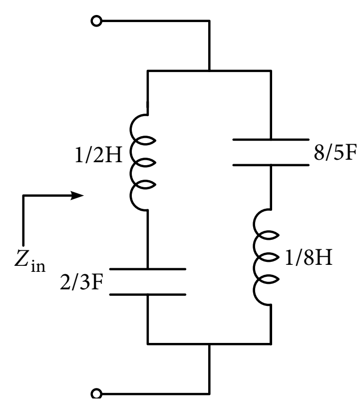

Figure 1.3 A simple LC circuit for illustrating definition of driving point function.

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Driving point impedance is a ratio of polynomials expressed in the complex frequency variable *s*. The degree of the numerator is one more that of the denominator as shown as an example in equation (1.4).

1.2.1 Transfer function

Consider Figure 1.2. Four out of the possible six network functions, which are defined for the two-port network, are such that they relate a voltage or a current of one port to the voltage or current of the other port. These functions are defined for a given driving force at the input port and a terminating condition at the output port; they are known as *transfer functions*. For the symbolic representation of the transfer function, a convenient convention is for the function to be subscripted first by the input and then by the output; for example, the voltage ratio transfer function is defined by the following relation:

$$V_2(s) \;/\; V_1(s) = G_{12}(s) \tag{1.5}$$

The current ratio transfer function is given by:

$$-I_2(s) \;/\; I_1(s) = \alpha_{12}(s) \tag{1.6}$$

The transfer impedance function and the transfer admittance function are, respectively, as follows:

$$V_2(s) \;/I_1(s) = Z_{12}(s) \text{ and } -I_2(s)/V_1(s) = Y_{12}(s) \tag{1.7}$$

In equations (1.6) and (1.7), $I_2(s)$ has a negative sign because of the conventionally chosen direction of the second-port current going into the port.

Since each transfer functions is a ratio of the Laplace transform of current/voltage, these functions are quotients of rational polynomials in the complex frequency variable *s*. For example, looking at Figure 1.4, the network's description given in the time domain is stated as follows:

$$L\left(di \,/\, dt\right) + iR + \left(1 \,/\, C\right)\int i \, dt = v_1\left(t\right) \tag{1.8}$$

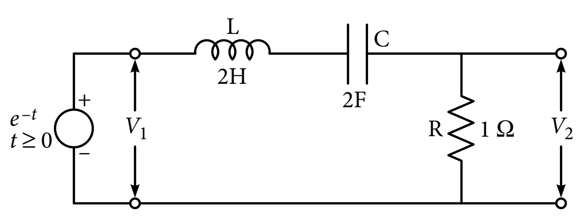

Figure 1.4 A simple RLC circuit for illustrating voltage-ratio transfer function.

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Taking the Laplace transform of equation (1.8) for the given element values of the circuit, we get:

$$\left(2s+1+\frac{1}{2s}\right)I(s)=\frac{1}{(s+1)} \tag{1.9}$$

As $V_2(s) = I(s) \times 1$, and $V_1(s) = 1/(s + 1)$, we get the following expression for the voltage ratio transfer function:

$$V_2(s) / V_1(s) = (2s)/(4s^2 + 2s + 1) \tag{1.10}$$

It is important to note that in equation (1.10), no component of the driving input or excitation contributes to the expression of the transfer function; in fact, the transfer function depends only on the element values and their interconnections. A general form of the transfer function may be expressed as a ratio of polynomials in the complex frequency s.

$$H(s)=\frac{N(s)}{D(s)}=\frac{a_m s^m + a_{m-1}s^{m-1} + \ldots\ldots\ldots + a_1 s + a_0}{b_n s^n + b_{n-1} + s^{n-1} + \ldots\ldots\ldots\ldots + b_1 s + b_0} \tag{1.11}$$

In equation (1.11), with $i = 0$ to m and $j = 0$ to n, the coefficients a_i and b_j are real constants, m is the degree of the numerator and n is the degree of the denominator. Factorization of both the polynomials $N(s)$ and $D(s)$ gives the following important form of the transfer function.

$$\frac{N(s)}{D(s)}=\frac{a_m}{b_n}\frac{(s-z_1)(s-z_2)\ldots\ldots\ldots(s-z_m)}{(s-p_1)(s-p_2)\ldots\ldots\ldots(s-p_n)} \tag{1.12}$$

Here, the roots of the numerator, $z_1, z_2, \ldots, z_m$ are called *zeros*, and the roots of the denominator $p_1, p_2, \ldots, p_n$ are called *poles* of the transfer function, respectively. Consideration of zeros and poles of a network function is extremely important as they describe the behavior of the network function in the frequency domain.

1.3 Basic Filtering Action

As mentioned earlier, network synthesis is the process of finding circuit elements, such that their values and interconnections conforms to the given relation between excitation and response. Since excitation and response are both conveniently represented by the transfer functions and y or z functions, the first step in the design of a *filter* is to convert the specified magnitude, phase or related entity as a function of frequency to a form of the transfer function. Electronic filters are circuits that perform signal processing in such a way that it separates signals on the basis of frequency. Ideally, the filter suppresses signals in a certain band of frequency, known as *stop* *band*, completely and passes signals in other bands of frequency, known as *pass bands*, without any attenuation or gain. Practically, however, it is not possible to suppress a signal completely in the stop band, though the suppression is not less than a permissible level, and signals may not pass without some attenuation in the pass band (unless intentional gain is added).

1.3.1 Types of filters

There are four commonly used types of filters: they are classified based on the terminology of pass band and stop band mentioned in the previous section. The types as shown in Figure 1.5(a–d) and are defined as follows:

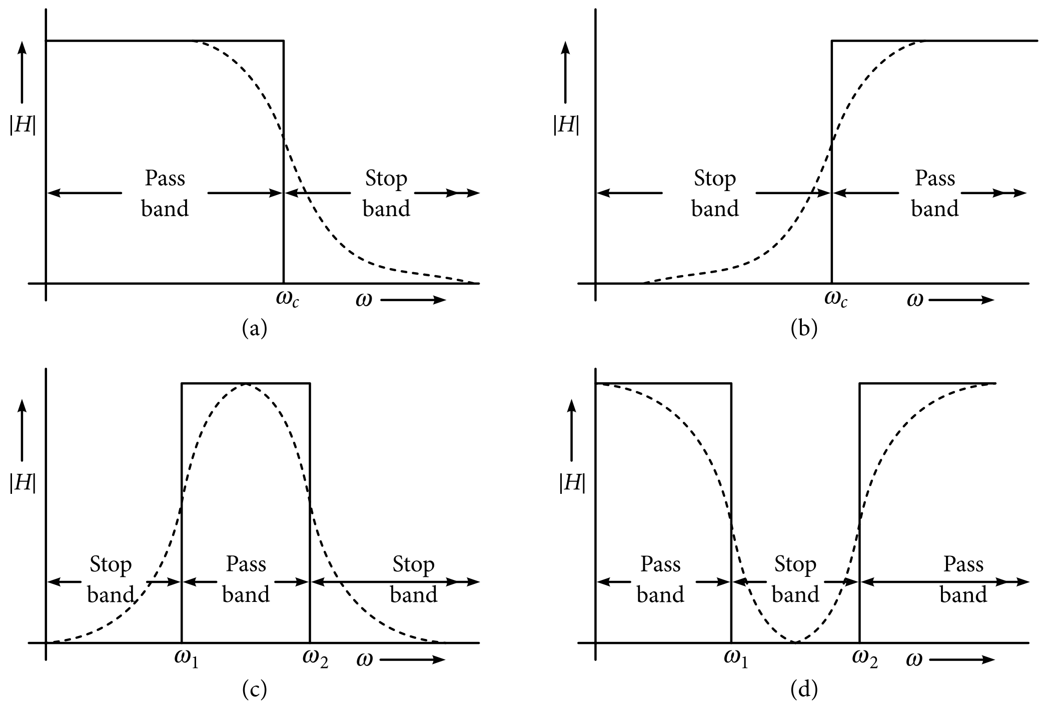

Figure 1.5 Firm lines show ideal response and dashed lines shows practical response for (a) low pass, (b) high pass, (c) band pass, and (d) band stop filters.

Low Pass Filters: When the frequency range of the pass band is from $\omega = 0$ to $\omega = \omega_c$ and the stop band extends from $\omega = \omega_c$ to ∞, the filter is known as a *low pass filter* (LPF) as shown in Figure 1.5(a). Here, ω_c is called the *cut-off frequency*.

LPFs are widely used in many diverse applications. For example, in acoustics, LPFs are used to filter out high frequency signals from the transmitting sound that would otherwise cause echo at higher sound frequencies. In audio speakers, LPF reduces the high frequency hiss sound produced in the system and inputs the clearer sound to the sub-woofers. LPFs are also

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used in equalizers and audio amplifiers. An important application of LPF is as an anti-aliasing filter placed before the analog to digital convertor (ADC) and as a reconstruction filter placed after the ADC. In a variety of applications, including those in medical instrumentation, LPFs are used to eliminate high frequency noise.

High Pass Filter: The complement of LPF is the *high pass filter* (HPF), where the stop band ranges from $\omega = 0$ to ω_c and the pass band extends from $\omega = \omega_c$ to ∞, as shown in Figure 1.5(b).

HPFs can be used wherever noise at low frequencies is to be eliminated, such as in medical instrumentation and audio systems. They find applications in loudspeakers to reduce the low-frequency noise. HPFs are also employed in those applications where high frequency signals are to be amplified. For example, a popular application is the *treble boost*. In some applications, combinations of HPFs with LPFs yield band pass filters or band stop filters. HPFs are used to pass high frequency signals to a tweeter and to block interfering and potentially damaging signals to loudspeakers. They are also used extensively in crossover of audio signals.

Band Pass Filters: In a *band pass filter* (BPF), the pass band ranges from ω_1 to ω_2; signals in the rest of the frequency range, which is known as the stop band, are stopped as shown in Figure 1.5(c).

BPFs are extensively used in all types of instruments, such as seismology and medical instruments like electroencephalograms and electrocardiograms. There is widespread use of BPFs in audio signal processing, where signals in a certain frequency range are to be passed (maybe with amplification) and signals in the remaining band of frequencies are to be rejected. In communication systems, the transmitter as well as the receiver employs BPFs to avoid interference from unwanted signals.

Band Stop Filter: A *band stop filter* (BSF) (or a notch filter) complements BPF with the stop band ranging between ω_1 and ω_2 and rest of the frequency band is the pass band as shown in Figure 1.5(d).

These filters are mainly used in public address systems and speaker systems to ensure rejection of power supply frequency interference. Similarly, BSFs are crucial for line noise reduction in telephonic signal transmission. These filters are also employed in many electronic communication devices to eliminate/reduce interference from harmonics. BPFs are used in the medical instruments, such as electrocardiogram machines, to reject unwanted signals.

In Figure 1.5, straight lines show the ranges of pass and stop bands for ideal filters. It has been proved mathematically in literature that such ideal characteristics are not realizable exactly with finite number of elements [1.1]. The approximated responses for the filter, which are practically realizable, are shown as dotted lines in Figure 1.5. The transfer function for such practically realizable filter networks are described by the *real rational function* given in equation (1.11). It is important to note that the transfer function expressed as the ratio of polynomials in the complex frequency s must satisfy certain conditions for the filter to be realizable practically. For example, the coefficients in the two polynomials in equation (1.11), $N(s)$ and $D(s)$, a_i, $i = 0$ to m and b_j, $j = 0$ to n are should be real numbers. In addition, the degree

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of the denominator n must be larger than or at least equal to the degree of the numerator, m, that is, $n \geq m$. Magnitude of the transfer function in equation (1.11) when evaluated at the $j\omega$-axis, that is, $|H(j\omega)|$, is a continuous function of frequency ω as shown by the dotted lines in Figure 1.5(a–d), rather than the characteristics given by the solid lines. It is to be noted that the sharpness of the curves in Figure 1.5 depends on the value of the coefficients in the denominator $D(s)$; hence, sharpness can be controlled by the designer.

It is a common practice in filter circuit design to represent the characteristics of the circuit in terms of linear output signal magnitude, or gain characteristic $|H(j\omega)|$ or in terms of the logarithmic attenuation characteristic $\alpha(\omega)$. Attenuation characteristics of the main four types of filters, corresponding to Figure 1.5(a–d), are shown in Figure 1.6(a–d). Here, logarithmic attenuation $\alpha(\omega)$ is related to the gain magnitude as follows:

$$\alpha(\omega) = 20 \log |H(j\omega)| \text{ dB} \tag{1.13}$$

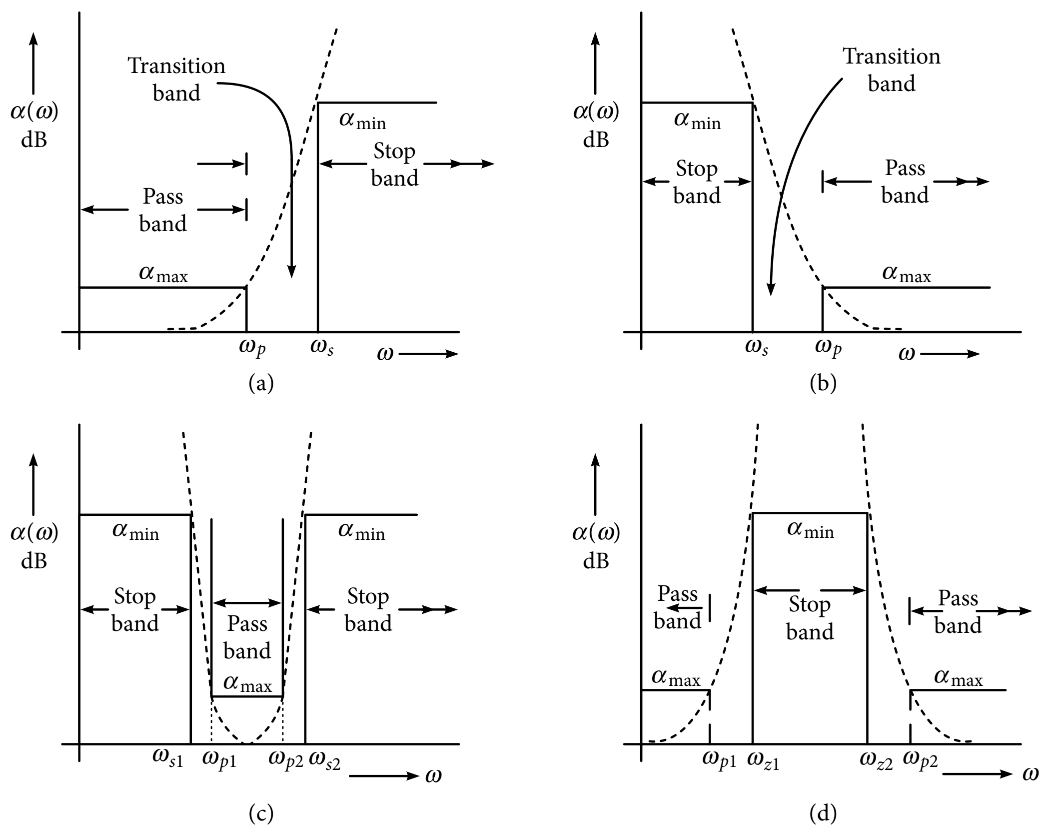

Figure 1.6 Ideal and practical characteristics in term of logarithmic attenuation for the (a) low pass, (b) high pass, (c) band pass, and (d) band stop filters.

It is to be noted that in the normal course, α will be positive for $|H(j\omega)| > 1$ and negative for $|H(j\omega)| < 1$. However, in filter design, the negative sign for α is normally not used even when

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$|H(j\omega)| < 1$. It is the context which informs the analyzer whether positive α implies gain during filtering or attenuation during loss in signal magnitude. The negative sign for α is used only when it becomes necessary to avoid any confusion.

Filter characteristics of Figure 1.5, which are drawn again in Figure 1.6, in terms of attenuation, show clearly that for a practical filter, the boundary of the pass and the stop band is separated by a definite region, which is called the *transition band*. This is not so in the case of the ideal filter response, where transition from the pass band to the stop band is instant and no transition region exists. It is the width of this transition region which decides the sharpness of the filter characteristic, an important parameter in filter design.

Characteristics of important four types of filter shown in Figure 1.6 are shown as solid lines and continuous dotted lines. The required practical characteristics are shown by the continuous dotted lines; however, the characteristics are not fixed, in the sense that they are governed by two parameters α_{max} and α_{min} shown on the solid lines. Here, α_{max} is the maximum attenuation and implies that the attenuation should always be less than this value in the pass band; α_{min} is the minimum attenuation and implies that the attenuation should always be more than this value in the stop band. Hence, the most common method of providing specifications for a filter is to mention α_{min}, α_{max} and the edge frequencies of the pass band and stop band ω_1 and ω_2, respectively. The mathematical process of finding the appropriate transfer function $H(s)$ in the form of equation (1.11) is known as *approximation*. There are different methods of approximations: classical forms, which are discussed in Chapter 3, and other methods based on the approximating phase or delay requirements of a filter, which are discussed in Chapter 4.

Another significant point to note is that the level of attenuation $\alpha(\omega)$ has been shown to reach the level of zero dB. In reality, the level may be at zero dB, below or above zero dB depending on the kind of circuitry used for filter realization and whether a gain (active) device has been used in the circuit or not. However, this level of attenuation does not violate the specifications α_{min}, α_{max} or pass band or stop band edge frequencies. Therefore, absolute value of the attenuation in the pass band is not a worrying factor for the designer as shall be evident later in the examples.

1.4 From Passive to Active Filtering

Over the years, passive elements have been used extensively for building *passive filters*. The filter could use all the three elements, resistance (R), inductance (L), and capacitance (C), or only two elements like RC, LC, or RL. Excellent literature is available on the conditions of realization, and methods that enable the realization of such passive filters [1.1]. There are quite a few advantages that such filters offer.

i. Passive filters do not require any power supply for operation.
ii. Two-port networks employing only LC components, commonly known as *lossless* networks as they did not dissipate any power, were found to be very useful as they could provide sharp transition between stop band and pass band.
iii. One of the most common forms of passive realization that was, and is still used, the *ladder structure*. LC ladder structures have been studied extensively and data, such as structural

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combination, element values, poles, and zero location for their transfer function, are available for different types of approximated filters [1.2]. One of the main advantages of the LC ladders is their low *element sensitivity* in the pass band. This means that realized filter parameters are comparatively much less affected due to changes (unavoidable or intentional) in the values of the elements used.

iv. Passive filters can and are still used at some higher frequencies where use of inductance is not very problematic and other types of filters face limitations.

v. Study of passive filters, especially the ladder structure, is important from the point of view of *active filter* realizations as well, as many active realizations use passive ladder structures as the starting point of synthesis because of their excellent sensitivity properties.

One of the major limitations of passive filters lies in its use of inductors, which are coils of wires on some kind of core material. Not only are the inductors bulky, radiate electromagnetic energy, and can result in parasitic mutual inductance, they are also practically not realizable in integrated circuit (IC) form. With the increase in the use of electronic filters in a big way and advances in technology, the use of inductors in filters has become a big limitation. The advent of active devices, especially the availability of operational amplifiers (OAs) in IC form, has reduced the need of inductors. These active filters use active device(s), mainly OAs (and later operational transconductance amplifiers, current conveyors and other active devices as well), capacitors and/or resistors. Such filter circuits can be implemented in a very small space in IC form, cutting down the cost of ICs heavily [1.3].

In passive filters, output power was always less than the input power; whereas, in active filters, use of active devices provides *gain*, which is very helpful especially when the input signal magnitude is small.

When OA (or some other active device) is used, and the output is taken at the output terminal of the OA, the output impedance of the active filter is (almost all the time) low. Such a condition is suitable for connecting these filter circuits in a chain, better known as *cascade*, even if the input impedance of a proceeding cascaded filter circuit is not very high. Cascades are commonly used in active filters for enhancing the filters' characteristics, resulting in *higher-order* filters (where n of equation (1.11) is large).

As mentioned earlier, passive filters are used at high frequencies; this needs a bit more explanation. Electronic filters can also be classified in terms of the reactive elements employed; these elements control the useful frequency range of operation of the filter. For example, resonant cavities are found suitable at microwave frequencies; whereas, excellent selectivity is provided by the piezoelectric crystal unit in the frequency range 5 to 150 MHz. The intermediate radio frequency reception range between 100 kHz and 10 MHz utilizes mainly ceramic filters. Even mechanical filters can provide medium to high selectivity with good tunability in the audio band down to about 0.1 Hz. Another important frequency spectrum lies between 20 kHz and 100 kHz, where lumped inductor and capacitors are used. However, as mentioned in Sections 1.4 and 1.5, active elements were introduced, particularly in the lower and sub-audio bands (though not limited to this frequency range). This effectively amounted to replacing passive inductors, thereby introducing the subject of this book: namely, active RC filters. As active

RC filters became widely used, the subject matter gained considerable momentum. However, with advancements in technology, digital and sampled data posed great challenges to active RC filters. Communication channels were progressively digitized and the filtering operation was done directly on digital signals. Without going into a discussion on the limitations (or comparison) of digital and sampled data filters, it can safely be said that analog filters are still in great demand. There are a number of applications where analog filters become essential or are preferred over digital or sampled data filters.

Active analog filters manufactured in IC form are in great demand in the field of communication, instrumentation, medical science and many more areas (some of the application areas were mentioned in Section 1.3.1), and their design is a specialized subject. However, this book provides an introduction to this topic in order to enable one to design filters with less demanding specifications in the domain of continuous-time filters and make the reader ready to undertake the specialized study.

Before proceeding further, let us keep in mind some of the following points while using or designing active analog filters.

(a) Active analog filters need dc power supply for active devices; the power is dissipated and it produces heat which needs to be removed without increasing the working temperature.

(b) The range of the operating frequency is limited by the type of active device used, its network topology and the magnitude of the input signal. Though the whole range of operating frequency may extend from around a fraction of Hz to a few GHz, the actual operating frequency range needs to be evaluated for each specific type of active filter with specific active device(s).

(c) Depending on the type of technology used for ICs, the tolerance of the values of passive elements and parameters of the active devices are different. Tolerance affects parameters and characteristics of the realized filter. It is for this reason that it is important to study the *sensitivity* effect on filter parameters due to the tolerance of the elements used for their practical utilization.

(d) As active analog filters provide gain, small magnitude signals can also be processed. However, active devices do generate noise and care has to be taken so that the signal does not get polluted by noise. In addition, rise in the level of signal due to gain should not go above the saturation level of the active device at any stage (final or intermediate) of the filtering; the devices operate in linear range.

1.5 Active RC Filters

To decrease the size of electronic filters in accordance with modern technology, replacement of inductors with appropriate elements becomes essential. Utilization of active devices, along with resistance and capacitance elements, which simulated inductance, led to active RC filters being considered as an alternative to passive filters. In the beginning, transistors were used as active devices; however, integration of OAs in 1970s gave active RC circuits a big boost. Parameters of OAs being close to ideal, active RC synthesis remained synonymous with OA–

RC network synthesis for a long time. Inspite of the frequency-dependent gain of OAs, and the advent of new active devices, OAs are still used in most voltage-mode filtering circuits. Depending on the application, either general purpose but cheaper OAs or specially designed but costlier OAs are used.

The new devices, which are also being increasingly used, are basically current-mode devices. They include operational transconductance amplifiers (OTAs), current conveyors (CCs) and its variants, and other current mode devices like current differencing transconductance amplifiers (CDTAs).

In linear active networks, the signal processing properties are largely dependent on the time constants, that is, RC products. Therefore, in IC filters, generally high-quality R and C elements are realized; effort is also made to obtain filter parameters in terms of element ratios as resistance or capacitance fabricated in ratio forms are more precise[1.3].

Whether we use OA-based design (OA–RC) or OTA based design (OTA–RC) or some other variations (these will be discussed at a later stage), the basic synthesis processes are similar. The processes are as follows:

i. Cascade form synthesis.

ii. Direct form synthesis using element substitution or through *operational simulation*.

In the cascade form active synthesis, the nth order transfer function, which was given in equation (1.11), is decomposed into second-order functions (if n is even, and one first/third-order sections if n is odd). These second-order functions are then individually realized and the resulting non-interactive blocks are cascaded to obtain the overall transfer function $H(s)$. Each second-order building block can be expressed as follows:

$$H_i(s) = \frac{N(s)}{b_2 s^2 + b_1 s + b_0} \tag{1.14}$$

where $N(s)$, the numerator polynomial of second-order functions, is chosen according to the required nature of the response of the section; whereas, the denominator polynomial defines the two important filter parameters, viz., the pole frequency ω_o and pole Q (Q). These are related to the denominator coefficient as follows:

$$\omega_o = (b_0 / b_2)^{1/2} \text{ and } Q = (b_0 b_2)^{1/2} / b_1 \tag{1.15}$$

There are some specific advantages while realizing second-order sections, hence great emphasis has been given to the optimum design of second-order sections. In addition, different forms of feedback and utilization of second-order (and first-order) sections have resulted in the *follow the leader feedback* type of filters as well.

In the element substitution approach of the direct form of synthesis technique, the network is directly realized from the transfer function $H(s)$ of equation (1.11). A number of direct form techniques are available; the following two are the most common.

(a) Inductance simulation (IS) approach.
(b) Frequency dependent negative resistance (FDNR) approach.

These techniques and their implementation shall be taken up in later chapters.

Operational Simulation of Ladders: Using passive RLC ladders as the starting point, active RC networks can also be realized in a slightly different direct form in what is known as *operational simulation*. In this approach, instead of simulating the elements, the process simulates the operation of the ladder by appropriately modeling the circuit equations and voltage–current relations of the elements used. Such circuit equations are represented by block diagrams in a signal flow graph.

1.5.1 Active R and active C filters

Conventionally, the design of active RC circuits assumes OA to be ideal with a large frequency-independent voltage gain ($A \rightarrow \infty$). However, OAs exhibit a frequency-dependent gain of low pass nature. This kind of non-ideality restricts the use of OA–RC circuits to mostly audio frequency range while using economically cheaper off-the-shelf OAs. Suitable compensating schemes have been developed to design filters with lesser dependence on the amplifier gain characteristics. In an alternate approach, the frequency-dependent gain of the OAs was directly used in the design; this helped in increasing the useful frequency range of operation to the MHz range while using commercially available OAs. The non-ideality of OAs has been used to develop active R and active C circuits, which employ OAs and resistors and OAs and capacitors only, respectively. However, because of certain limitations, these types of filter structures have not been manufactured in bulk. The limitations included the large tolerance of the bandwidth of OAs and their dependence on temperature and biasing voltage. Suggested methods to overcome these limitations in active R and active C filters are also available.

1.6 Sensitivity Concepts

Due to the availability of a number of choices for the synthesis of a particular transfer function, it is possible to use a number of active circuits for a single specification. Hence, the designer/user has to decide on the *best possible* choice amongst the available designs. To take this decision, certain criterion is chosen, such as number of active and passive elements used, the useful frequency range of operation and ease of design. However, while realizing the circuit in practice, one significant problem is that element values differ from their nominal values due to the fabrication process tolerance. In addition, the elements' values change with working temperature and variation in voltage and environment. Inaccurate modeling of the passive as well as active elements and parasitic elements also contribute towards increasing the complexities of the circuit. Since all the coefficients of the transfer function, hence its poles and zeros, depend on the passive and active component values, it is obvious that the gain and characteristics of the transfer function also deviate from the specified form. It is important to evaluate the deviations in different performance parameters caused by the possible change in

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circuit element values. The amount of introduced error depends on the amount of component tolerance and the *sensitivity* of the circuit's performance parameter to these tolerances. As a result, while comparing different available active circuits to choose the best possible alternative, one of the important considerations is the study of *sensitivity*.

1.7 Operational Amplifier

OA is the most prominent active device used in active filters. It is expected that the reader is familiar with the basics of OAs. Therefore, here, the discussion will be restricted to only those aspects of OAs that are directly related to their usage in analog filters.

Figure 1.7 shows the pin connection diagram of the most commonly used OA, type 741. It needs dual power supply and has two terminals, one for inverting and non-inverting inputs and one for the output. Dual OA and quad OA ICs are also available with matching characteristics. OA is basically a high-gain differential amplifier that can be modeled in the simplest form as shown in Figure 1.8. Output voltage of the OA is the difference of the two input voltages multiplied by the high gain factor A; it can be expressed as follows:

$$V_o = A(V_+ - V_-) \tag{1.16}$$

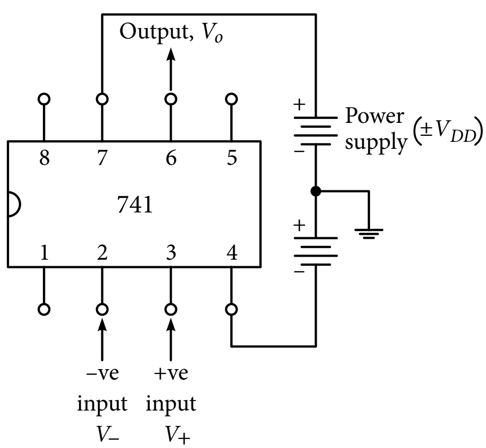

Figure 1.7 Signal input/output and power supply connections for an operational amplifier.

The differential gain A is frequency dependent in a practical OA. Therefore, it is represented by the single-pole roll-off model given here:

$$A(s) = \frac{A_o \omega_a}{s + \omega_a} = \frac{B}{s + \omega_a} \simeq \frac{B}{s} \tag{1.17}$$

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Here, A_o, the ratio of the single-ended output voltage to the differential input voltage, is called *open-loop gain*, or dc gain (i.e., gain at $\omega = 0$), ω_a is the open-loop bandwidth and $B = A_o\omega_a$ is the gain–bandwidth product. For a practical OA, $A_o \approx 10^5$, $\omega_a \approx 2\pi \times 10$ rad/s, that is, $B = 2\pi \times 10^6$ rad/s and the approximated expression in equation (1.17) is valid for all $\omega \gg \omega_a$; input resistance at each input (R_i) is normally more than 1 MΩ and the output resistance (R_o) is less than 100 Ω. For the purpose of analysis, initially, OA is assumed to be ideal with the following parameters: $A_o = \infty$, $\omega_a = 0$, $B = \infty$, $R_i = \infty$ and $R_o = 0$; appropriate corrections are made at a later stage. However, during simulation, an appropriate model has to be used, which would involve a large number of parameters depending on the type of model used for the OA.

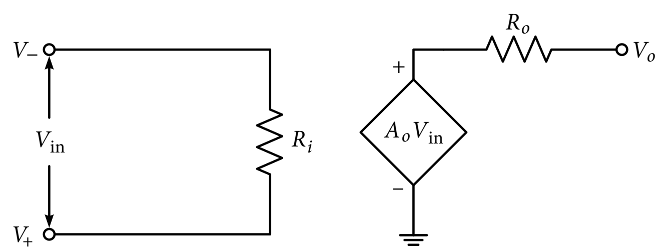

Figure 1.8 Simple model of a practical operational amplifier.

Open-loop Gain and Bandwidth: It is generally preferred to have the value of the open-loop gain A as high as practically feasible. Obviously, during analysis, assuming the value of gain as infinite will create less error if its absolute value is higher. Figure 1.9 shows the typical variation of the open-loop gain with frequency; this is a replica of the frequency response of a general-purpose OA like type 741. Roll-off at the rate of 20 dBs/decade is due to the presence of a compensating capacitor in the OA; the roll-off begins at a small frequency ω_a. Figure 1.9 also depicts the idea of the gain bandwidth of the OA, that is, the frequency where the gain falls to unity or 0 dB. As the gain bandwidth product is a constant, use of OA in the close-loop form increases its 3 dBs, or half-power bandwidth, while decreasing its close-loop gain.

Slew Rate: Signal magnitude needs to be controlled so that the level of output remains below the supply voltage ($\pm V_{DD}$), otherwise, the signal will be clipped and distorted. Another important reason of distortion in OAs is due to its slew rate (SR) limitation. Because of the current driving capability of the transistors used in OAs, output voltage cannot change at a faster rate than specified. The maximum rate of change of the output voltage in terms of SR is as follows:

$$\mathrm{SR} = |dv_o(t)/dt|_{\max} \tag{1.18}$$

Therefore, for undistorted output signal, the bandwidth becomes limited. Its relation with SR is given as:

$$\text{Bandwidth} = \mathrm{SR}/(\pi V_{pp}) \tag{1.19}$$

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Here V_{pp} is the peak-to-peak output voltage. For a typical value of slew rate for the 741 type OA, 0.5V/μs, if maximum useful frequency is fixed at 100 kHz, V_{pp} shall be limited to $(0.5 \times 10^6/\pi 10^5) = 1.59$ volts.

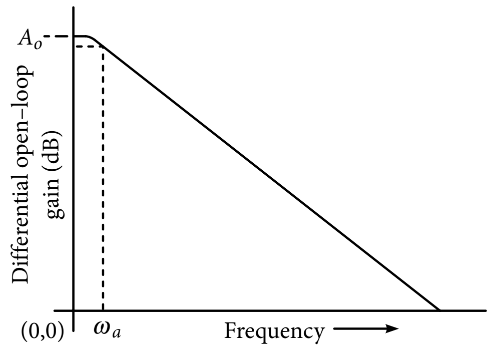

Figure 1.9 Typical variation of open-loop gain of operational amplifier with frequency.

Input Currents: OAs have a differential stage and transistors at this stage require bias current to flow so that the transistors can operate in the saturation region. For bias current to flow, a dc path must be made available for both the inputs to either ground or to the output of the OA. Normally, the mentioned path remains available; but in circuits involving only capacitors/ switches, precaution is needed, as shall be shown later.

1.7.1 Analysis of circuits using operational amplifiers

For accurate analysis, simulation of filters using OAs needs better and appropriate models. However, at the first instance, manual analysis assumes ideal OA with infinite open-loop gain and input resistance and zero output resistance. This means that both input terminals are forced to be at the same potential and no current flows in them. Hence, when the non-inverting terminal is grounded, which happens in a majority of circuits, the inverting terminal is forced to be at the ground potential; the inverting terminal is now said to be at *virtual ground*. This concept of *virtual grounds* has been found to be very useful during analysis. With the suggested conditions of the OA, it is preferable to employ Kirchhoff's current law (KCL) at the input/output terminals of the OAs, making analysis simpler.

The analysis procedure mentioned here gives fairly accurate results at low frequencies (up to a few kHz range). Deviations from the expected results start taking place when the working frequency is increased. Hence, the next step is to consider finite values of the open-loop gain A, resulting in the appearance of a non-zero differential voltage between the inverting and non-inverting input terminals. The gain A can be substituted by an appropriate numerical value or by its frequency-dependent expression in equation (1.17). In specific cases where the effect of finite R_i and R_o needs to be analyzed, there values are taken into consideration; otherwise,

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these are taken as ideal. Considering A to be infinite is very helpful in manual analysis when conventional circuit analysis methods are used.

In the next section, two-integrator circuits are analyzed using the aforementioned method, with $A \to \infty$ as well as A being represented by the frequency-dependent model.

1.8 Integrators Using Operational Amplifiers

A large number of simple and not so simple applications of OAs are in practice. However, the integrator is probably the most common OA application in active filtering. The integrating function has been employed for simulating inductors (say in ladder structure of any order) or for the realization of second-order filter sections. Assuming an ideal OA with infinite open-loop gain, for the circuit shown in Figure 1.10, with no signal current going into the OA, application of KCL at the inverting terminal gives:

$$\frac{(V_{\text{in}} - V_x)}{R} = \frac{(V_x - V_{\text{out}})}{(1/sC)} \tag{1.20}$$

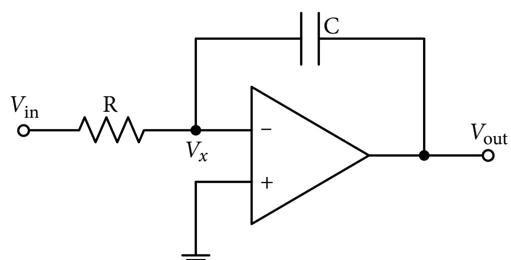

Figure 1.10 Operational amplifier as inverting integrator.

Since $V_x = 0$, the transfer function of the circuit, working as an inverting integrator, is obtained as:

$$\frac{V_{\text{out}}}{V_{\text{in}}} = -\frac{1}{sCR} \tag{1.21}$$

However, with the single-pole roll-off model of equation (1.17) having frequency-dependent finite OA gain, the inverting terminal potential in Figure 1.10 shall be $(-V_o/A)$ instead of virtual zero. Simple analysis gives the closed-loop transfer function:

$$\frac{V_{\text{out}}}{V_{\text{in}}} = -\frac{1}{sCR}\frac{1}{1+\frac{1}{A}\left(1+\frac{1}{sCR}\right)} \tag{1.22}$$

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Expression in equation (1.22) will obviously reduce to equation (1.21) for $A \to \infty$. Hence, a larger value of A is always preferred as it means lesser deviation from the ideal condition.

Another commonly used variation of the integrator is shown in Figure 1.11(a). For a non-ideal OA, with A being finite, the transfer function is obtained as follows:

$$\frac{V_o}{V_{\text{in}}} = -\frac{R_2}{R_1\left(1+sCR_2\right)} \frac{1}{1+\dfrac{1}{A}\left\{1+\dfrac{R_2}{R_1(1+sCR_2)}\right\}} \tag{1.23}$$

$$\simeq -\frac{1}{R_1C\left(s+1/CR_2\right)} \text{ for } A \to \infty \tag{1.24}$$

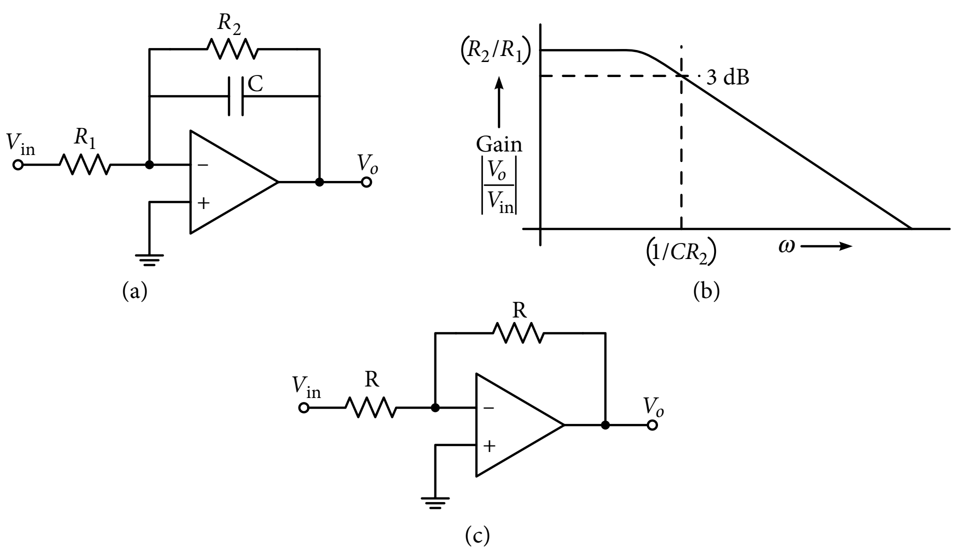

Figure 1.11 (a) Operational amplifier as lossy inverting integrator, (b) its frequency response, and (c) OA as inverter.

Obviously, equation (1.23) will reduce to equations (1.22) and (1.21), respectively, for $R_2 \to \infty$ and further with $A \to \infty$. At high A, the response of equation (1.24) is like a first-order low pass filter (LPF) shown in Figure 1.11(b) with 3 dB frequency (cutoff frequency) and the dc gain as given here:

$$\omega_{\text{LP}} = 1/CR_2 \tag{1.25}$$

$$h_{dc} = (R_2/R_1) \tag{1.26}$$

A non-inverting integrator can be obtained by cascading the inverting integrator with an OA based inverter. For equal value resistors in the inverter shown in Figure 1.11(c), gain is (–1)

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when $A \to \infty$. However, with A represented by the approximated frequency-dependent model of equation (1.17), the circuit's gain is given as:

$$\text{Gain} = -\frac{1}{1+2/A(s)} \to -\frac{1}{1+2s/B} \tag{1.27}$$

This gain expression implies that when an inverter is added to obtain a non-inverting integrator, it adds a bit more non-ideality to the inverting integrator. Compensation methods are used for both inverting and non-inverting integrators to minimize the amount of non-ideality. This will be studied in more detail in Chapter 7.

1.9 Passive Components in Monolithic Filters

In the early stages of the development of active filters, OAs and discrete resistors and capacitors were mounted on a printed circuit board (PCB). The next stage of miniaturization was called *hybrid technology*. In hybrid technology, RC components were fabricated using the (i) thick film, or (ii) thin film technique and the OAs and RC components were bonded in a single package reducing the overall size considerably and improving the reliability of the interconnections between OAs and RC components. In the thick film technique, resistors are fabricated by depositing a composite paste of conducting material and glass on an insulating surface. Depending on the conducting material used, sheet resistance (resistance per square of sheet or strip forming the resistance) of the paste is in the range of 1 Ω/square and 100 MegΩ/square. Hence, resistors in the range of 0.5 Ω to 1 GΩ could be realized with tolerance in the range of 20% to 50%. Reduction in tolerance became possible in the thin film technique through deposition of metal films on insulating material. Fabricated resistance using thin film was much smaller than that realized using thick film as its sheet resistance was in the range of 10 Ω to 10 MegΩ. Capacitances were also fabricated in hybrid technology using thin and thick films.

With advancements in fabrication technology employing bipolar, MOS and CMOS technologies leading to system on chip (SOC) expertise, efforts were taken in the direction of the fabrication of monolithic filters in which passive elements could also be integrated simultaneously with active devices. At present, further advancements are taking place in the fabrication of passive elements in integrated form using new materials and improved physical configurations.

Resistors: Technologies may differ, but the most common method of fabricating resistance is through layering rectangular sheets of semiconducting material. The value of the realized resistance is equal to the number of squares multiplied by the sheet resistance. Obviously, a larger value resistance requires a larger chip area.

While fabricating a transistor, a diffusion layer has also been used to realize resistors without adding any separate processing step. Usually, the base diffusion resistor is the most commonly used resistor in a bipolar process. For such resistors, a matching tolerance of +0.2% between resistors is possible. If the base diffusion area is pinched by diffusing an *n*+ diffusion layer over a *p*-type base diffusion, sheet resistance is increased to 2–10 kΩ/square. Resistances so obtained are called *pinched resistors*; they realize larger resistance, but tolerance becomes worse, ranging up to 50%.

An advanced technique of resistance fabrication is through ion implantation. A very thin layer of implant (0.1–0.8 μm) leads to a very high value of sheet resistance (100–1000 Ω/square). Matching tolerance is also good, being approximately 2%.

In a different technique, resistances are fabricated through active devices. Observing the current–voltage relation of a BJT (bipolar junction transistor) or a MOS (metal oxide semiconductor) transistor, it is easy to see that for a certain range of operation, these devices behave as quite a stable resistance. In MOS or CMOS (complementary MOS) technology, transistors operating in the linear region are used to realize *active resistors*. Resistance at the fabrication/design stage is controlled by the width to length (W/L) ratio of transistors, and at a later stage through gate voltage V_G.

Important advantages of active resistors compared to passive resistors are as follows: (i) required chip area in active resistors is very small for the same value of passive resistance, and (ii) value of the resistance is easily controllable.

Capacitors: Quite a few techniques are available for the fabrication of capacitors in monolithic IC technology: for example, using pn junctions, MOSFETs (metal oxide semiconductor field effect transistors) and polysilicon capacitors. In the BJT process, capacitors are formed between semiconductor junctions. However, the obtained value of the capacitor is very small: 0.05–0.5 pF/mil^2 and large chip areas are needed for even small value capacitors.

It is well known that reverse bias semiconductor junctions create a depletion region. This region acts as an insulator sandwiched between doped silicon on two sides, resulting in a capacitor depending on the width of the depletion region. Along with some parasitic capacitors, a depletion region capacitance of 0.001 pF/μm^2 can be realized.

In the MOS technology process, an MOS capacitor is formed between the *n*+ diffusion regions, while forming the channel region, a polysilicon layer, and a thin layer of silicon dioxide or silicon nitride between them.

Polysilicon capacitors are the most commonly used ones, as the fabrication process suits MOS technology. In this technique, basically, the gate of the transistor is made of polysilicon. Thin oxide is deposited on top of a polysilicon layer which acts as the insulating layer over another bottom polysilicon plate. This type of capacitor too has parasitic capacitance and shall be considered later while discussing switched capacitor circuits in Chapter 15.

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[1.3] Moschytz, G. S. 1974. *Linear Integrated Circuit Fundamentals* Part I. New York: Van Nostrand Reinhold.

Practice Problems

1-1 Find the driving point impedance function $Z(s)$ for the network shown in Figure P1.1 and find its poles and zeroes.

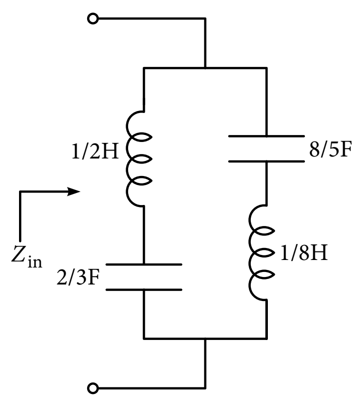

Figure P1.1

1-2 Plot the magnitude and phase of the following transfer functions for $s = j\omega$, $0 \le \omega \le 10$.

(a) $\dfrac{1}{(s+2)(s+4)}$ (b) $\dfrac{s}{(s+3)(s+6)}$

(c) $5\dfrac{(s-2)}{(s+2)}$ (d) $\dfrac{s^2}{(s^2+s+1)}$

1-3 Find the transfer function for the network shown in Figure P1.2.

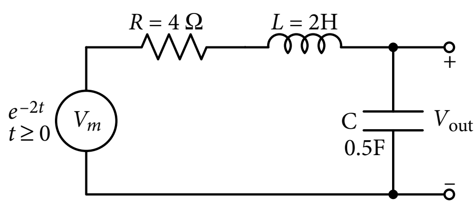

Figure P1.2

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1-4 Sketch the following ideal responses in one figure and classify them.

(i) pass band from 0 to 10 krad/s; rest is stop band.

(ii) pass band from 15 krad/s to infinity; rest is stop band.

1-5 Repeat problem 1.4 for the following:

(i) pass band from 10 to 12 krad/s; rest is stop band.

(ii) stop band from 18 to 20 krad/s; rest is pass band.

1-6 Sketch the magnitude responses for the following values of attenuations and classify them.

(i) α_{max} = 2 dBs, from 0 to 10 krad/s and α_{min} = 40 dBs from 20 krad/s to infinity.

(ii) α_{max} = 1 dBs, from 40 krad/s to infinity and α_{min} = 50 dBs from 0 to 20 krad/s.

(iii) α_{max} = 0.5 dBs, from 14 to 18 krad/s and α_{min} = 40 dBs from 0 to 8 krad/s and from 24 krad/s to infinity.

(iv) α_{max} = 1 dB, from 0 to 6 krad/s and from 14 krad/s to infinity, α_{min} = 50 dBs from 8 to 12 krad/s

1-7 Apply (1/s) transformation to the network shown in Figure P1.2. Determine the new element values and fine its voltage-ratio transfer function again. Does it remain same as before in problem 1-3 or not?

1-8 Determine the input impedance of the network of Figure P1.3, (a) with OA as ideal and (b) when OA is represented by its first-pole roll-off model. Note: Unless specified, approximated model of OA, $A \approx B/s$ shall be used.

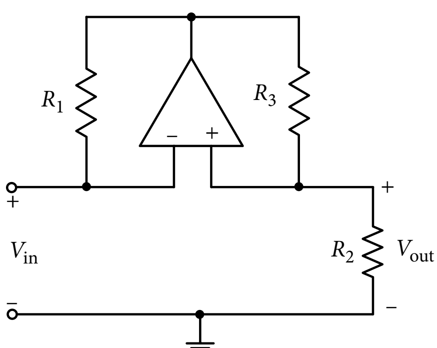

Figure P1.3

1-9 A 741 type OA employing power supply of ± 15 volt was used to construct an inverting amplifier with voltage gain of (− 5). Sinusoidal signal input frequency was 50 kHz. Find the largest input signal before it gets distorted due to the slew rate limitation; assume standard value of SR.

1-10 (a) Find the transfer function for the amplifier shown in Figure P1.4 ($Z_1 = Z_2$ = 5 kΩ) and (Z_1 = 5 kΩ and Z_2 = 50 kΩ). The amplifier is non-ideal and is modeled with single-pole roll-off model with $A_o = 10^5$ and the first pole ω_a = 2 rad/s. Determine the 3-dB bandwidth of the amplifiers.

(b) Repeat problem in part (a) for Figure P1.5.

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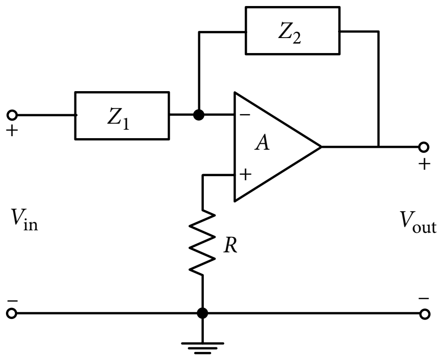

Figure P1.4

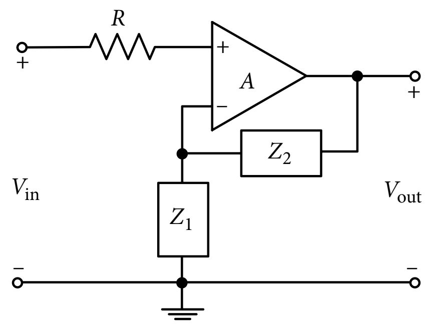

Figure P1.5

1-11 Derive the transfer function of the integrator circuit shown in Figure P1.6. Find its critical frequency when OA used is near ideal and non-ideal with unity gain bandwidth of 10^5 rad/s and $R_1 = R_2 = R_3 = 10$ k Ohm, $R_4 = 5$ k Ohm and $C = 0.1$ nF.

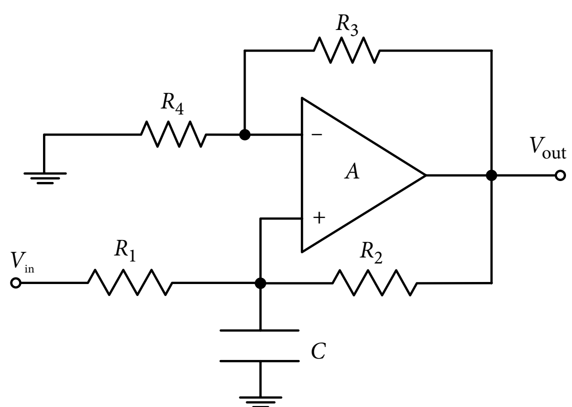

Figure P1.6

1-12 Determine the transfer functions of the circuits shown in figures P1.7 and P1.8, considering OA as (a) ideal and (b) non-ideal with unity gain bandwidth of 10^5 rad/s. In Figures P 1.7 and P 1.8 capacitors $C_1 = C_2 = 1$ pF, $R_1 = 10$ k Ohm, $R_2 = 5$ k Ohm, $R = 1$ k Ohm and $C = 1$ nF. Find critical frequencies when OA is considered ideal and non-ideal.

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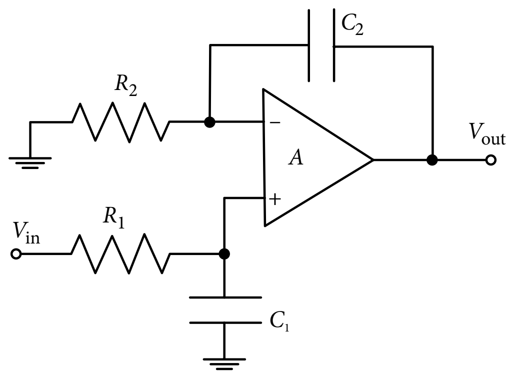

Figure P1.7

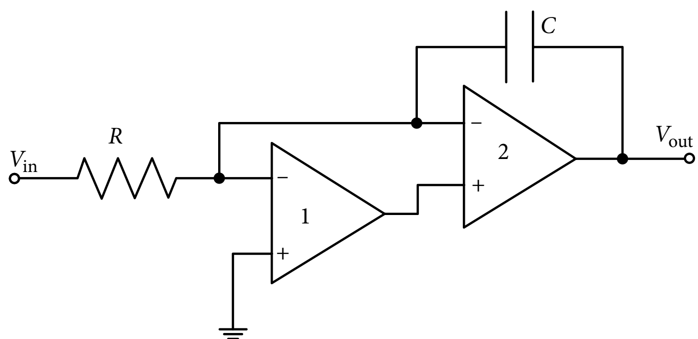

Figure P1.8

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