Chapter 7

Single Amplifier Second-order Filters

7.1 Introduction

Depending on the specifications, the required order of the filter may be small or big. For less challenging requirements, a second-order or even a first-order section may be adequate. However, higher-order sections may become necessary in other situations. For higher-order filter requirements, either the direct form approach is used, or a combination of second-order second sections is used which are connected in a cascade form or in a multiple-loop feedback form. Hence, the study of and realization methods of second-order sections assumes importance as they are used either stand-alone or form the building blocks for higher-order filters. Since the overall performance of the higher-order filter depends on the individual second-order building blocks, it is very important that these building blocks have desirable and attractive properties; the following are some such requirements.

(c) Low sensitivities of the two important filter parameters, pole frequency ω_o and pole-Q, with respect to the circuit elements are highly desirable.

(d) Utilization of a lesser number of passive elements (active elements as well, though in present discussion, it is assumed that only one active component is used) makes the realization economical in discrete, as well as in integrated form.

(e) Values of the passive components should be in the practical range of integrated circuit (IC) fabrication, and the component spread is preferred to be as small as possible in order to make them attractive in IC form.

(f) Independent tunability of the parameters ω_o and Q is important as analog filters require some tuning.

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(g) To properly connect the second-order blocks in cascade or in multiple feedback system, their input impedance should be as high as possible and the output impedance as low as possible at all working frequencies. Otherwise, it can change the characteristics of the overall higher-order filter.

A large number of second-order active building blocks are available with each one claiming advantages over the others. Study of the development procedure of such sections might help the readers to gain more insight towards selecting the optimal section or help in designing new circuits/improving upon the existing ones.

Hence, we begin the study of development of second-order sections using a single passive feedback in Section 7.2; the effect of using different types of passive structures in the feedback is also included. Versatility of the configuration is shown to be improved with multiple feedbacks in Section 7.3. Certain constraints of the single input are overcome in differential-mode input. Next, general active RC feedback is studied in Section 7.5 to gain more understanding of the factors responsible for controlling the sensitivities. It was found that the well-known circuits of Sallen–Key and Delyiannis–Friend, which will be studied in Sections 7.7 and 7.8, are, in fact, special cases of the general active RC feedback structure discussed in Section 7.5.

7.2 Single-feedback Basic Biquadratic Section

A simple and common configuration for generating a biquadratic (biquad) section is shown in Figure 7.1. It uses a single-ended operational amplifier (OA) with its non-inverting input grounded, a passive RC two-port (three terminals) section (N_P) in the negative feedback path and another RC two-port section in the feed-forward path (N_z). Let the two-port sections be represented by the admittance parameter y_z and y_p as follows:

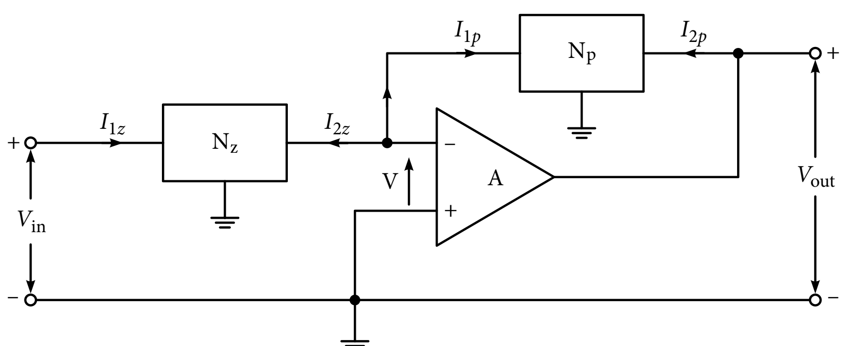

Figure 7.1 Basic biquad configuration using single feedback with a single-ended operational amplifier.

$$\begin{bmatrix} I_{1z} \\ I_{2z} \end{bmatrix} = \begin{bmatrix} y_{11z} & -y_{12z} \\ -y_{21z} & y_{22z} \end{bmatrix} \begin{bmatrix} V_{in} \\ V \end{bmatrix} \tag{7.1}$$

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$$\begin{bmatrix} I_{1p} \\ I_{2p} \end{bmatrix} = \begin{bmatrix} y_{11p} & -y_{12p} \\ -y_{21p} & y_{22p} \end{bmatrix} \begin{bmatrix} V \\ V_{\text{out}} \end{bmatrix} \tag{7.2}$$

With the inverting input terminal of the OA at the virtual ground potential, and its input current neglected being small, summation of currents at the inverting terminal gives:

$$I_{2z} + I_{1p} = 0 \rightarrow -y_{21z} V_{\text{in}} + y_{22z} V + y_{11p} V - y_{12p} V_{\text{out}} = 0 \tag{7.3}$$

If the OA is considered ideal, $A \rightarrow \infty$, $V = 0$ and the transfer function is obtained as:

$$(V_{\text{out}}/V_{\text{in}}) = (-y_{21z}/y_{12p}) \tag{7.4}$$

The networks N_z and N_p being RC-only networks, their natural frequencies lie on the negative real axis of the complex frequency s plane. If the two networks are selected in such a way that their natural frequencies cancel each other, then they will not affect the overall transfer function. In that case, if the transfer function is represented as a ratio of two polynomials in s as in equation (7.5), then selection of an arbitrary polynomial $Q(s)$ representing the natural frequencies of the two RC networks shall be given as in equation (7.6).

$$(V_{\text{out}}/V_{\text{in}}) = \{N(s)/D(s)\} \tag{7.5}$$

$$y_{21z} = -\{N(s)/Q(s)\} \text{ and } -y_{12p} = -\{D(s)/Q(s)\} \tag{7.6}$$

It is obvious from equation (7.5) and (7.6) that the zeros of the transfer function are decided only by the zeros of the transfer admittance y_{21z} of the feed-forward network. Since an RC network without inductors can have transmission zeros anywhere in the complex frequency plane except on the positive real axis, all types of stable transfer functions like LP (low pass), HP (high pass), BP (band pass) or BR (band reject) can be realized except an odd order all pass function. Again, from equation (7.5) and (7.6), poles of the transfer function are decided by the zeros of the transfer admittance y_{12p}, with the same location restrictions.

We can use the following general expression for a normalized biquad:

$$H(s) = \frac{N(s)}{s^2 + (\omega_o / Q) + \omega_o^2} \tag{7.7}$$

From the aforementioned discussion, it is obvious that realization of the transfer function reduces to the synthesis of the two RC two ports N_z and N_p. Conventional methods of RC synthesis are to be used for the realization of the RC networks. However, if $N(s)$ and $D(s)$ contain complex roots, then the synthesis is a little involved. For the second-order admittance functions, Table 7.1 can also be used, though the choice is not limited to this table.

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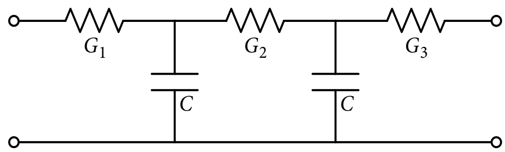
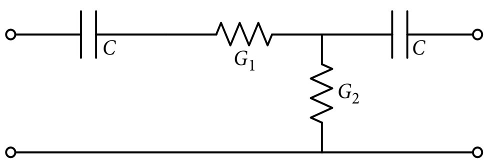
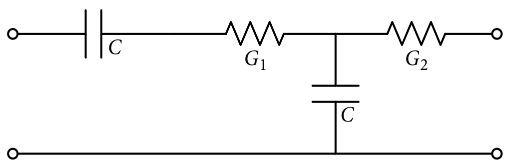
Table 7.1 Short circuit transfer admittances of some common RC two ports sections

Type of function	$H(s)$	Circuit	Element values
Low pass	$\dfrac{a}{D(s)}$		$G_1 = 2a/\delta_2(\delta_1-\delta_2)$ $G_2 = a/\delta_2^2$ $C = 2a/\delta_2{}^2(\delta_1-\delta_2)$
High pass	$\dfrac{as^2}{D(s)}$		$G_1 = a$ $G_2 = \dfrac{(\delta_1-\delta_2)^2 \pm (\delta_1+\delta_2)\Sigma}{(2\delta_1\delta_2)/a}$ $C = \dfrac{(\delta_1\delta_2)\pm\Sigma}{(2\delta_1\delta_2/a)}$ With $\Sigma = \left\{(\delta_1+\delta_2)^2 - 8\delta_1\delta 2\right\}^{\frac{1}{2}}$
Band pass	$\dfrac{as}{D(s)}$		$G_1 = \Sigma_1 \pm \Sigma_2$ $G_2 = \Sigma_1 \mp \Sigma_2, C = a/\delta_1\delta_2$ $\Sigma_1 = \left(\dfrac{1}{\delta_1}+\dfrac{1}{\delta_2}\right)$ $\Sigma_1 = \left(\Sigma_1^2 - 8\delta_1\delta_2\right)^{\frac{1}{2}}$

Example: 7.1 Realize a second-order BPF (band pass filter) section with a pole-Q of 5, a pole frequency of 5 kHz and a center band gain of 20 dB using a single-ended OA based configuration.

Solution: The following will be the frequency-normalized (ω_o = 1) transfer function of a second-order BPF while using the configuration of Figure 7.1.

$$\frac{V_{\text{out}}}{V_{\text{in}}} = \frac{-hs}{s^2 + bs + 1} \tag{7.8}$$

$(\omega_o/Q) = b$, gives $b = 1/5$, and with $s = j\omega$, the gain magnitude at the center band of 20 dBs gives 20 $\log(h/b) = 10$, or $h = 2$. Assuming a tertiary polynomial $Q(s) = (s + \delta)$, for the N_z block, we get the parameter y_{21z} as:

$$y_{21z} = -hs/(s + \delta) \tag{7.9}$$

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A suggested circuit for realizing equation (7.9) is shown in Figure 7.2(a) for which:

$$y_{21z} = (-s/R)/(s + 1/RC) \tag{7.10}$$

Comparing equations (7.9) and (7.10), we get:

$$R = 1/h = 0.5\ \Omega \text{ and } C = h/\delta \tag{7.11}$$

To cancel the arbitrary polynomial $Q(s)$ of equation (7.6) and to find the two-port N_p, it is assumed that:

$$RC = C_2R_1R_2/(R_1 + R_2) \tag{7.12}$$

With selected $Q(s)$ and using the denominator of the transfer function in equation (7.8), $-y_{12p}$ can be written in compact and, then in expanded form as:

$$-y_{12p} = \frac{s^2 + bs + 1}{s + \delta} = s + (b - \delta) + \frac{(1 + \delta^2 - \delta b)}{(s + \delta)} \tag{7.13}$$

Equation (7.13) represents a parallel combination of a capacitor, a resistor and a bridged-T network circuit as shown in Figure 7.2(b). Expressions for the elements shown in the figure are compared with the denominator in equation (7.8); it gives, $a_2 = 1$, $a_1 = b$ and $a_0 = 1$. The bridged-T network is simplified by choosing $\delta = b = 0.2$, which results in R_3 being infinite. With R_3 open circuited, the T network has the following expression for y_{12}:

$$y_{12} = -\frac{(1/R_1R_2C_2)}{s + \frac{1}{C_2}\left(\frac{1}{R_1} + \frac{1}{R_2}\right)} \tag{7.14}$$

A parallel combination of the admittance of a T network and capacitance C_1 gives the admittance of the feedback network $-y_{12p}$ as:

$$-y_{12p} = \frac{s^2 + s\frac{(R_1 + R_2)}{R_1R_2}\frac{1}{C_2} + \frac{1}{R_1R_2C_1C_2}}{\frac{s}{C_1} + \frac{R_1 + R_2}{R_1R_2C_1C_2}} \tag{7.15}$$

Further, from equation (7.13) and Figure 7.2(b), with the following element values and relations ($C_1 = a_2 = 1$ F, $a_1 = b = 0.2$, $a_0 = 1$ and $C = h/\delta = 10$ F), the obtained normalized resistance values are:

$$G_1 = G_2 = 2\,(1 - 0.2 \times 0.2 + 1 \times 0.2 \times 0.2)/0.2 = 10 \text{ Mho} \rightarrow R_1 = R_2 = 0.1\ \Omega \tag{7.16a}$$

Using equation (7.12) and $R = 0.5\ \Omega$, we get the value of $C_2 = (4/\delta^2) = 100$ F. Dividing equation (7.10) by equation (7.15), (y_{21z} by $-y_{12p}$), we obtain the transfer function of the filter for which the normalized pole frequency, $\omega_o = 1/\sqrt{R_1R_2C_1C_2}$ = 1 rad/s. De-normalization with frequency

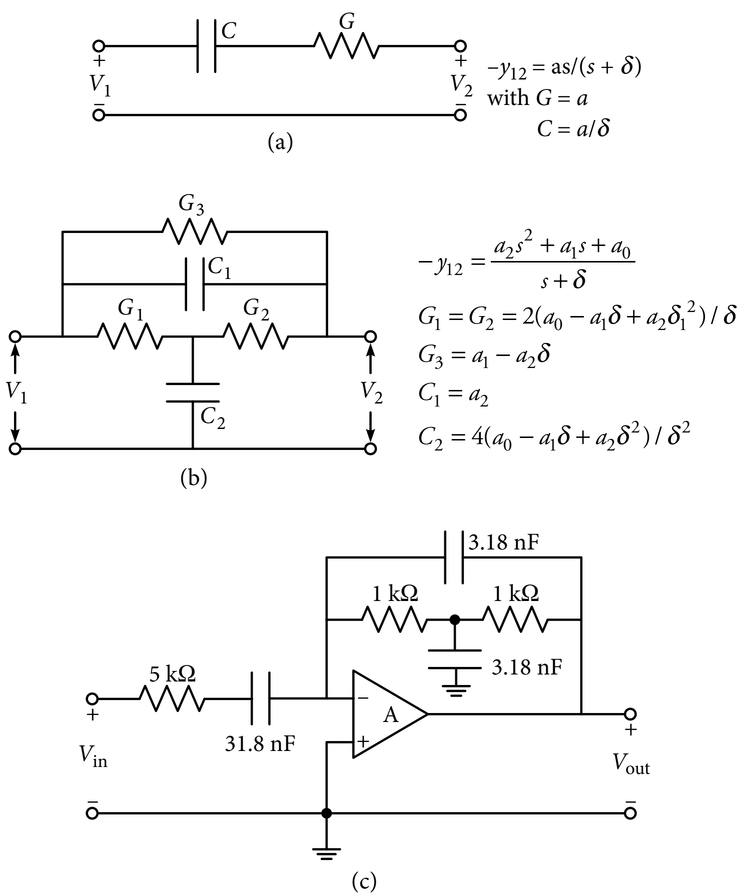
$\omega_o = 2\pi \times 5 \times 10^3$rad/s will give the element values as:

$$C_1 = (1/\pi \times 10^4) = 31.8\ \mu\text{F},\ C_2 = 3.18\ \text{mF and } C = (10/\pi \times 10^4) = 318\ \mu\text{F} \tag{7.16b}$$

Further, for realizing resistance and capacitance values appropriate for IC fabrication, the values obtained can be impedance scaled (say) by a factor of 10^4. The resulting element values will be as follows:

$$C_1 = 31.8\text{nF},\ C_2 = 318\ \text{nF},\ C = 31.8\ \text{nF and } R_1 = R_2 = 1\ \text{k}\Omega;\ R = 5\ \text{k}\Omega \tag{7.17a}$$

The complete second-order BP section using bridged-T network in feedback path is now shown in Figure 7.2(c). Figure 7.2(d) shows the network's PSpice simulated response having a mid-band gain of 9.975 at a center frequency of 4.86 kHz. Bandwidth being 945.3 Hz results in pole-Q = 5.14.



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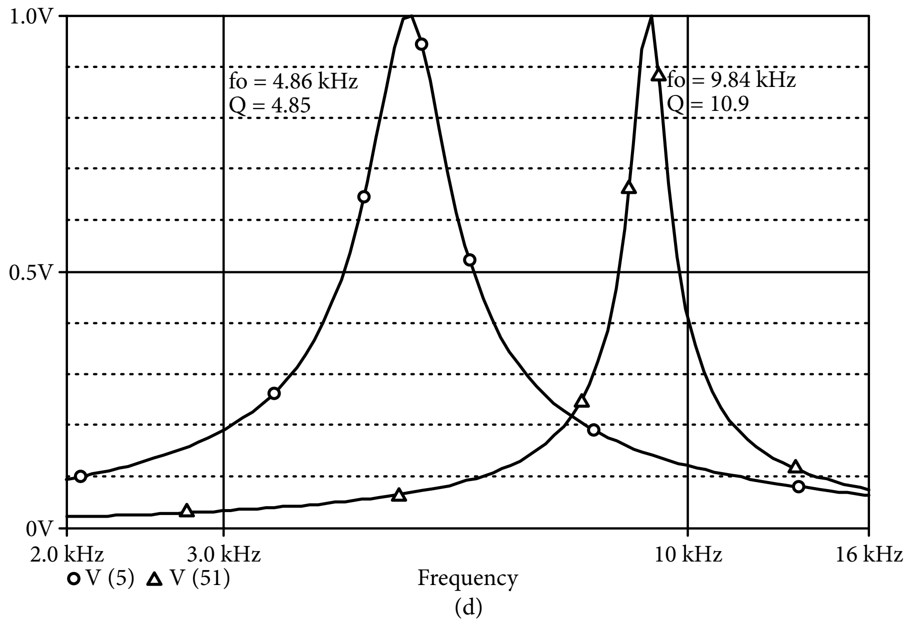

Figure 7.2 (a) An admittance function realization for Example 7.1. (b) An admittance function realization using bridged-T network. (c) Second-order band pass filter section for Example 7.1. (d) Magnitude responses of a band pass filter using a bridged-T network for Example 7.1.

Another BPF for a center frequency of 10 kHz with a mid-band gain of 10 and Q = 10 is designed using the same structure. The required element values in this case are as follows:

$$C_1 = 1 \text{ nF}, C_2 = 0.1 \text{ nF}, C = 40 \text{ nF and } R_1 = R_2 = 7.96 \text{ k}\Omega; R = 159.2 \text{ k}\Omega \tag{7.17b}$$

The simulated response is also shown in Figure 7. 2(d); value of the center frequency is 9.12 kHz; mid-band gain is 9.84 and Q = 10.9.

It is to be noted that the element for the LP, HP and BP responses can be obtained directly from the relations given in Table 7.2.

Table 7.2 Element values for the multi-loop feedback structure

Elements shown in Figure 7.5 (explained later)	Element values of a low pass section	Element values of a high pass section	Element values of a band pass section
Y_1	$G_1 = h/\sqrt{b_0}$	$C_1 = h$	$G_1 = h$
Y_2	$C_2 = \dfrac{(2b_0 + h)}{b_1\sqrt{b_0}}$	$G_2 = \dfrac{b_0(2+h)}{b_1}$	$G_2 = \dfrac{2b_0}{b_1} - h$
Y_3	$G_3 = \sqrt{b_0}$	$C_3 = 1$	$C_3 = 1$

Contd.

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Y_4	$G_4 = G_3$	$C_4 = C_3$	$C_4 = C_3$
Y_5	$C_1 = \dfrac{b_1 \sqrt{b_0}}{(2b_0 + h)}$	$G_5 = \dfrac{b_1}{(2+h)}$	$G_5 = \dfrac{b_1}{2}$

7.2.1 Signal conditioning modules: application example

Signal conditioning modules (SCMs) are an essential part of circuits for measuring process control variables, such as temperature, pressure, strain, and so on. SCMs are subject to a number of noise signals which are both electrically and magnetically induced. Elimination of noise from the mixed signal requires appropriate filtering.

One such module, namely, the *DSCA 43 DIN rail analog input module, 4-way isolation, multiple LP, anti-aliasing filter* is discussed here as an application example [7.1] of a single feedback biquadratic configuration. The module requires LPF with a 4 Hz, 3 dB frequency at a roll-off of 120 dBs per decade. Such a filter attenuates the 50/60 Hz noise by 100 dBs.

To get the required roll-off, order of the LPF can be calculated: it is found to be six. Hence, from Table 3.1, the pole locations of the sixth-order Butterworth characteristics are as follows:

$$-0.2588 \pm j0.9659,\ -0.7071 \pm j0.7071,\ -0.9659 \pm j0.2588. \tag{7.18}$$

Since it will be an all pole filter, the following will be the transfer functions of the three second-order sections that will be connected in cascade.

$$H_1(s) = 1/(s^2 + 0.5176s + 1) \tag{7.19a}$$

$$H_2(s) = 1/(s^2 + 1.41s + 1) \tag{7.19b}$$

$$H_1(s) = 1/(s^2 + 1.9318s + 1) \tag{7.19c}$$

A number of circuits are available for realizing second-order LPFs. Figure 7.1 shows the structure of the single feedback with a single input op amp circuit that will be used for this application example. The following is a standard form of the normalized transfer function of a second-order LPF:

$$\frac{V_{\text{out}}}{V_{\text{in}}} = -\frac{h_{lp}}{\left(s^2 + bs + 1\right)} \tag{7.20}$$

For the first LP section, using equation (7.19), $b = 0.5176$ and with dc gain of unity, $h_{lp} = 1$.

Following the procedure of Example 7.1, a tertiary polynomial $Q(s) = (s + \delta)$. The following will be the parameter y_{21z} for the block N_z:

$$y_{21z} = -\,a/(s + \delta) \tag{7.21a}$$

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A suggested circuit for realizing equation (7.21a) is shown in Figure 7.3(a), for which:

$$G = 2a/\delta \text{ and } C = 4a/\delta^2 \tag{7.21b}$$

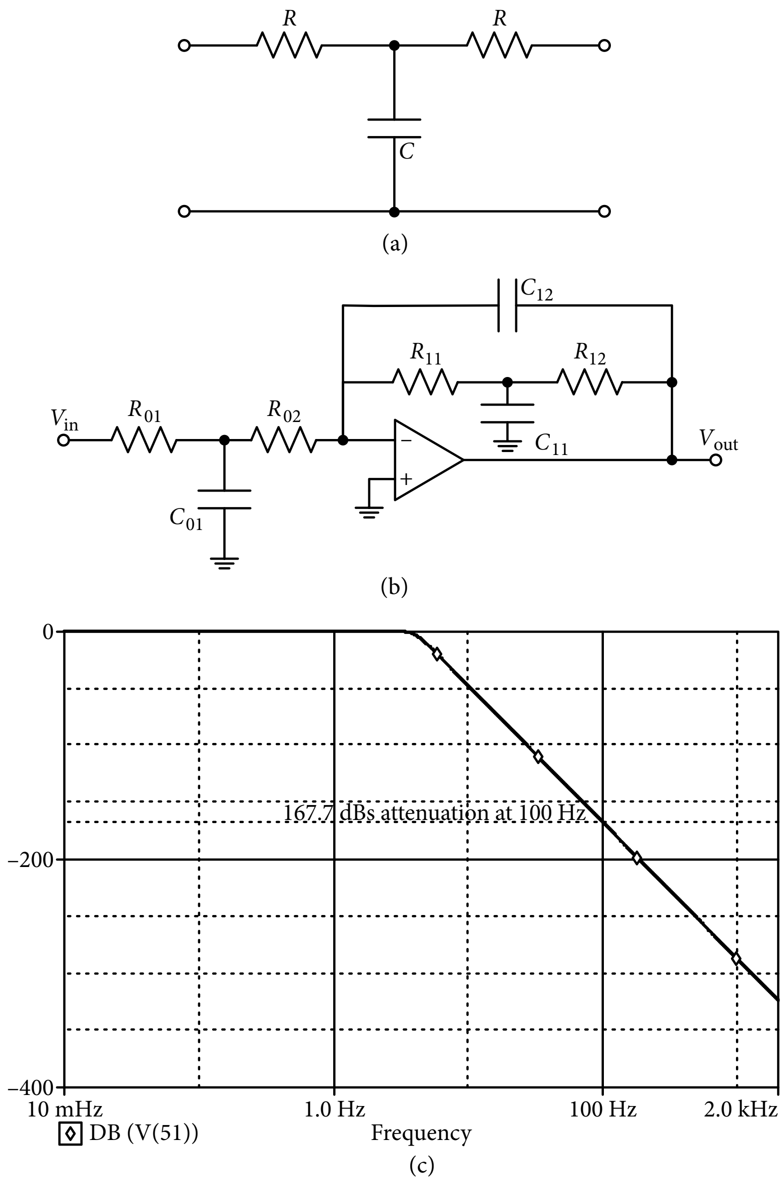

Figure 7.3 (a) A suggested circuit for the realization of equation (7.21a). (b) A second-order low pass section obtained while employing a single feedback biquadratic section. (c) Simulated magnitude response of the sixth-order filter shown in Figure 7.3(b).

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To cancel the arbitrary polynomial $Q(s)$ of equation (7.21a) and find the two-port N_p, the bridged-T network of Figure 7.2(b) is selected. With $a_2 = 1$, $a_1 = b = 0.5176$ and $b_o = 1$, we get $C_1 = 1$ F, $C_2 = \dfrac{4}{0.5176^2} = 14.936\,\text{F}$. The resistance values are: $R_1 = R_2 = \delta/2 = 0.2588$, $R_3 = \infty$, with $\delta = b$.

For the T network of Figure 7.3(a), $R_{01} = \delta/2h = 0.2588$ and $C_{01} = 4h/\delta^2 = 4/0.5166^2 = 14.936$ F.

The component values are de-normalized using an impedance scaling factor of 10 kΩ and frequency scaling factor of 24.143 rad/s (4 Hz). The element value for the first second-order section is given in equation (7.22a) and the corresponding first LP section with element symbols is shown in Figure 7.3(b).

$$R_{01} = 2.588\ \text{k}\Omega = R_{11} = R_{12},\ C_{01} = 59.378\ \mu\text{F} = C_{11},\ C_{12} = 3.977\ \mu\text{F} \tag{7.22a}$$

Similar steps were taken to design the other two second-order sections. De-normalized values of the corresponding elements are as follows:

$$R_{02} = 7.705\ \text{k}\Omega = R_{21} = R_{22},\ C_{02} = 7.954\ \mu\text{F} = C_{21},\ C_{22} = 3.977\ \mu\text{F} \tag{7.22b}$$

$$R_{03} = 9.659\ \text{k}\Omega = R_{31} = R_{32},\ C_{03} = 4.262\ \mu\text{F} = C_{31},\ C_{32} = 3.977\ \mu\text{F} \tag{7.22c}$$

The three sections are cascaded with $H_1(s)$ being the first section with minimum pole-Q followed by $H_2(s)$ and $H_3(s)$. The simulated magnitude response of the cascaded filter is shown in Figure 7.3(c). Its 3-dB frequency occurs at 3.9989 Hz and with an attenuation of 47.726 dBs at 10 Hz and 167.73 dBs at 100 Hz, the obtained roll-off is 120.3 dBs per decade.

7.2.2 Bridged twin-T RC network

In a number of applications, it becomes necessary for a BP section to be more selective. In such cases, the filter requires a high value of pole-Q (or a small value of the coefficient b in equation (7.8)). It was observed in Example 7.1 that even for $Q = 5$, the component spread is high, which is not attractive in IC implementation. Hence, instead of using the bridge-T network shown in Figure 7.2(b), it is advised to use a bridged twin-T network which is shown in Figure 7.4(a). A bridged twin-T network is in effect a parallel combination of an HP-T, an LP-T network and a resistance R^*. For the HP-T network, which comprises C_1, C_2 and R_3, transfer admittance, $-y_{12H}$ is given as:

$$-y_{12H} = \frac{\left\{(C_1C_2)/(C_1+C_2)\right\}s^2}{s+\dfrac{1}{R_3(C_1+C_2)}} \tag{7.23a}$$

However, for the LP-T network comprising R_1, R_2 and C_3, transfer admittance is given as:

$$-y_{12L} = \frac{1/(R_1R_2C_3)}{s+\dfrac{1}{C_3}\left(\dfrac{1}{R_1}+\dfrac{1}{R_2}\right)} \tag{7.23b}$$

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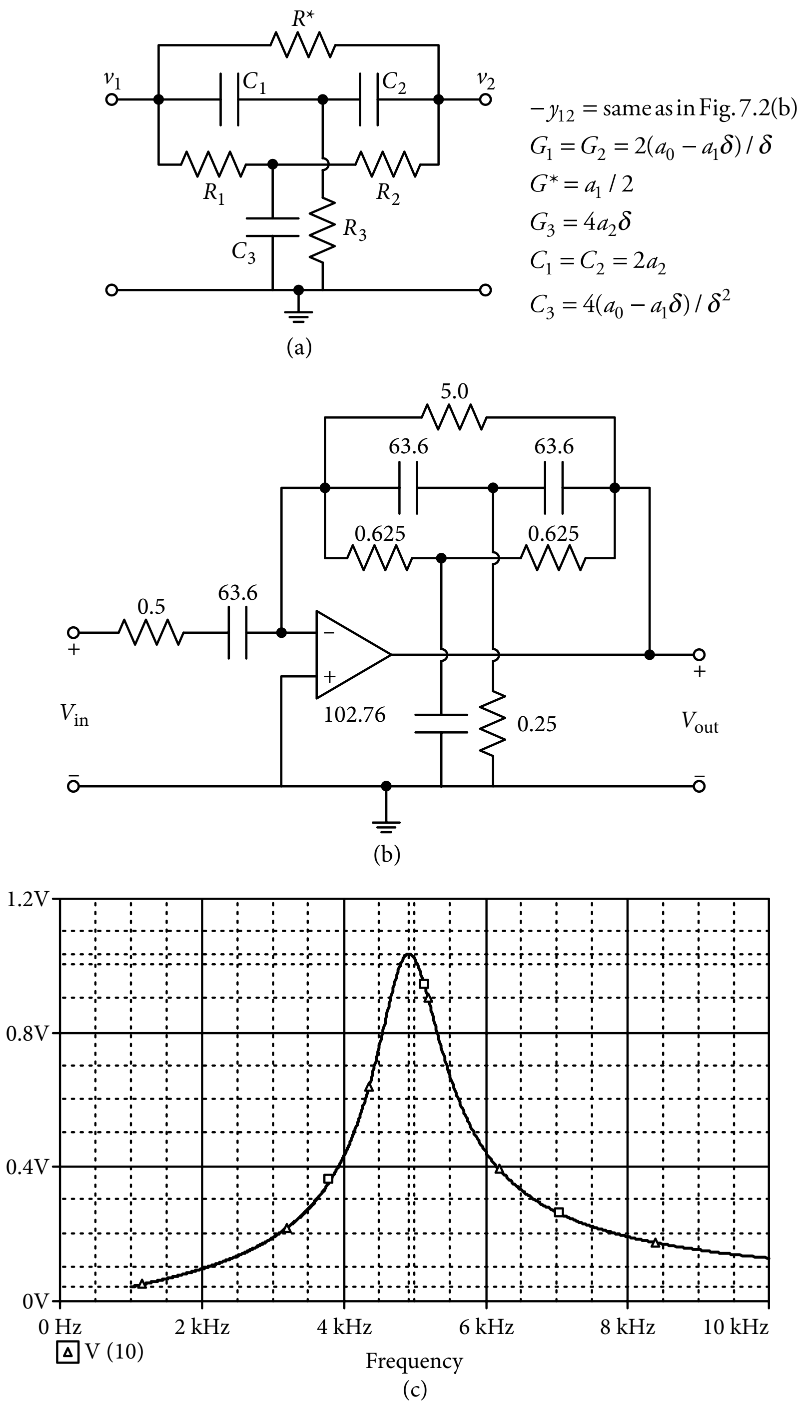

Figure 7.4 (a) A bridged twin-T network. (b) Circuit realization for the band pass filter of Example 7.2. Capacitors are in nF and resistors are in kΩ. (c) Magnitude response of the band pass filter using twin-T feedback for Example 7.2.

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If the twin-T network is symmetrical, $R_1 = R_2$ and $C_1 = C_2$, a factor of half appears in the simplified expression of equation (7.23a, b), which forces:

$$b \text{ to be halved} \rightarrow \text{effective } R^* \text{ is halved.} \tag{7.24}$$

Utilization of the twin-T network in the feedback path of the basic configuration of Figure 7.1 requires the following form of representation of the transfer admittance $-y_{12p}$:

$$-y_{12p} = \frac{s^2}{(s+\delta)} + \frac{(1-\delta b)}{s+\delta} + b \tag{7.25}$$

Again, expressions of the elements in Figure 7.4(a) and the denominator of equation (7.8) are compared. It is worth noting that the twin-T network is a third-order section, but can be reduced to a second-order section, according to our present requirement by single pole–zero cancellation through the selection of δ as:

$$\delta = \frac{1}{R_3(C_1+C_2)} = \frac{1}{C_3}\left(\frac{1}{R_1}+\frac{1}{R_2}\right) \rightarrow \frac{1}{2R_3C_1} = \frac{2}{C_3R_1} \tag{7.26}$$

The following example will illustrate the procedure and the advantage in terms of getting a low component spread.

Example 7.2: Redesign the BP section of Example 7.1 while using the twin-T network of Figure 7.4(a).

Solution: With $Q = 5$, $a_1 = b = 0.2$, and as in Example 7.1, $h = 2$, $R = (1/h) = 0.5\ \Omega$ and $C = h/\delta = 2$ F. For δ assumed to be unity and using the expressions for elements in Figure 7.4(a), with $a_0 = 1$, $a_1 = 0.2$ and $a_2 = 1$.

$$C = (h/\delta) = 2\text{ F} \tag{7.27a}$$

$$R_1 = R_2 = \{\delta/2(a_0 - a_1\delta)\} = 0.625\ \Omega \text{ and } R_3 = 1/4a_2\delta = 0.25\ \Omega \tag{7.27b}$$

$$R^* = (2/0.2)/2 = 5\Omega,\ C_1 = C_2 = 2a_2 = 2\text{ F},\ C_3 = 4\,(a_0 - a_1\delta)/\delta^2 = 3.2\text{ F} \tag{7.27c}$$

The component spread comes down significantly for capacitance from 100 to 1.6, though the resistance spread has increased to 20. For $\omega_o = 2\pi \times 5$ krad/s, capacitance values will change as follows:

$$C_1 = C_2 = 63.6 \times 10^{-6}\text{ F},\ C_3 = 102.76 \times 10^{-6}\text{ F and } C = 63.6 \times 10^{-6}\text{ F} \tag{7.27d}$$

To bring all components in the preferable range for IC integration, impedance scaling by a factor of 10^3 is performed to give the following element values.

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$$R_1 = R_2 = 625\ \Omega,\ R_3 = 250\ \Omega,\ R^* = 5\ \text{k}\Omega,\ C_1 = C_2 = 63.6\ \text{nF},\ C_3 = 102.76\ \text{nF and } C = 63.6\ \text{nF} \tag{7.27e}$$

The complete circuit is shown in Figure 7.4(b) and its PSpice simulated response is shown in Figure 7.4(c). Its center frequency is 4.93 kHz, bandwidth is 938 Hz resulting in Q = 5.25, and a mid-band gain of 10.33; these results are close to the design specifications with improved capacitor spread.

7.3 Multiple Feedback Single Amplifier Biquad (SAB)

A large number of configurations are available in literature in which a number of two-terminal passive elements are connected in the negative feedback path of an OA. The multiple feedbacks can provide realizations of biquadratic and all pole functions with different levels of advantages and constraints. A widely used multi feedback single amplifier arrangement obtained from a multiple-loop feedback arrangement [7.2], is a double-ladder or Rauch structure shown in Figure 7.5. Application of Kirchhoff's current law (KCL) at nodes 1 and 2 with node 1 at virtual ground gives the following equations, respectively.

$$V_{out} Y_5 + V_2 Y_3 = 0 \tag{7.28a}$$

$$(V_{in} - V_2) Y_1 + (V_{out} - V_2)\ Y_4 + (-V_2) Y_3 - V_2 Y_2 = 0 \tag{7.28b}$$

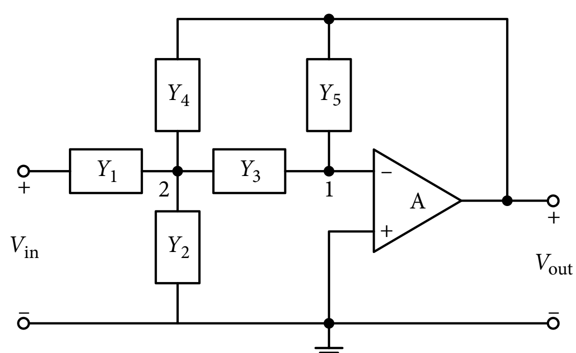

Figure 7.5 Double ladder multiple feedback single amplifier second-order section.

Hence, the voltage ratio transfer function is obtained as follows:

$$\frac{V_{out}}{V_{in}} = \frac{-Y_1 Y_3}{Y_3 Y_4 + Y_5\ (Y_1 + Y_2 + Y_3 + Y_4)} \tag{7.28c}$$

Selection of elements for the admittances Y_1 to Y_5, either as a resistor or capacitor, can give a large number of circuits. Their selection decides whether the filter would be an LP, HP, BP or

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any other type. Instead of using a single-element, a combination of RC components can also be used: this provides more versatility to the network. For any specific selected combination of elements, the network can be studied to determine the kind of response that can be obtained and any constraint, such as the achievability of a quality factor, or the level of sensitivity of the parameters with respect to the passive and active elements. Out of the many choices available, if Y_1 and Y_3 are conductances, Y_5 is a capacitor and either Y_2 or Y_4 is a capacitor as shown in Figure 7.6, an LP response with the following transfer function is obtained.

$$\frac{V_{out}}{V_{in}} = -\frac{G_1G_3}{C_2C_5s^2 + C_5(G_1 + G_3 + G_4)s + G_3G_4} \tag{7.29}$$

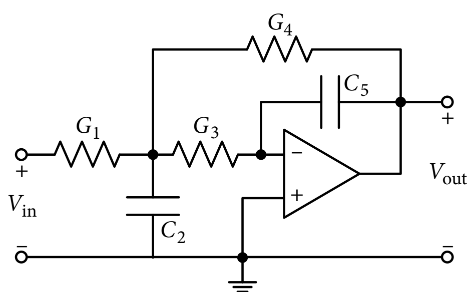

Figure 7.6 Second-order low pass function realization of equation (7.29).

Further showing the versatility of the structure, a BP function can be realized with either Y_1 or Y_3 as a capacitor, and the other one a conductor. If $Y_1 = G_1$ and $Y_3 = sC_3$, Y_4 needs to be a capacitor, Y_5 a conductor and Y_2 can be either a capacitor or conductor (shown in Figure 7.7). Hence, selecting $Y_2 = G_2$, the transfer function shall be that of a BPF:

$$\frac{V_{out}}{V_{in}} = \frac{-G_1C_3\,s}{C_3C_4s^2 + G_5(C_3 + C_4)s + G_5(G_1 + G_2)} \tag{7.30}$$

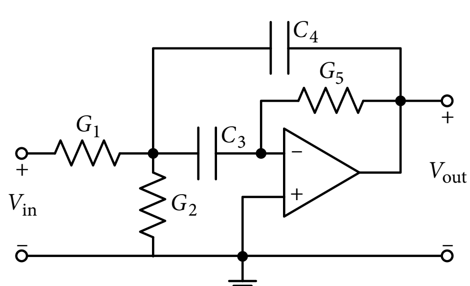

Figure 7.7 Second-order band pass function realization of equation (7.30).

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For the realization of an HP function, the numerator will have a term with s^2. Hence, selecting both Y_1 and Y_3 as capacitors, Y_4 also needs to be a capacitor. With Y_2 and Y_5 as conductors, the circuit will be as shown in Figure 7.8(a) with the following HP function.

$$\frac{V_{\text{out}}}{V_{\text{in}}} = \frac{-C_1C_3s^2}{C_3C_4s^2 + G_5(C_1 + C_3 + C_4)s + G_2G_5} \tag{7.31}$$

Designing of the filter section can easily be performed using the coefficient matching technique. Consider the HP second-order voltage ratio transfer function:

$$\frac{V_{\text{out}}}{V_{\text{in}}} = -\frac{hs^2}{b_2s^2 + b_1s + b_0} \tag{7.32}$$

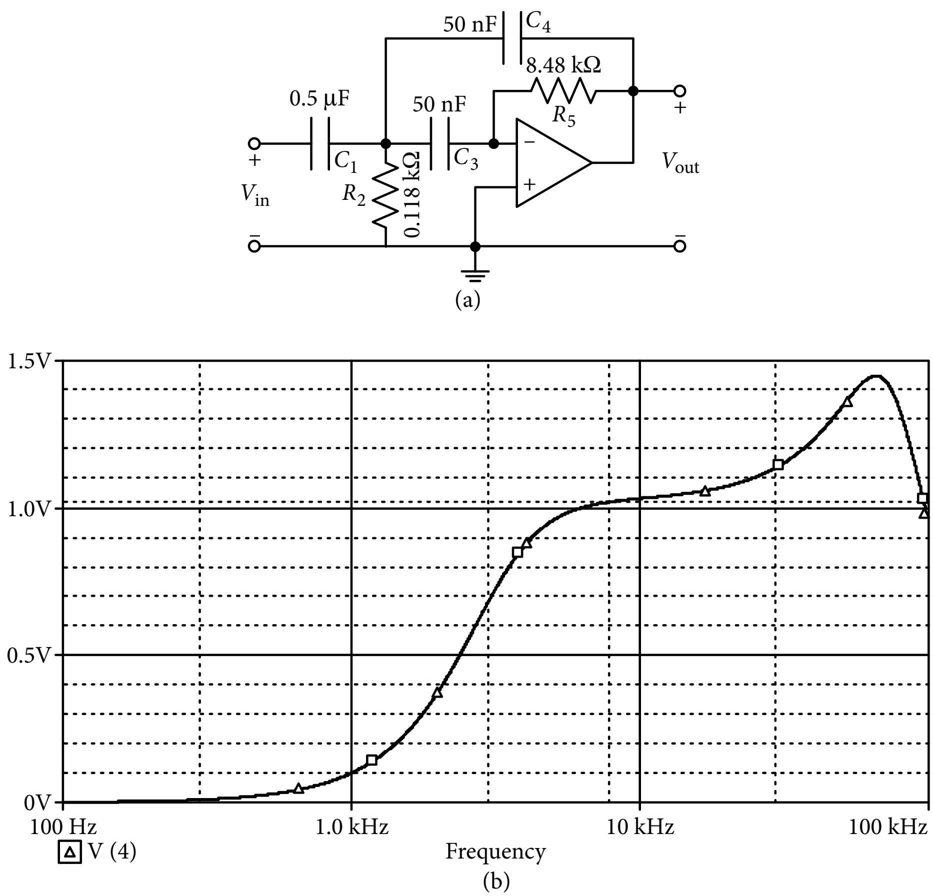

Figure 7.8 (a) Second-order high pass function realization of equation (7.31). (b) Second-order high pass filter response for the circuit shown in Figure 7.8(a) for Example 7.3.

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By comparing the coefficients for the HP function of equation (7.31) with equation (7.32), the following design equations can be obtained.

$$h = C_1\, C_3,\; b_0 = G_2 G_5 \tag{7.33a-b}$$

$$b_1 = G_5(C_1 + C_3 + C_4),\; b_2 = C_3\, C_4 \tag{7.33c-d}$$

Selection of element values usually put a constraint on the performance parameters. For example, if equal value capacitors are used, that is, $C_1 = C_3 = C_4 = C$, with $b_2 = h$, h will also be unity. If we use equal value capacitors, having h as unity is not a severe constraint, rather it is an advantage. However, these capacitors may result in a high spread of resistance; it may also require an additional amplifier to get the required gain (if needed).

Example 7.3: Design a normalized second-order HP Butterworth filter using multiple feedback in a single OA configuration, with a pass band gain of 20 dBs. De-normalize the filter for a cut-off frequency of 3.2 kHz. Use suitable element values and obtain the filter's response.

Solution: The network structure is shown in Figure 7.8(a) and the desired Butterworth function has $h = 10$, $b_1 = \sqrt{2}$ and $b_0 = 1$. As mentioned earlier, many possibilities are available for the selection of element values. Solving equations (7.32) and (7.33) for equal value capacitors and without using an extra amplifier for getting $h = 10$, two possible sets of relations and resulting elements are as follows:

Set 1: With $C_1 = C_3 = C$, $h = 10 = C^2 \rightarrow C = \sqrt{10}$, $b_o = G_2 G_5$, $b_1 = \sqrt{2} = G_5\left(2\sqrt{10} + C_4\right)$ and $b_2 = 1 = \sqrt{10}C_4 \rightarrow C_4 = 1/\sqrt{10}, G_5 = \sqrt{20}/21, G_2 = 21/\sqrt{20}$

Set 2: With $C_3 = C_4 = C$, $b_2 = 1$ $C^2 \rightarrow C = 1$, $h = 10 = C_1$, $b_1 = \sqrt{2} = G_5(10 + 1 + 1) \rightarrow G_5 = \sqrt{2}/12, b_0 = 1 = G_2\sqrt{2}/12 \rightarrow G_2 = 12/\sqrt{2}$

Element values for the normalized functions are in ohms and farads, respectively. De-normalizing the filter section with $\omega = 2\pi \times 3.2$ krad/s and using an impedance scaling factor of 10^3, value of the elements for the second set are as follows:

$$C_3 = C_4 = 50 \text{ nF},\; C_1 = 0.5\ \mu\text{F},\; R_2 = 0.118 \text{ k}\Omega \text{ and } R_5 = 8.48 \text{ k}\Omega$$

Figure 7.8(b) shows the PSpice simulated response of the HP filter. The filter's output flattens at 1.028 volts for an input voltage of 0.1 volt, providing a high frequency gain of 10.028. The cut-off occurs at 3.19 kHz; a very close response to the design. A peaking occurs at higher voltage due to the frequency dependent gain of the OA.

7.3.1 Power line communication: automatic meter reading

Automatic meter reading (AMR) is one of the most well-known applications of power line communication (PLC). PLC is now considered an optimal solution to provide communication

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between a residential or industrial consumer and power distributers. However, the frequency response of the electric grid for a certain consumer is different from any other consumer due to various reasons; because of this reason, a system level solution is required.

The European regulatory committee responsible for allowing the communication requirement (CENELEC) has provided five different frequency bands and stipulated the maximum transmission and distribution levels when transmitting data over the power line. The frequency range for the signal transmission of signals has been divided into five bands, and the frequency range for the distribution company use and their licensees is from 9 kHz to 95 kHz. A fourth-order LP Butterworth filter having a cut-off frequency of 95 kHz have been employed [7.3]. Though other circuit structures can be used, the multiple feedback topology of Figure 7.6 was selected; two such stages may be connected in cascade.

From Table 3.1, location of poles for a fourth-order Butterworth filter are as follows:

$$s_{1,2} = -0.38268 \pm j0.9238,\ s_{3,4} = -0.9238 \pm j0.38268$$

Hence, the normalized transfer function for the two second-order stages will be as follows:

$$H_1(s) = \frac{2.5}{s^2 + 0.76536s + 1} \text{ and } H_2(s) = \frac{2.0}{s^2 + 1.9576s + 1}$$

To get an overall dc gain of 5 (14 dBs), the numerators of the transfer functions were selected as 2.5 and 2.0, respectively. Both the transfer functions are compared with equation (7.29) and the elements are de-normalized with a frequency scaling factor of 95(2π) krad/s and an impedance scaling factor of 1 kΩ. The resulting element values for the two sections are as follows:

$$R_{11} = R_{31} = 3.135\ \text{k}\Omega,\ R_{41} = 7.839\ \text{k}\Omega,\ C_{21} = 1.674\ \text{nF},\ C_{51} = 68.1\ \text{pF}$$

$$R_{12} = R_{32} = 1.277\ \text{k}\Omega,\ R_{42} = 2.554\ \text{k}\Omega,\ C_{22} = 1.674\ \text{nF},\ C_{52} = 0.5132\ \text{nF}$$

The simulated frequency response of the filter is shown in Figure 7.9, having a 3 dB frequency of 93.1 kHz.

Another PLC module for the distribution of signals in the CENELEC band of frequencies is available [7.4]. The module's receiver signal path is shown in Figure 7.10. It consists of a passive HPF with a cut-off frequency of 25 kHz, an active LPF with a cut-off frequency of 125 kHz, with two in-between programmable gain amplifiers (PGAs). The third-order passive HPF with element values is shown in Figure 7.11(a). The third-order LPF introduces 6 dBs of attenuation to keep the dc level at 1.65 V according to the specifications. For the third-order active LP filter, a multiple feedback circuit as shown in Figure 7.6 is used. The value of poles for a third-order Butterworth filter from Table 3.1 are as follows:

$$s_{1,2} = -0.5 \pm j0.866 \text{ and } s_3 = -1.0$$

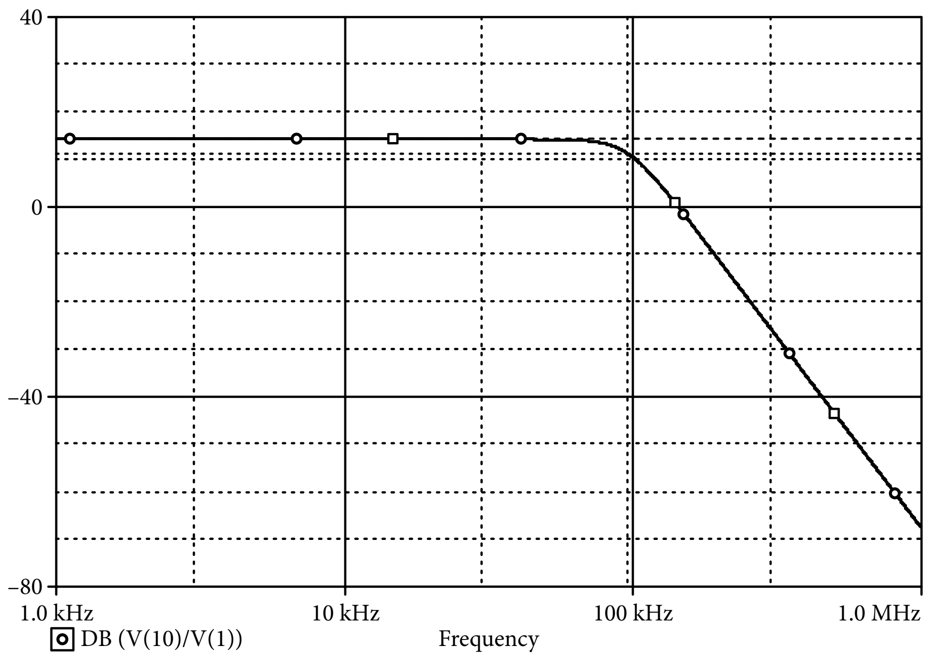

Figure 7.9 Magnitude response of a fourth-order low pass filter for power line communication.

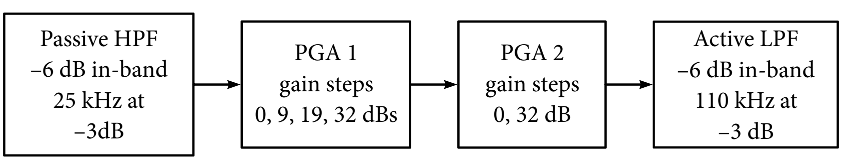

Figure 7.10 Receiver signal path for a PLC module [7.4].

Hence, the transfer function of the LPF with a dc gain of 0.5 (-6dB) will be as follows:

$$H_{LP}(s) = \frac{0.5}{(s^2+s+1)(s+1)} = \frac{0.5}{(s^2+s+1)}\frac{1}{(s+1)}$$

For the first-order LP section, the normalized elements will be $R_0 = 1\Omega$ and $C_0 = 1$ F. Using equation (7.29), the normalized elements for the second-order section are $R_1 = R_3 = 4\Omega$, $R_4 = 2\Omega$, $C_2 = 1$F, $C_5 = 0.125$ F. De-normalization with a frequency scaling factor of $125 \times (2\pi)$ krad/s and an impedance scaling factor of $10^3\ \Omega$ results in the following element values.

$$R_0 = 1\ \text{k}\Omega,\ C_0 = 0.12727\ \text{nF},\ R_1 = R_3 = 4\ \text{k}\Omega,\ R_4 = 2\ \text{k}\Omega,\ C_2 = 0.12727\ \text{nF},\ C_5 = 0.159\ \text{nF}$$

The third-order LPF is shown in Figure 7.11(b). The filter needs to remove signals having a frequency above 110 kHz with a 6 dBs in-band attenuation. The simulated frequency response

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of the HPF and the LPF is shown in Figure 7.11(c) with a 3 dB frequency of 125.3 kHz, in conformity with specifications.

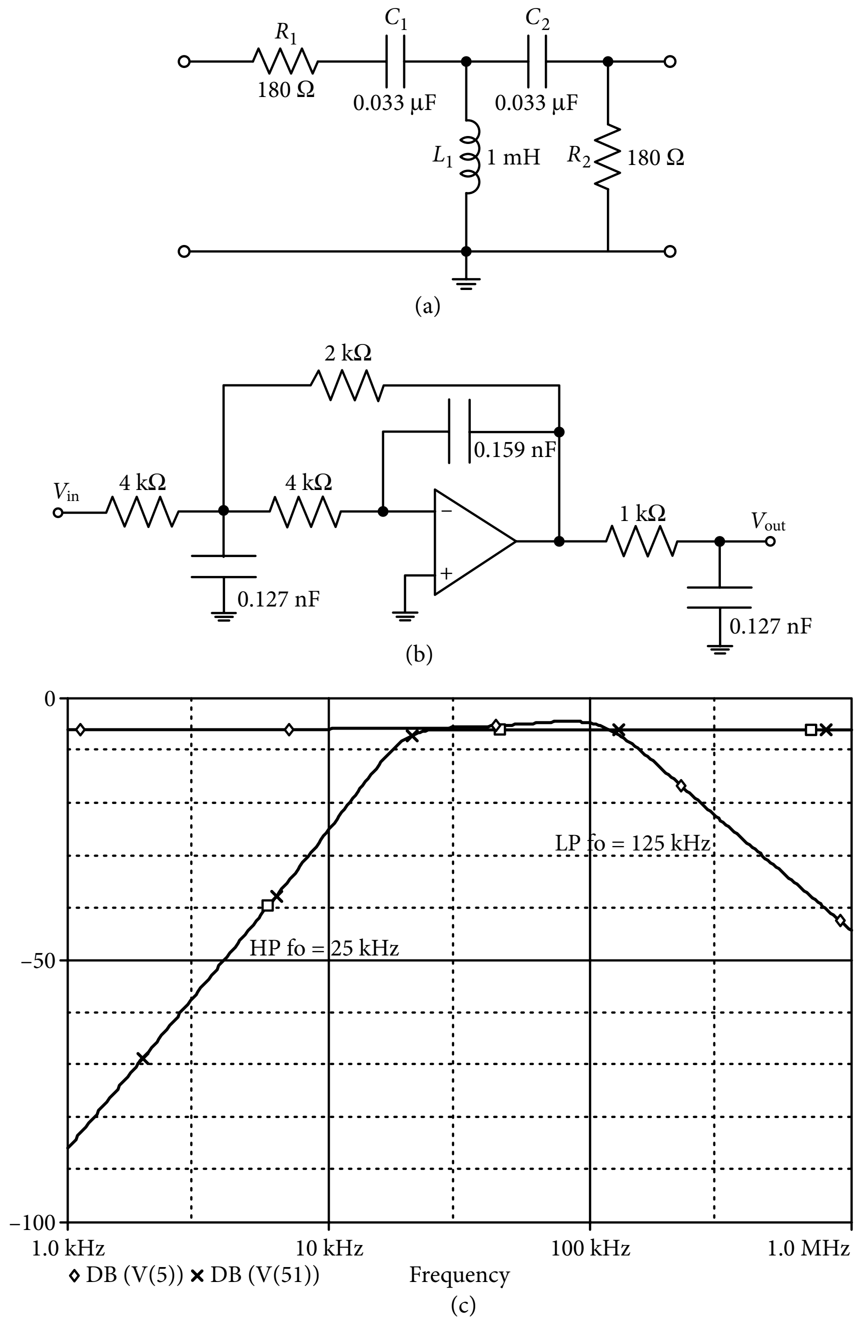

Figure 7.11 (a) Passive high pass filter structure for Figure 7.10 [7.4]. (b) Third-order low pass filter. (c) Responses of the third-order active low pass and passive high pass for CENELEC module.

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7.4 Differential Input Single Amplifier Filter Sections

Single amplifier single-ended sections explained in Sections 7.2 and 7.3 are good examples of efficient and economical implementation of second-order sections. However, there are certain general constraints when using these sections, which include their inability to realize complex conjugate zeros or their need for a specific choice of elements. It has been observed that utilization of the differential nature of OAs increase the versatility of the filter section. In the following sub-sections, we will discuss this nature in more detail.

7.4.1 Differential input single feedback biquad

Figure 7.12(a) shows an OA in differential mode with single feedback for realizing the same range of transfer functions as that in the sections explained in Sections 7.2 and 7.3 [7.2]. The open circuit transfer functions of the passive RC networks N_z and N_p are $H_z(s)$ and $H_p(s)$, respectively. With the open-loop gain of the OA being A, we get:

$$V_{\text{out}} = A(V_1 - V_2) = A(H_z V_{\text{in}} - H_p V_{\text{out}})$$

$$(V_{\text{out}}/V_{\text{in}}) = H_z/(H_p + 1/A) = H_z/H_p \text{ for } A \to \infty \tag{7.34}$$

If the poles of $H_z(s)$ and $H_p(s)$ are chosen in such a way that they cancel each other, the synthesis of the transfer function resolves into the separate synthesis of two-port RC networks. For the normalized second-order case, the following $H_z(s)$ can be realized using a number of simple ladder structures, whereas the $H_p(s)$ of equation (7.36) may be a bridged-T or twin-T structure as explained earlier.

$$H_z(s) = N(s)/(s^2 + bs + 1) \tag{7.35}$$

$$H_p(s) = \frac{s^2 + (1/Q)s + 1}{s^2 + bs + 1} \tag{7.36}$$

For finite A, substituting equations (7.35) and (7.36) in equation (7.34), we get:

$$\frac{V_{\text{out}}}{V_{\text{in}}} = \frac{1}{1 + 1/A} \frac{N(s)}{s^2 + \left(\frac{1}{Q}\right)\left(1 + \frac{Q_p}{A}\right)s + 1} \tag{7.37}$$

This equation gives the effective selectivity or quality factor as:

$$Q_e = Q/(1 + Q_p/A) \tag{7.38}$$

In equations (7.37) and (7.38), $Q_p = 1/b$, the pole-Q of the denominator from equation (7.36). While using the bridged twin-T network of Figure 7.2(b) for $H_p(s)$, the selected element values are as follows:

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$$R_1 = R_2 = R,\ R_3 = (R/2),\ C_1 = C_2 = C,\ C_3 = 2C \text{ and } R^* = (R/a)$$

This gives the transfer function of $H_p(s)$ as:

$$\frac{V_{\text{out}}}{V_{\text{in}}} = \frac{N(s)}{s^2 + \left(\frac{4}{A} + 2a\right)s + (1+2a)} \tag{7.39}$$

From equation (7.39), expressions of the center frequency and quality factor, respectively, are as follows:

$$\omega_{oe} = \omega_o (1+2a)^{½} \tag{7.40}$$

$$Q_e = \frac{(1+2a)^{½}}{(2a + 4/A)} \cong \frac{1}{2a} \tag{7.41}$$

The small shift in the pole frequency given by equation (7.40) is easily corrected by a pre-distortion of the twin-T design. Structure of the H_z network needs to be in the form shown in Figure 7.12(b); otherwise, the response functions will be restricted. The advantage of using OA in differential mode is that it avoids increase in sensitivity due to interaction between the two-port networks H_z and H_p. Order of sensitivity of the overall network is mainly decided by the choice of the network H_p.

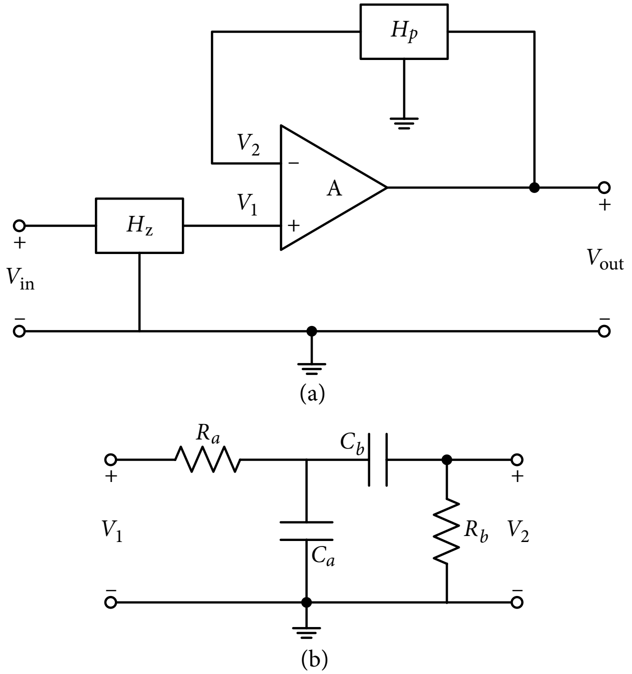

Figure 7.12 (a) Differential input single feedback single operational amplifier biquad. (b) A preferred RC two port network for realizing network H_Z shown in Figure 7.12(a).

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7.4.2 General differential input single OA biquad

The biquad realizations discussed in the previous sections have the restriction that there are no positive real poles and zero for their transfer functions. To overcome this restriction, additional single-ended infinite gain amplifiers are used. A more economical way to avoid this constraint is to use a general differential input configuration as shown in Figure 7.13. Applying Kirchhoff's current law at the terminals V_p and V_z, we get the following relations for obtaining the transfer function [7.5].

$$(V_p - V_{\text{in}})\, y_1 + V_p y_2 + (V_p - V_{\text{out}})\, y_3 = 0 \tag{7.42a}$$

$$(V_z - V_{\text{in}})\, y_a + V_z y_b + (V_z - V_{\text{out}})\, y_c = 0 \tag{7.42b}$$

$$V_{\text{out}} = (V_z - V_p)A \tag{7.43}$$

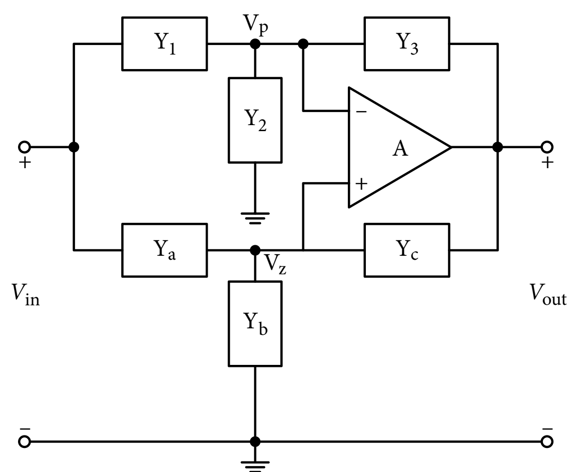

Figure 7.13 General differential input single operational amplifier biquad configuration.

The obtained transfer function for the general differential input single OA is as follows:

$$\frac{V_{\text{out}}}{V_{\text{in}}} = \frac{y_a\left(y_1 + y_2 + y_3\right) - y_1\left(y_a + y_b + y_c\right)}{y_3\left(y_a + y_b + y_c\right) - y_c\left(y_1 + y_2 + y_3\right) + (1/A)\left(y_1 + y_2 + y_3\right)\left(y_a + y_b + y_c\right)} \tag{7.44}$$

When $A \to \infty$, the transfer function of equation (7.44) is considerably simplified as shown in equation (7.46), provided the condition presented in equation (7.45) is satisfied.

$$(y_a + y_b + y_c) = (y_1 + y_2 + y_3) \tag{7.45}$$

$$\frac{V_{\text{out}}}{V_{\text{in}}} = \frac{y_a - y_1}{y_3 - y_c} \tag{7.46}$$

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Equation (7.46) ensures that any real transfer function $N(s)/D(s)$ can be realized from the configuration shown in Figure 7.13.

Procedurally, the numerator and denominator are to be divided by a polynomial $Q(s)$ with simple negative real roots, which has order at least one less than that of $N(s)$ or $D(s)$. The problem will then reduce to realizing the driving point admittances (y_1, y_3, y_a and y_c) using RC elements. The rest of the driving point function y_2 and y_b are obtainable using equation (7.45) as follows:

$$y_b - y_2 = (y_3 - y_c) - (y_a - y_1) \tag{7.47}$$

$$= \{D(s) - N(s)\}/Q(s) \tag{7.48}$$

Example 7.4: Realize the following BP transfer function using the configuration of Figure 7.13.

$$H(s) = \frac{s}{s^2 + 0.20s + 1.01} \tag{7.49}$$

Solution: Selecting auxiliary polynomial $Q(s) = (s + 1)$ and an order less than $H(s)$, we get:

$$\frac{N(s)}{Q(s)} = \frac{s}{(s+1)} = \frac{s}{s+1} - 0 = y_a - y_1 \tag{7.50}$$

$$\frac{D(s)}{Q(s)} = \frac{s^2 + 0.2s + 1.01}{(s+1)} = (s + 1.01) - \frac{1.81s}{s+1} = y_3 - y_c \tag{7.51a}$$

$$\frac{D(s) - N(s)}{Q(s)} = \frac{s^2 - 0.8s + 1.01}{(s+1)} = \ (s + 1.01) - \frac{2.81s}{(s+1)} = y_b - y_2 \tag{7.51b}$$

Here, y_1 is an open circuit, y_a, y_c and y_2 are a series combination of R and C elements and y_3 and y_b have parallel RC elements. The normalized element values from equations (7.50–51) are as follows:

> R_a = 1 Ω in series with C_a = 1 F, R_c = 0.5525 Ω in series with C_c = 1.81 F, R_2 = 0.3558 Ω in series with C_2 = 2.81 F, R_3 = R_b = 0.99 Ω in parallel with C_3 = C_b = 1 F, respectively.

In order to get the center frequency at 10^4rad/s, the frequency is de-normalized with a factor of $(1.01)^{-½} \times 10^4$. Further, an impedance scaling of 10^4 gives the following element values, which are also shown in the filter circuit presented in Figure 7.14(a).

$$R_a = 10\text{k}\Omega,\ C_a = 10.0498\text{ nF},\ R_c = 5.525\text{ k}\Omega,\ C_c = 18.19\text{ nF},\ R_2 = 3.558\text{ k}\Omega,$$
$$C_2 = 28.24\text{ nF},\ R_3 = R_b = 9.9\text{ k}\Omega,\text{ and } C_3 = C_b = 10.0498\text{ nF}. \tag{7.52}$$

Figure 7.14(b) shows the PSpice simulated magnitude and phase response of the BPF. The obtained mid-band gain is 5.0. Center frequency f_o= 1.595 kHz and a bandwidth of 318 Hz

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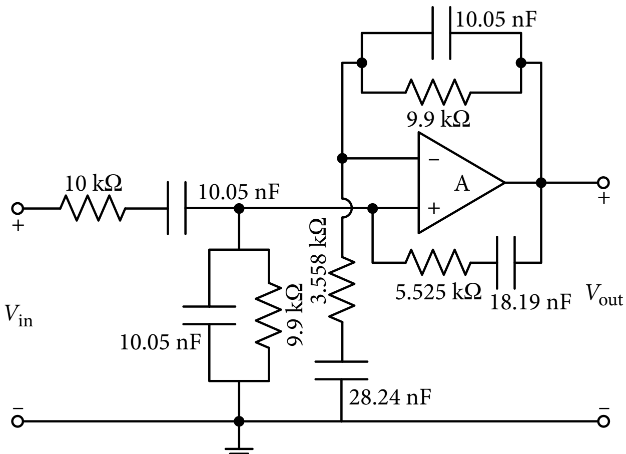
give the value of pole-Q as 5.017; this confirms the design parameter values. In the phase response, a 0° phase shift occurs at 1.6 kHz as per design.



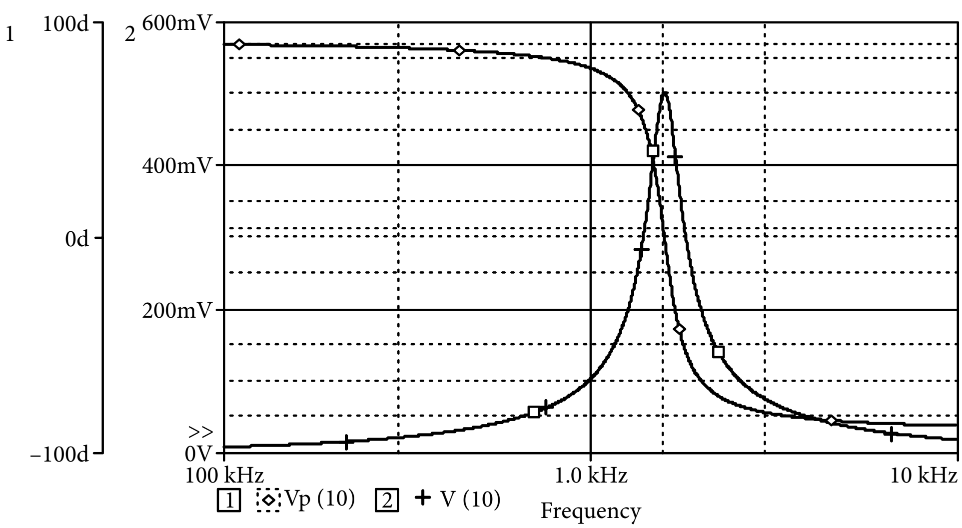


Figure 7.14 (a) Second-order band pass filter section using a differential mode operational amplifier for Example 7.4. (b) Magnitude and phase response of the differential input single operational amplifier band pass filter depicted in Figure 7.14(a).

Example 7.5: Design a BR filter with the following normalized transfer function using a general differential input single OA biquadratic circuit.

$$H(s)=\frac{N(s)}{D(s)}=\frac{s^2+0.25}{s^2+0.09s+0.83}$$

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Solution: Selecting the auxiliary polynomial $Q = (s + 1)$, we get:

$$\frac{N(s)}{Q(s)} = \frac{s^2 + 0.25}{s+1} = (s + 0.25) - \frac{1.25s}{s+1} \rightarrow y_a = s + 0.25 \text{ and } y_1 = 1.25s/(s+1).$$

$$\frac{D(s)}{Q(s)} = \frac{s^2 + 0.09s + 0.83}{s+1} = s + 0.83 - \frac{1.74s}{s+1} \rightarrow y_3 = s + 0.83 \text{ and } y_c = 1.74s/(s+1)$$

$$\frac{D(s) - N(s)}{Q(s)} = \frac{0.09s + 0.58}{s+1} = 0.58 - \frac{0.49s}{s+1} \rightarrow y_b = 0.58 \text{ and } y_2 = 0.49s/(s+1)$$

Admittances y_1, y_c and y_2 are a series combination of R and C elements, y_a and y_3 are parallel RC elements and y_b is only resistive. The normalized element values are as follows:

R_a = 4.0 Ω, C_a = 1 F, R_b = 1.724 Ω, R_c = 0.5747 Ω, C_c = 1.74 F, R_1 = 0.8 Ω, C_1 =1.25 F, R_2 = 2.0408 Ω, C_2 = 0.49 F, R_3 = 1.2048 Ω and C_3 = 1 F

Frequency de-normalization by a factor of $0.83^{-0.5} \times 10^4$ and an impedance scaling by 10^4 will result in a notch frequency of $(\sqrt{0.25}\sqrt{}/\sqrt{0.83}) \times 10^4$ rad/s (873.1 Hz), a dc gain of 0.3012 and a high frequency gain of unity with the following element values, which are shown in Figure 7.15.

R_a = 40 kΩ, C_a = 9.1104 nF, R_b = 17.24 kΩ, R_c = 5.747 kΩ,
C_c = 15.852 nF, R_1 = 8.0 kΩ, C_1 = 11.388 nF, R_2 = 20.408 kΩ,
C_2 = 4.461 nF, R_3 = 12.048 kΩ and C_3 = 9.1104 nF

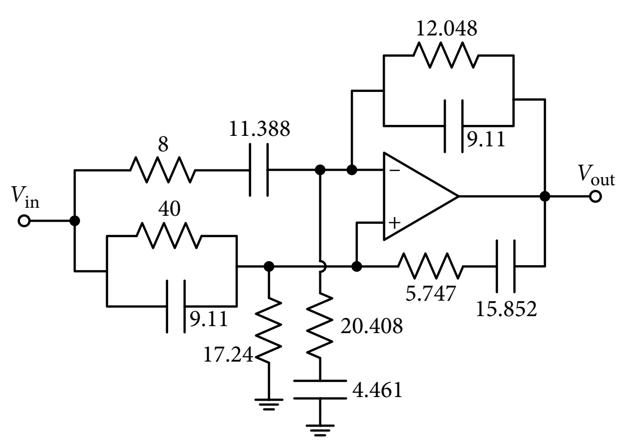

Figure 7.15 Second-order band stop filter using a differential mode operational amplifier for Example 7.5. All capacitors are in nF and resistors in kΩ.

Figure 7.16 shows the PSpice simulated magnitude and phase responses of the BR filter. Its gain at dc is 0.3011(0.3012) and gain at high frequencies is 0.998 (1.0); notch occurs at 875.04

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Hz (873.1 Hz) and a peak gain of 7.14 occurs at 1.595 kHz (1.591 kHz). This response is very close to the design.

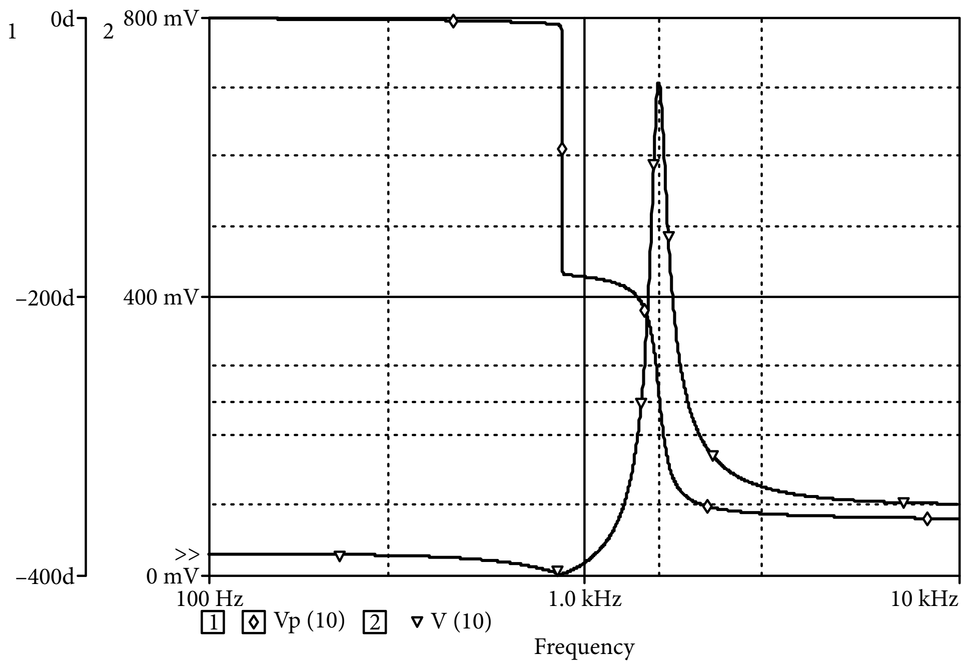

Figure 7.16 Magnitude and phase responses of the band reject filter for Example 7.5.

7.5 General Active RC Feedback Single Amplifier Biquad

In Sections 7.2 to 7.4, single amplifier biquad generation using OAs in single-ended mode and in differential mode were discussed. Obviously, a large number of circuit configurations are available and many more are likely to be found. Such circuits are selected for a particular application depending on their performance characteristics. However, the criterion that is very important for all circuits is the sensitivity factors with respect to the elements used. Study of a general active RC feedback circuit is helpful in gaining an understanding of the factors responsible for increase (decrease) in sensitivities. Such a study also helps in a systematic generation of good quality single amplifier RC biquads. It can be seen that the circuits studied in the previous sections were in effect special cases of such a general case.

Consider a general active RC single amplifier block configuration shown in Figure 7.17. With H_{ij} being the transfer function corresponding to the output and input terminals i and j, we get

$$V_{out} = A(s)[V_{out}\{H_{42}(s) - H_{32}(s)\}] + A(s)[V_{in}\{H_{41}(s) - H_{31}(s)\}] \tag{7.53a}$$

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With OA being ideal (infinite gain), the overall transfer function is obtained as:

$$H(s)=\frac{V_{\text{out}}}{V_{\text{in}}}=\frac{H_{41}(s)-H_{31}(s)}{H_{32}(s)-H_{42}(s)} \tag{7.53b}$$

From equation (7.53b), the following two important inferences are obtained.

(a) For $H(s)$ to be a second-order function, the RC network should also be second-order.

(b) Transmission zeros of the active RC network are determined by the feed-forward path, whereas poles (natural frequencies) are set by the feedback path; this confirms an inference already seen in the previous sections.

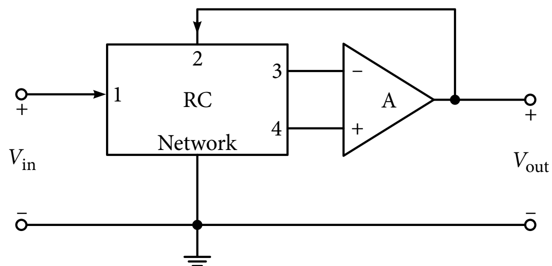

Figure 7.17 General active RC single amplifier configuration for biquad realization.

Theoretically, there is no problem in realizing a transfer function using the configuration shown in Figure 7.17, but it requires a three-port RC network, which is rather involved. In order to make the realization simpler, the network is broken into two-port RC networks as shown in Figure 7.18. With transfer function $H_{ij}(s) = (V_i/V_j)$ notation, output is obtained as follows:

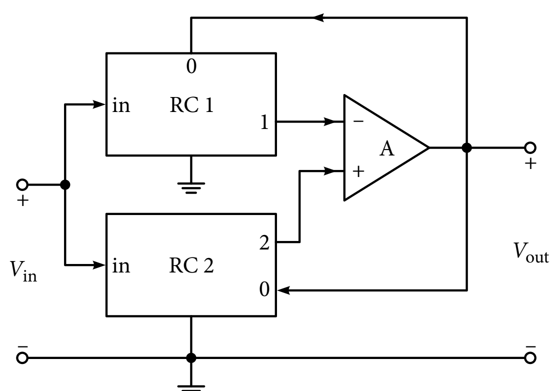

Figure 7.18 General active RC single amplifier configuration from Figure 7.17 with two two-port networks: RC1 and RC2.

$$\frac{V_{\text{out}}}{V_{\text{in}}}=\frac{H_{2\text{in}}-H_{1\text{in}}}{\left(H_{10}-H_{20}\right)+(1/A(s))} \tag{7.54}$$

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$$= \frac{1}{D_{1\text{in}} D_{2\text{in}}} \frac{\left(N_{2\text{in}} D_{1\text{in}} - N_{1\text{in}} D_{2\text{in}}\right) D_{10} D_{20}}{\left(N_{10} D_{20} - N_{20} D_{10}\right)} \quad \text{For } A(s) \text{ nearing infinity} \tag{7.55}$$

Poles of the transfer function, from equation (7.55), are given by the following equation:

$$(N_{10}D_{20} - N_{20}D_{10}) = 0 \tag{7.56}$$

In order to get a second-order polynomial for getting only two roots in equation (7.56), it is essential that the transfer functions H_{10} and H_{20} are selected in such a way that the denominator D_{10} and D_{20} are same and cancel each other. Otherwise, the order of the polynomial will become more than two. In fact, even selecting $D_{10} = D_{20}$ is not sufficient to make the polynomial of equation (7.56) a second-order one since practically there is always some mismatch between the components of D_{10} and D_{20}. Therefore, it is advised to select either RC1 or RC2 as frequency independent. (The same argument is valid for the configuration of Figure 7.12(a)). Selecting RC2 (say) as a purely resistive network as shown in Figure 7.19, $H_{20} = (k - 1)/k$. Hence, the poles are now determined from equation (7.57), which is obtained using equation (7.56).

$$H_{10} - \frac{(k-1)}{k} = 0 \rightarrow N_{10} - \frac{k-1}{k} D_{10} = 0 \tag{7.57}$$

In equation (7.57), if $k = 1$, poles of the realized transfer function depend on the numerator polynomial of the negative feedback network in the same way as for the structure shown in Figure 7.5 and Figure 7.12(a). For $k > 1$, some positive feedback is also applied which causes enhancement in the pole -Q, the main RC feedback goes to the inverting terminal of the OA. Hence, this structure is also called *enhanced negative feedback (ENF) circuit*.

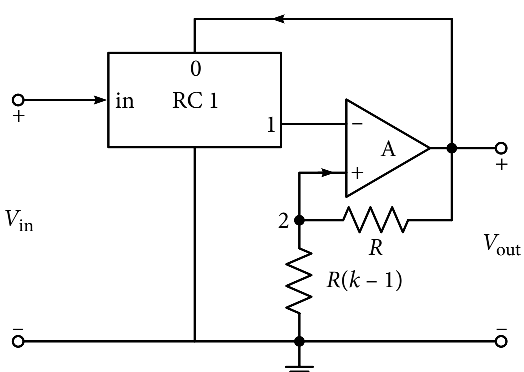

Figure 7.19 Simplified structure from Figure 7.18 while ensuring that it is a second-order active RC network.

Instead of making RC2 resistive, RC1 network can be made resistive with RC2 as a frequency dependent structure. Such a configuration is called an *enhanced positive feedback*

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(EPF) circuit. The EPF configuration is shown in Figure 7.20. With $H_{10} = 1/k$, poles are obtained from equation (7.54) using the following relation:

$$\frac{1}{k} - H_{20} = 0 \rightarrow \frac{D_{20}}{k} - N_{20} = 0 \tag{7.58}$$

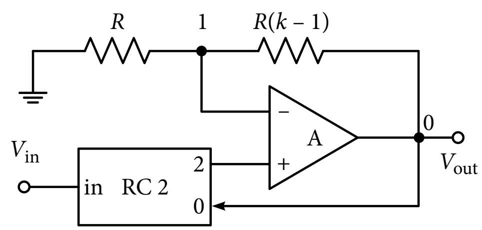

Figure 7.20 Enhanced positive feedback circuit obtained from Figure 7.18.

Special cases of these structures can be obtained with $k = 1$ as well. Figures 7.21(a) and (b) show the modified forms of Figures 7.19 and 7.20, respectively, with $k = 1$; it is essential that these forms are studied to understand the limitations of similar structures.

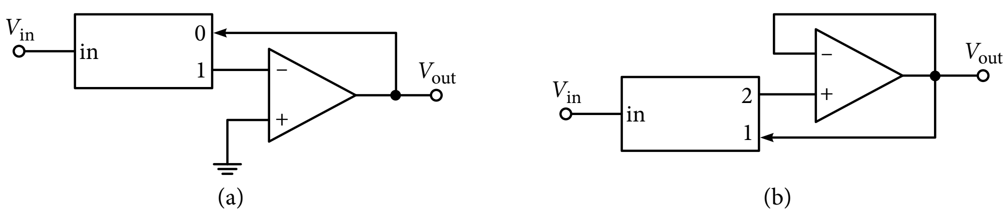

Figure 7.21 (a) Infinite gain negative feedback (NF), and (b) unity gain positive feedback (PF) configuration.

7.6 Coefficient Matching Synthesis Technique

Before taking up specific cases of general RC feedback discussed in the previous section, a significant method of network synthesis needs to be formalized, though we are already using a method without mentioning it as a procedure.

In the *coefficient matching synthesis technique*, a network is available for which the transfer function is determined by the conventional method. Coefficients in the transfer function are obviously in terms of the elements used in the filter structure. The coefficients of the transfer function are then equated to the function to be realized. This results in a set of simultaneous algebraic equations in terms of normalized (or sometimes de-normalized) values of the components. In general, such equations are lesser in number than the number of elements used. This allows assumption of values for some of the elements, which affects the performance, sensitivity, and component spread. Therefore caution is necessary.

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As a technique, the coefficient matching method is particularly useful for second-order (or at most third-order) sections, and has been extensively used. Obviously, the choice of the initial filter structure decides the usefulness, advantages or limitations of the filter; many choices are available for any particular type of filtering action. For example, a large number of circuits can be used in order to obtain a passive RC network in the ENF and EPF configurations. At the same time, as mentioned earlier, the selection of elements or certain element ratios (depending upon the degree of freedom in design) also affects the filter performance. The technique becomes difficult for higher orders, hence, it is seldom used.

7.7 Sallen and Key Biquad

The Sallen and Key biquad circuit, introduced in 1955, was probably the first set of circuits that could realize almost all types of second-order responses [7.6]. Figure 7.22 shows one of these circuits realizing a LP response. Incidentally, it happens to be an ENF circuit where the passive circuit RC1 is realized by R_1, R_2, C_1, and C_2. Obviously, a different combination of these elements or some other passive structure may give another type of response. The transfer function of the circuit in Figure 7.22 can be obtained using equation (7.54) or (7.55). However, a direct application of KCL at nodes 1 and 3 gives the transfer function more easily, as shown here. While considering OA as ideal:

$$V_1 = V_2 \text{ and } V_2 = \{(k-1)/k\} V_{out} \tag{7.59}$$

$$V_1 (G_2 + sC_2) - V_3 G_2 = 0 \tag{7.60}$$

$$V_3 (G_1 + G_2 + sC_1) - V_{in} G_1 - V_1 G_2 - V_{out} sC_1 = 0 \tag{7.61}$$

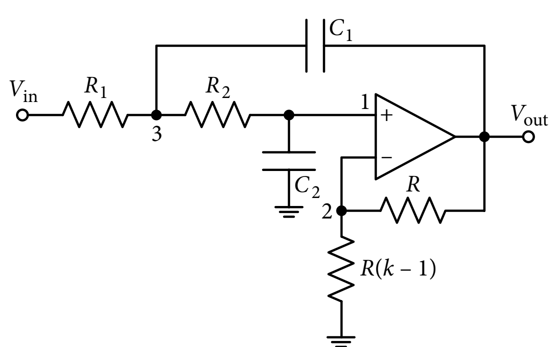

Figure 7.22 Sallen and Key second-order low pass filter section.

Substituting V_3 from equation (7.60) and V_1 and V_2 from equation (7.59) in equation (7.61) and simplifying, we get:

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$$\frac{V_{\text{out}}}{V_{\text{in}}} = \frac{(k/k-1)(G_1G_2/C_1C_2)}{s^2 + \left\{\frac{(G_1+G_2)}{C_1} - \frac{G_2}{(k-1)C_2}\right\} + \frac{G_1G_2}{C_1C_2}} \tag{7.62}$$

Comparing equation (7.62) with the standard format of a second-order LP section shown again in equation (7.63) and applying the coefficient matching technique:

$$H_{LP}(s) = \frac{h_{lp}\omega_o^2}{s^2 + (\omega_o/Q)s + \omega_o^2} \tag{7.63}$$

The following relations are obtained.

$$\omega_o^2 = \frac{G_1G_2}{C_1C_2} \tag{7.64}$$

$$\frac{\omega_o}{Q} = \frac{G_1+G_2}{C_1} - \frac{G_2}{(k-1)C_2} \rightarrow Q = \frac{(k-1)(C_1C_2G_1G_2)^{½}}{(k-1)(G_1+G_2)C_2 - G_2C_1} \tag{7.65}$$

$$h_{lp} = k/(k-1) \tag{7.66}$$

From equation (7.66), it is obvious that the dc gain h_{lp} shall will always be more than unity. It is not a serious problem but shows one of the limitations connected with dc gain. The issue can be resolved by cascading an amplifier, though this may increase the cost of the filter. Alternatively, if it is necessary to have a dc gain less than that obtained as such, only a resistive potential divider at the input or output can serve the purpose, provided the divider's loading effect is taken care of.

There are three design parameters, h_{lp}, ω_o and Q against four passive elements, G_1, G_2, C_1, and C_2, and coefficient k which affects h_{lp} as well as Q. Hence, there are options available for pre-selecting some element values or their ratios. To make the equations suitable for integration, we can select $C_1 = C_2 = C$, then, equations (7.64)–(7.66) modify as:

$$\omega_o^2 = \left(G_1G_2/C^2\right) \tag{7.67}$$

$$Q = (k-1)(G_1G_2)^{½}/\{(k-1)G_1 + (k-2)G_2\} \tag{7.68}$$

Further, if $G_1 = G_2 = G$, $\omega_o = G/C$ (7.69)

$$Q = (k-1)/(2k-3) \tag{7.70}$$

Example 7.6: Design an LP Chebyshev second-order filter having a ripple width of 0.5 dB and a corner frequency of 3.18 kHz using the Sallen and Key circuit. It should have maximum gain of unity.

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Solution: From Table 3.4, the pole location for the desired specifications is as follows:

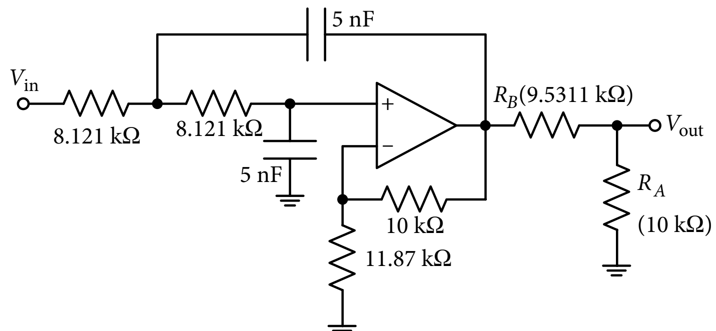
$$s = -0.7128 \pm j1.004$$

This gives the normalized transfer function of the second-order filter as:

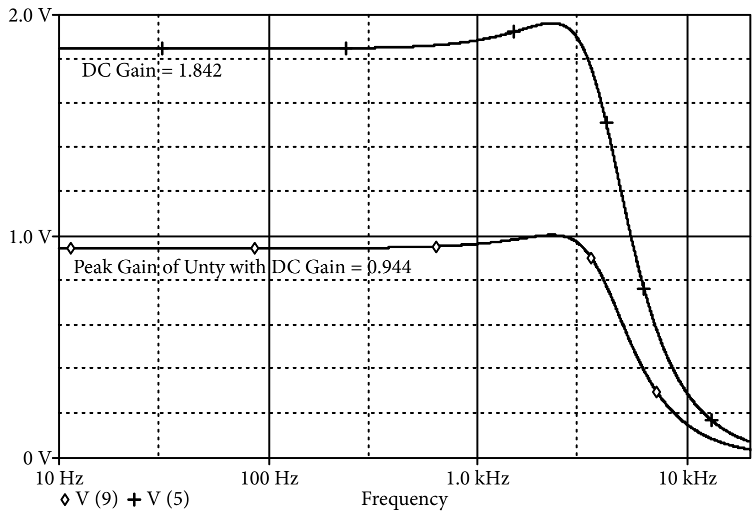
$$H(s) = \frac{1.5161 \times 0.944}{s^2 + 1.4256s + 1.5161} \tag{7.71}$$

In the numerator of equation (7.71), 0.944 comes from the fact that for an even order Chebyshev filter, the dc gain drops from normalized unity by the ripple width; which is 0.5 dB in this case.

With $\omega_o = \sqrt{1.5161} = 1.2313$, equation (7.71) gives $Q = 1.2313/1.4256 = 0.8637$



(a)



(b)

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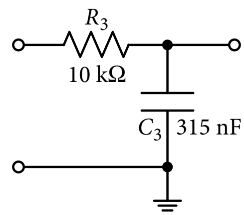
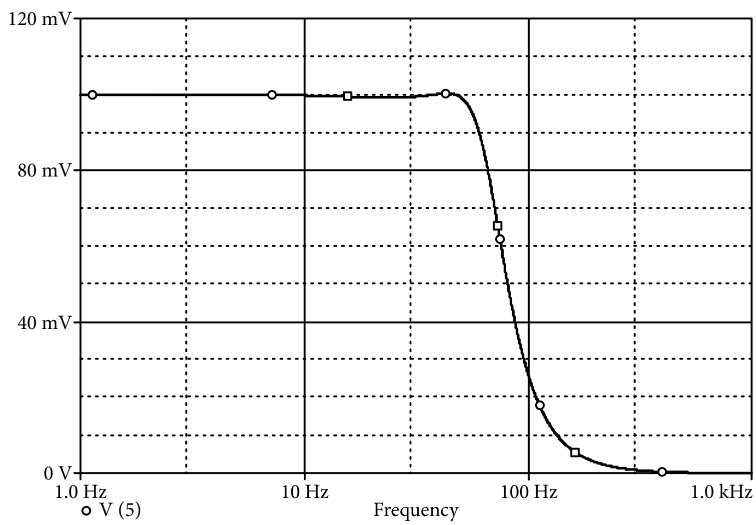


(c)



(d)

Figure 7.23 (a) Sallen and Key low pass circuit with element values for Example 7.5. (b) Second-order low pass Chebyshev filter response using the Sallen–Key circuit shown in Figure 7.23(a). (c) First-order filter section component in a moving air vehicle model. [with permission from N. Chattaraj et al. [7.7]]. (d) Simulated response of the filter employed with moving air vehicle.

Selecting normalized $C = 1$, equation (7.67) gives $G_1 \times G_2 = 1.5161$. Further, with $G_1 = G_2 = G = 1.2313$, and using $Q = 0.8637$, the value of k is obtained from equation (7.70) as:

$$0.8637 = \{(k-1)/(2k-3)\} \rightarrow k = 2.18738$$

The value of the dc gain obtained from equation (7.66) is 1.842, and in order to get a dc gain of 0.944 and a maximum gain of unity, a potential divider at the output as shown in Figure 7.23(a) is used with a ratio of R_A and R_B as 1: 0.9513.

De-normalizing the elements with a frequency scaling factor of 3.18 kHz and an impedance scaling factor of 10^4, the element values are:

$$C_1 = C_2 = 5 \text{ nF},\ R_1 = R_2 = 8.121 \text{ k}\Omega,\ R = 10 \text{ k}\Omega,\ R(k-1) = 11.873 \text{ k}\Omega,\ R_A = 10 \text{ k}\Omega \text{ and } R_B = 9.513 \text{ k}\Omega$$

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The circuit in Figure 7.23(a) shows these element values, and the simulated response is shown in Figure 7.23(b). The pass band edge frequency is 3.21 kHz and the ripple width is 0.51 dB, both of which are in very close agreement with the design. The effect of the passive potential divider is also clear in the two responses, bringing the final gain to unity.

The component spread is an important consideration in the selection of filters while implementing in integrated form. Another important issue is the sensitivity expressions and values for the passive and active elements used with respect to the design parameters. In the present case, the component spread is ideal as far as R_1, R_2, C_1 and C_2 are concerned. R_A and R_B depend on the amount of reduction in the filter gain (if reduction is required) but the value of k decides the final component spread and it also affects the sensitivity of Q as will be shown here.

Using equations (7.64) and (7.66), the sensitivity of ω_o and h_{lp} is found to be:

$$S^{\omega_o}_{R_1,\,R_2,\,C_1,\,C_2} = -\frac{1}{2}\,,\, S^{\omega_o}_{k} = 0$$

$$S^{h_{lp}}_{R_1,\,R_2,\,C_1,\,C_2} = 0,\; S^{h_{lp}}_{k} = -\frac{1}{k-1} = -0.533$$

From equation (7.65), evaluation of the sensitivity of Q is a bit involved, but for the present case, without losing any generality, from equation (7.70):

$$S^{Q}_{k} = \frac{k}{Q}\frac{\partial Q}{\partial k} = \frac{k}{(k-1)(2k-3)} = 1.34$$

which means that for the present design, with $k = 2.187$, Q will change 1.34% for a 1% change in the value of Q. The value of Q-sensitivity is not alarming but the condition will become worse if k reaches values near 1.5 for which the denominator in Q-sensitivity will increase alarmingly.

7.7.1 Micro air vehicle: a case study

Flapping wing micro air vehicles (MAVs) technology has attracted considerable attention in recent times. Advances in micro-electromechanical systems have led to the development of a number of new miniature devices such as infrared sensors and cameras, and MAVs can act as platform for these micro sized systems. Piezoelectrically actuated flapping wing MAVs is an emerging technique. Hence, MAVs based on piezoelectrical actuators are now subjects of intense study. In one such study [7.7], it was suggested that an active filter based unipolar high voltage driver be used to obtain the desired result. Analysis and synthesis of a Chebyshev active LPF was done for this purpose.

The average flapping wing frequency of insects, that were observed for inspiration, lies below the 50 Hz range. Hence, in this case, a flapping frequency range of 30–50 Hz was considered for designing a driver of the piezoelectric actuator.

The evaluated specifications for the LPF in this case were as follows: maximum gain = 1, minimum allowable gain = 0.99, pass band edge frequency $\omega_p = 2\pi(50)$ rad/s. Hence, the required order of the Chebyshev filter was three for which the following was the normalized transfer function.

$$H(s) = \frac{1.77}{(s+1.006)(s^2+1.006s+1.76)} \tag{7.72}$$

The Sallen–Key circuit shown in Figure 7.22 was selected for the second-order section with the following values for the de-normalized elements: $R_1 = R_2 = 10$ kΩ, $C_1 = 633$ nF, $C_2 = 91$ nF. The circuit was followed by an RC circuit shown in Figure 7.23(c), which represents the first-order component of the transfer function; this models the capacitance of the piezoelectric bimorph actuator in series electrical connection.

The simulated response of the filter is shown in Figure 7.23(d).

7.8 Delyiannis–Friend Biquad

The classic and widely reported second-order filter circuit given by Deliyannis [7.9] and Friend [7.10] (D&F circuit) is an excellent example of an EPF configuration. It is convenient to study the circuit first without positive feedback and later introduce the feedback. An example of a second-order BP section was illustrated in Section 2.9 of Chapter 2. It is basically the same as a D&F circuit, except for a small difference in the formation of input resistance, which obviously reflects in some difference in the expressions of center frequency ω_o and pole-Q with the D&F circuit shown in Figure 7.24.

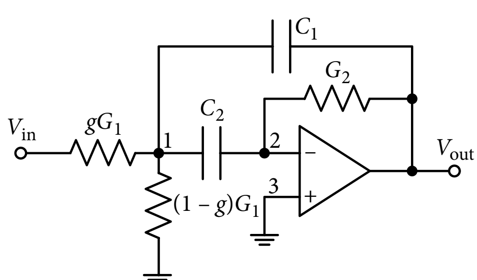

Figure 7.24 Deliyannis and Friend's second-order band pass filter circuit.

Applying KCL at terminals 1 and 2, respectively, we get:

$$V_1(gG_1 + G_1 - gG_1 + sC_1 + sC_2) - V_{in}gG_1 - V_2sC_2 - V_{out}sC_1 = 0 \tag{7.73}$$

$$V_2(sC_2 + G_2) - V_1sC_2 - V_{out}G_2 = 0 \tag{7.74}$$

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For the OA open-loop gain being finite:

$$V_{\text{out}} = A(V_3 - V_2). \tag{7.75}$$

If the effect of finite A is to be investigated, equation (7.75) will be used; otherwise, considering OA as ideal, $V_3 = V_2 = 0$. With OA as ideal, equation (7.73) and (7.74) are simplified and combined to obtain the transfer function as:

$$\frac{V_{\text{out}}}{V_{\text{in}}} = -\frac{g\left(G_1/C_1\right)s}{s^2 + s\dfrac{\left(C_1 + C_2\right)}{C_1C_2}G_2 + \dfrac{G_1G_2}{C_1C_2}} \tag{7.76}$$

Comparing this equation with the standard format of a second-order BP section of equation (7.77):

$$H_{\text{BP}}\left(s\right) = \frac{h_{bp}\left(\omega_o/Q\right)s}{s^2 + \left(\omega_o/Q\right)s + \omega_o^2} \tag{7.77}$$

The coefficient matching technique gives the relations of important parameters as:

$$\omega_o^2 = \left(G_1G_2/C_1C_2\right) \tag{7.78}$$

$$\frac{\omega_o}{Q} = \frac{\left(C_1 + C_2\right)}{C_1C_2}G_2 \rightarrow Q = \left(C_1C_2\right)^{½}\left(G_1/G_2\right)^{½}/\left(C_1 + C_2\right) \tag{7.79}$$

$$h_{bp} = g(1 + C_2/C_1)Q^2 \tag{7.80}$$

Ordinarily, equations (7.78)–(7.80) are used to find element values for the given specifications ω_o, Q and h_{bp}. However, without losing generality, a good choice is $C_1 = C_2 = C$, which modifies these equations as equations (7.78)–(7.80), respectively.

$$\omega_o^2 = G_1G_2/C^2 \rightarrow \omega_o = \left(G_1G_2\right)^{½}/C \tag{7.81}$$

$$Q = ½\left(G_1/G_2\right)^{½} \tag{7.82}$$

$$h_{bp} = 2gQ^2 \tag{7.83}$$

From equations (7.78)–(7.80), the expressions of the elements are found in terms of the specifications:

$$R_2 = (2Q/\omega_oC),\ R_1 = 1/2Q\omega_oC, \text{ and } g = h_{bp}/2Q^2 \tag{7.84}$$

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Equation (7.84) shows the high resistance spread as $(R_1/R_2) = 1/4Q^2$; the mid-band gain is proportional to Q^2. These characteristics were reflected in Example 2.7; for $Q = 10$ and effective $g = 0.5$, resistance spread was 200 and mid-band gain was 100. If the mid-band gain is to be reduced, a smaller value of coefficient g has to be used.

7.8.1 Enhanced-Q circuit

It is observed that resistance spread can be considerably reduced when the D&F circuit is modified in the form of an EPF configuration as shown in Figure 7.25. Denominator of its transfer function can be formed using equation (7.58). However, there is an alternate method which uses the application of KCL as earlier; with OA taken as ideal:

$$V_2 = V_3 = V_{\text{out}}(k-1)/k \tag{7.85}$$

$$V_1(gG_1 + G_1 - gG_1 + sC_1 + sC_2) - V_{\text{in}}gG_1 - V_2 sC_2 - V_{\text{out}} sC_1 = 0 \tag{7.86}$$

$$V_2(sC_2 + G_2) - V_1 sC_2 - V_{\text{out}} G_2 = 0 \tag{7.87}$$

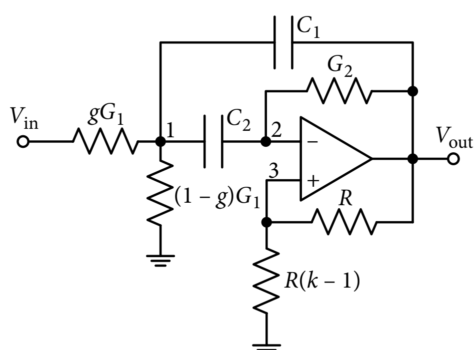

Figure 7.25 Deliyannis and Friend's circuit with Q-enhancement in ENF configuration.

Simplifying the aforementioned three equations, we get the transfer function.

$$H(s) = -\frac{gG_1 ks/C_1}{s^2 + \left\{\frac{G_1(1-k)}{C_1} + \frac{C_1 + C_2}{C_1 C_2} G_2\right\} s + \frac{G_1 G_2}{C_1 C_2}} \tag{7.88}$$

Once again, we can select $C_1 = C_2 = C$, and the transfer function will simplify as:

$$H(s) = -\frac{gG_1 ks/C}{s^2 + \left\{\frac{G_1(1-k)}{C} + \frac{2}{C} G_2\right\} s + \frac{G_1 G_2}{C^2}} \tag{7.89}$$

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From equation (7.89), the expression for center frequency remains the same; expression of the enhanced $Q(Q_e)$, and gain $h(h_e)$ are obtained as follows:

$$\omega_o = (G_1G_2)^{½}/C \tag{7.90}$$

$$\frac{\omega_o}{Q_e} = \frac{(1-k)G_1 + 2G_2}{C} \rightarrow Q_e = ½\frac{(G_1/G_2)^{½}}{½(1-k)(G_1/G_2)+1} \tag{7.91}$$

$$h_e = \frac{kgG_1}{(1-k)G_1 + 2G_2} \tag{7.92}$$

Expression of Q_e in equation (7.91) and h_e in equation (7.92) can be written in terms of the quality factor Q from equation (7.82).

$$Q_e = \frac{Q}{2Q^2(1-k)+1} \tag{7.93}$$

$$h_e = \frac{gk}{(1-k)+\dfrac{1}{2Q^2}} \rightarrow 2gkQ_eQ \tag{7.94}$$

Equations (7.90) to (7.94) can be used to find the expressions or relations for element values. However, before discussing these relations, we need to determine the constraints, which can also be helpful in the design of the filter. Since coefficient $k > 1$ (when $k = 1$, it does not remain EPF form of the circuit), from equation (7.93), we get that:

$$2Q^2(1-k) < 1 \rightarrow Q < \left(\frac{1+k}{2}\right)^{½} \tag{7.95}$$

To keep the resistance ratio small while forming the potential divider, the value of k should not be large, as even with a small value of Q, sufficiently large values of Q_e can be realized, as can be seen from equation (7.93). From equation (7.82), it can be observed that a small value of Q means a small ratio between R_1 and R_2 resulting in smaller resistance spread.

The first step in the filter design is to assume a small arbitrary value for k; then, equation (7.95) is used to find the upper limit for Q. A value for Q is assumed which is less than its upper limit and equation (7.82) is used to get a ratio between G_1 and G_2 and their normalized values. Since the expression for ω_o remains unchanged in the EPF case, equation (7.90) is used to give a nominal value of C for a selected nominal value of G_1 and G_2.

Equation (7.93) now gives the value of k for a specified value of Q_e and the assumed value of Q. The obtained value of k gives resistance values for the potential divider and equation (7.94) gives the value of coefficient g for a specified value of h_e.

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Example 7.7: Design a BPF having a central frequency of 20 krad/s, pole-Q = 10 and mid-band gain of 10 employing an enhanced Q circuit. Repeat the design for pole-Q = 20 and a mid-band gain of 20.

Solution: Selecting arbitrarily k = 3, a small value, from equation (7.95):

$$Q < \left(\frac{1+3}{2}\right)^{½} = 1.414$$

Hence, we select Q = 1.2, and from equation (7.82):

$$Q = 1.2 = \frac{1}{2}\sqrt{(G_1 / G_2)} \rightarrow \text{for}\, G_2 = 1, G_1 = 5.76 \tag{7.96}$$

Next, using equation (7.90), for normalized center frequency = 1

$$\sqrt{(G_1 G_2)} = C = 2.4 \tag{7.97}$$

For Q_e = 10, applying equation (7.93) with Q = 1.2:

$$Q_e = 10 = \frac{1.2}{2*1.2*1.2*(1-k)+1} \rightarrow k = 1.30555 \tag{7.98}$$

And then, using equation (7.94), we get:

$$h_e = 10 = 2g \times 1.30555 \times 1.2 \times 10 \rightarrow g = 0.31915,\ (1 - g) = 0.68085 \tag{7.99}$$

Applying a frequency scaling of 20 krad/s and an impedance scaling of 10 kΩ, element values are:

$$R = 10\,\text{k}\Omega,\ R(k-1) = 3.0555\,\text{k}\Omega,\ \frac{R_1}{g} = 5.4395\,\text{k}\Omega,\ \frac{R_1}{1-g} = 2.55\,\text{k}\Omega,\ R_2 = 10\,\text{k}\Omega,\ C_1 = C_2 = 12\,\text{nF} \tag{7.100}$$

Using these element values, the circuit shown in Figure 7.25 is simulated and its response is shown in Figure 7.26. Its center frequency is 3.1623 kHz (19.877 krad/s); a bandwidth of 313.22 Hz gives Q_e = 10.096 and the mid-band gain h_e is 10.007.

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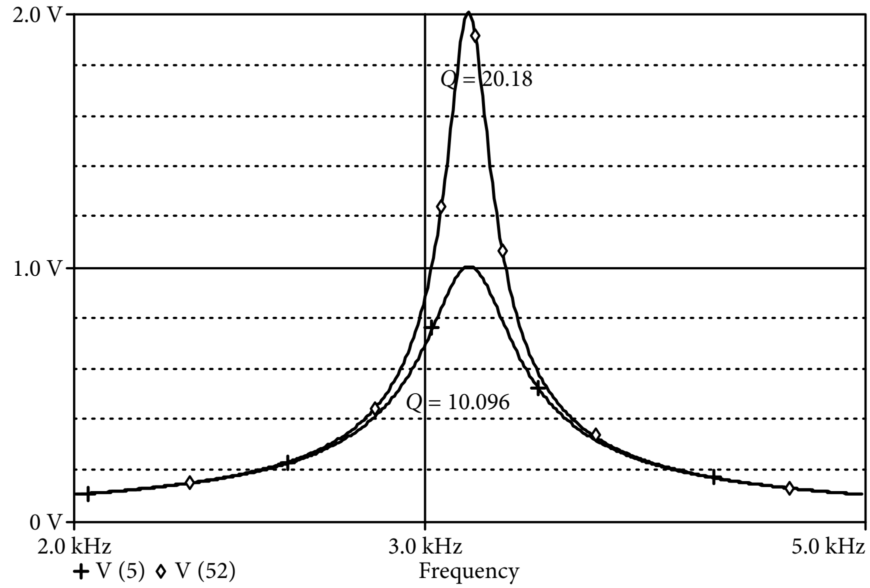

Figure 7.26 Simulated response of the band pass filter, with theoretical $Q = 10$ and $Q = 20$, using the enhanced Q circuit shown in Figure 7.25. Input to the filters was 0.1 V.

Not only is the obtained parameter very close to the design, the resistance spread is less than 4.

Using the same procedure for the second case with $Q_e = 20$ and $h_e = 20$, the obtained value of $k = 1.3264$ and $g = 0.3141$. With capacitor values remaining the same, resistance values are:

$$R = 10\,\mathrm{k}\Omega,\ R(k-1) = 3.2638\,\mathrm{k}\Omega,\ \frac{R_1}{g} = 5.5266\,\mathrm{k}\Omega,\ \frac{R_1}{1-g} = 2.5312\,\mathrm{k}\Omega,\ R_2 = 10\,\mathrm{k}\Omega \quad (7.101)$$

The simulated response is also shown in Figure 7.26. The center frequency is the same with realized pole-$Q = 20.18$ and mid-band gain = 20.

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Practice Problems

7-1 Design and test a second-order BP filter using a single OA and a bridged-T network in its feedback path. Its center frequency is to be 25 krad/s, mid-band gain should be of 20 dBs and pole-Q of 2.5.

7-2 (a) Derive two-port y parameters for the circuit shown in Figure P7.1.

(b) Design and test a second-order LP filter using a bridged-T RC network of Figure 7.3(a) in the feedback path of an OA, and the RC network of part (a). Cut-off frequency of the filter is 100 krad/s and the dc gain 0.5.

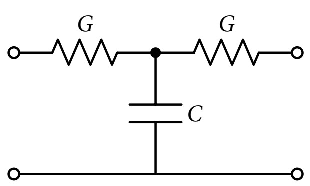

Figure P7.1

7-3 Repeat Problem 7-2 (b) for a maximally flat LP filter for which dc gain is to be unity, and the gain drops by 1 dB at 10 krad/s.

7-4 (a) Derive two-port parameters for the circuit shown in Figure P7.2.

(b) Design and test a second-order HP filter using the bridged-T RC network in the feedback path of an OA, and the RC network of part (a). The cut-off frequency of the filter is 50 krad/s and the high frequency gain is 0.5.

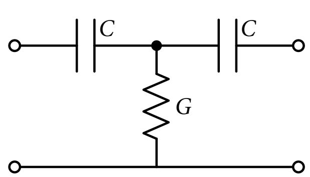

Figure P7.2

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7-5 Design and test a second-order BP filter using a single OA and a bridged twin-T network in its feedback path. Its center frequency is to be 10 kHz, mid-band gain 5 and pole-Q 20.

7-6 Using multiple feedback with a single OA, design and test a second-order LP filter with a dc gain of 6 dBs and a cut-off frequency of 100 krad/s.

7-7 Repeat Problem 7-6 for a 1 dB drop occurring at 50 krad/s; the filter should have a dc gain of 12 dBs.

7-8 Using multiple feedback with a single OA, design and test a second-order HP filter with a high frequency gain of unity and an attenuation of 2 dB at a frequency of 60 krad/s.

7-9 Using multiple feedback with a single OA, redesign the BPF with specifications in Problem 7-1.

7-10 Realize and test the following transfer function using the configuration of Figure 7.13.

$$H(s) = \frac{4\times 10^4 s}{s^2 + 2\times 10^3 s + 10^8}$$

7-11 Design and test a BR filter for the following transfer function using the general differential input single OA configuration shown in Figure 7.13.

$$H(s) = \frac{s^2 + 1.44}{s^2 + 0.1s + 1.21}$$

De-normalize with a frequency scaling factor of 50 krad/s and a suitable impedance scaling factor to bring all components in a range compatible with integration.

7-12 Redesign and test $H(s)$ of Problem 7-11, if the constant term in the numerator is 0.25.

7-13 Design a second-order AP section using the configuration as in Problem 7-11 such that its phase shift becomes $-180°$ at 25 krad/s.

7-14 Design a Sallen and Key LP second-order filter with the following specifications:

Ripple width in the pass band = 1 dB, pass band edge frequency = 100 krad/s.

Test the circuit with OA having a very large bandwidth, as if OA is ideal.

7-15 Design and test a maximally flat Sallen and Key LP circuit for f_o = 1.59 kHz and Q = 2. Modify the circuit to get dc gain of unity. Find the incremental sensitivities of ω_o and Q with respect to the passive elements.

7-16 (a) Replace the passive components in Figure 7.24 by the circuit shown in Figure P7.3, and show that it realized a second-order LP filter. Obtain the expressions of the parameters (a) considering OA as ideal and (b) as non-ideal.

7-17 Design and test a circuit obtained in problem 7-16 (a) with cut-off frequency of 20 krad/s and Q = 2.5 using OA 741. Justify the difference in the peak magnitude value with the value of Q.

7-18 (a) Design and test the D&F circuit of Figure 7.24 with the following specifications:

ω_o = 10 kHz, Q = 10 and mid-band gain = 5

(b) Find the incremental sensitivities of the parameters ω_o and Q with respect to the passive elements.

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7-19 Redesign and test the BP filter of Problem 7-18 using the Q-enhancement circuit of Figure 7.25. Compare the component spread with that in Problem 7-18.

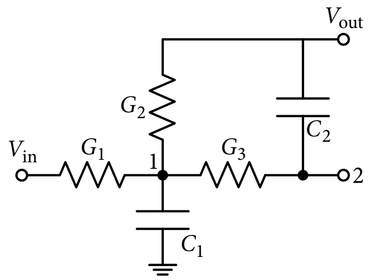

Figure P7.3

7-20 Replace the passive components in Figure 7.24 by the circuit shown in Figure P7.4, and show that it realizes a second-order HP filter. Obtain the expressions of the parameters (a) considering OA as ideal and (b) as non-ideal.

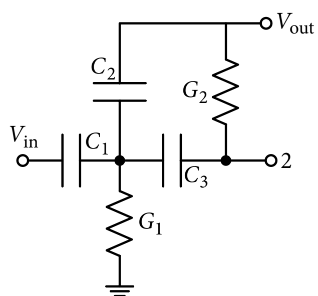

Figure P7.4

7-21 Design and test the circuit obtained in Problem 7-20 (a) with a cut-off frequency of 20 krad/s and $Q = 2.5$ using OA741. Justify the difference in the peak magnitude value with the value of Q.

7-22 Repeat Problem 7-19 using an enhanced-Q D&F circuit with increased value of Q as 20. Also find the sensitivities of the parameters with respect to the elements.

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