Chapter 5

Frequency and Impedance Transformations

5.1 Introduction

In Chapter 3, magnitude approximations were studied for the low pass (LP) response. Specifications for the LP response contained maximum attenuation in the pass band α_{max}, minimum attenuation in the stop band α_{min}, and corner frequencies of the pass band Ω_p and stop band Ω_s (or the selectivity factor = (Ω_p/Ω_s)); for a normalized LP filter, the value of the selectivity factor becomes Ω_s. Such approximation methods are not commonly available for other types of filter responses like high pass (HP), band pass (BP), or band reject (BR). However, this is not too much of an issue as *frequency transformations* are available, which can convert all the important characteristics of an LP filter to that of any other type of filter response and vice-versa. This process of using frequency transformations is a longer procedure compared to direct approximation of other types of responses, but it has some basic advantages. Instead of using different approximation procedures for the different types of filters, extensively available charts and tables for the LP response are used for the maximally flat-Butterworth, Chebyshev, inverse Chebyshev, and the elliptic forms of approximations. The values of poles and zeros, the expression of the transfer function and structure in ladder form with element values are available for small and large order n of the LP filter. The procedure of frequency transformation is lengthy because it involves conversion of the specifications of the *filter to be designed* (FTD) in terms of a corresponding *low pass prototype* (LPP). After designing the LPP, its transfer function is then converted back to that of the FTD.

Transformation of an LPP to the prototype HP, BP, and BR is described in Sections 5.3 to 5.5; the respective transformation factors are also described. Besides these transformations, the level of impedance has to be changed to control the values of the passive components that are

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allowed to be used in a practical circuit. Impedance scaling and conversion of an LPP to an LP filter of some other frequency is studied in Section 5.2.

For convenience and to avoid confusion between the frequency axes of the FTD and LPP, different symbols are used. For the LPP, the complex frequency variable is expressed in capital letters $S = \Sigma + j\Omega$; small letters $s = \sigma + j\omega$ are used for the FTD. To transform the transfer function of the LPP, $H_{LP}(S)$ in terms of the transfer function of either HP: $T_{HP}(s)$, BP: $H_{BP}(s)$, or band elimination (stop): $H_{BE}(S)$ functions, we need to find an appropriate functional relation as follows:

$$\Omega = f(\omega) \tag{5.1}$$

The function $f(\omega)$ has to be selected in such a way that the approximated magnitude function of LPP, $|H_{LP}(j\Omega)| = |H_{LPP}\{f(j\omega)\}| = |H_{HP}(j\omega)|$ (say) for HPF.

It is important to note that transformation through equation (5.1) affects only the frequency axis. The magnitude on the y-axis is not affected; therefore, the amount of variation of gain in the pass and stop band will remain the same.

There is an alternate method of converting the LPP magnitude response to other responses known as the *network transformation* method. It will be shown that this method is more convenient as it can use the available extensive networks and element values of doubly terminated LPP ladders for any arbitrary specifications.

5.2 Frequency and Impedance Scaling

We have discussed *normalized* and *de-normalized* frequency earlier. Study of approximation can be in both forms, but doing it in normalized form is comparatively easy. Changing from one frequency level to another is called *frequency scaling*; changing the frequency level from unit frequency to another frequency (usually higher) is called *frequency de-normalization*. In this section, we will express frequency and impedance scaling in a formal way and also observe their effect on the location of poles and zeros of the transfer function of the prototype filter.

5.2.1 Frequency scaling

The simplest form of frequency transformation is a frequency scaling operation which is given in terms of the frequency scaling parameter ω_o as follows:

$$S = (s/\omega_o) \rightarrow s = \omega_o S \tag{5.2}$$

The transformation in equation (5.2) converts an LPP response to another LP response at a different frequency level: from normalized to de-normalized, with S being considered as a normalized frequency.

Use of the transformation equation (5.2) changes a numerator factor $(S - z_i)$ to $(s - \omega_o z_i)$ and a denominator factor $(S - p_j)$ to $(s - \omega_o p_j)$ in the factorized form of an LPP transfer function. This means that poles and zeros for the new LPF are simply multiplied by the

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frequency transformation factor ω_o. Hence, expressions for the new zeros and poles will be as follows:

$$(z_i)_s = (\omega_o z_i)_S, (p_j)_s = (\omega_o p_j)_S \tag{5.3}$$

While transforming an LPP to another LP, the normalization frequency and its mirror image in the negative x-axis, $\pm\Omega$ converts to the frequency $\pm\omega_o\Omega$. However, the transfer function retains the same magnitude. Hence, as shown in Figure 5.1(a) and (b), magnitude of the LPP and LP are equal, that is, $|T_{\text{LPP}}(j\Omega)| = |H_{\text{LP}}(j\omega)|$ and the pass band edge frequency $\Omega_p = 1$ rad/s and stop band edge frequency Ω_S rad/s gets converted to $\omega_p = \omega_o$ rad/s and $\omega_s = \omega_o\Omega_S$ rad/s, respectively.

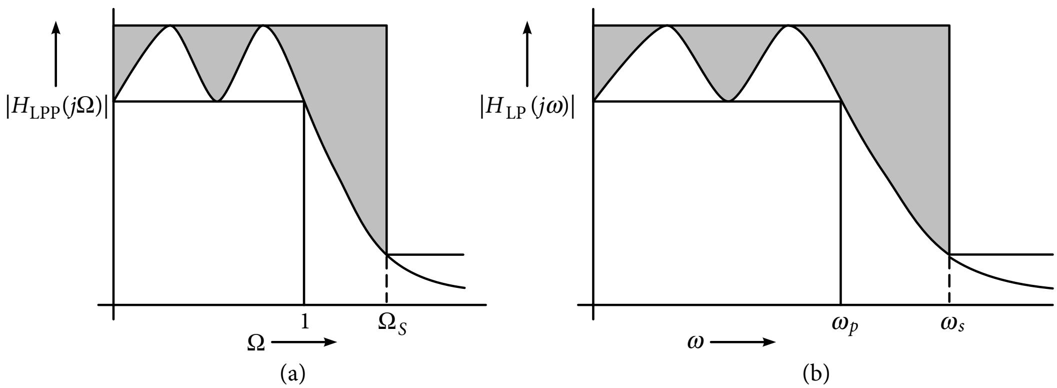

Figure 5.1 Transformation of frequency level from (a) low pass prototype to (b) another low pass.

A filter section is realized either using only passive elements or including active elements. It is only the inductors and capacitors which are frequency dependent; hence, only the values of these elements are affected during frequency transformations. Use of the frequency transformation equation (5.2) converts the inductor impedance of LPP, $Z_L = j\Omega L$ ohms to $Z_L' = (j\Omega L / \omega_o)$ ohms and the capacitor impedance of LPP, $Z_c = 1/j\Omega C$ ohms to $Z_C' = (\omega_o / j\Omega C)$ ohms. Thus, the value of the inductance and the capacitance are divided by the factor ω_o, as illustrated in Figure 5.2(a) and (b).

5.2.2 Impedance scaling

After designing a filter, the circuit configuration is to be selected. Element values of the selected circuit depend on the selected architecture, specifications of the filter and the frequency range of operation. The resulting circuit has to be realized in either discrete form or in an integrated circuit (IC) form. For either form of practical realization, element values should be in a practical range. For example, in IC form, capacitance values should be as small as possible, preferably below the nF range and the resistances should be of the order of or less than a few kilo ohms range. To convert the element values of the designed circuit to within the practically desirable range, *impedance scaling* is almost essential, where all impedances of the network are scaled by a common factor say k. It is important to note that while performing impedance scaling the voltage ratio transfer function of the circuit is not affected because it is dimensionless.

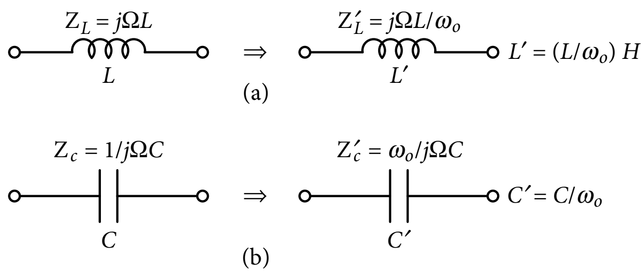

Figure 5.2 Change in the value of (a) inductor and (b) capacitor, due to the frequency transformation.

Similar to the frequency scaling, during impedance scaling also, the initial circuit whose impedance level is to be changed is called the *normalized impedance circuit* (NIC) and the circuit after the impedance scaling is called the *de-normalized impedance circuit* (DIC). Hence, impedance scaling operation is expressed as:

$$z(\text{DIC}) = k \times Z(\text{NIC}) \tag{5.4}$$

Application of equation (5.4) changes the impedance level of resistor (R), inductance (L), capacitance (C), transconductance gain coefficient (G_m) and transresistance gain coefficient (R_n). The respective changed expressions for the DIC are as follows.

$$kR,\ Z(\omega kL),\ Z\left(1/\omega\frac{C}{k}\right),\ Z\left(\frac{G_m}{k}\right),\ \text{and } ZkR_n \tag{5.5}$$

These changed expressions result in a change in the respective circuit element values:

$$r = kR\ \Omega,\ l = kL\ H,\ c = C/k\ F,\ g_m = G_m/k\ \text{mho, and } r_n = kR_n\ \Omega \tag{5.6}$$

Example 5.1: Apply frequency scaling by a facto $\omega_o = 10$ krad/s and impedance scaling factor $k = 10^5$ to the LPP passive ladder structure of a seventh-order Chebyshev filter with a 1 dB pass band ripple width (shown in Figure 5.3(a)), and find the element values after the scaling.

Solution: In the original ladder structure, values of the elements are:

$$\begin{aligned} &R_1^* = R_2^* = 1\Omega, C_1^* = C_7^* = 2.1666F, C_3^* = C_5^* = 3.0936F, \\ &L_2^* = L_6^* = 1.1115H,\ L_4^* = 1.735H \end{aligned} \tag{5.7a}$$

Application of frequency scaling will not change the resistor values, but inductances and capacitances will be divided by 10^4; hence, the frequency scaled elements are as follows:

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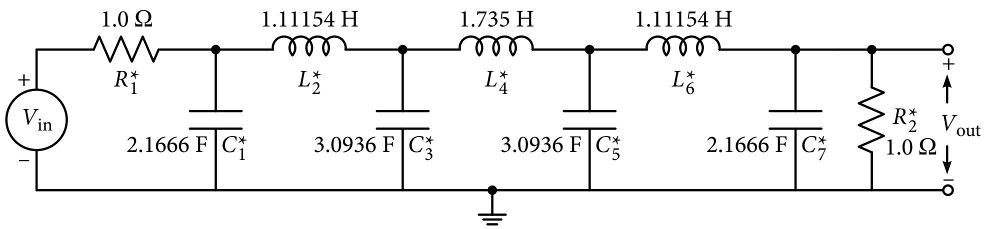
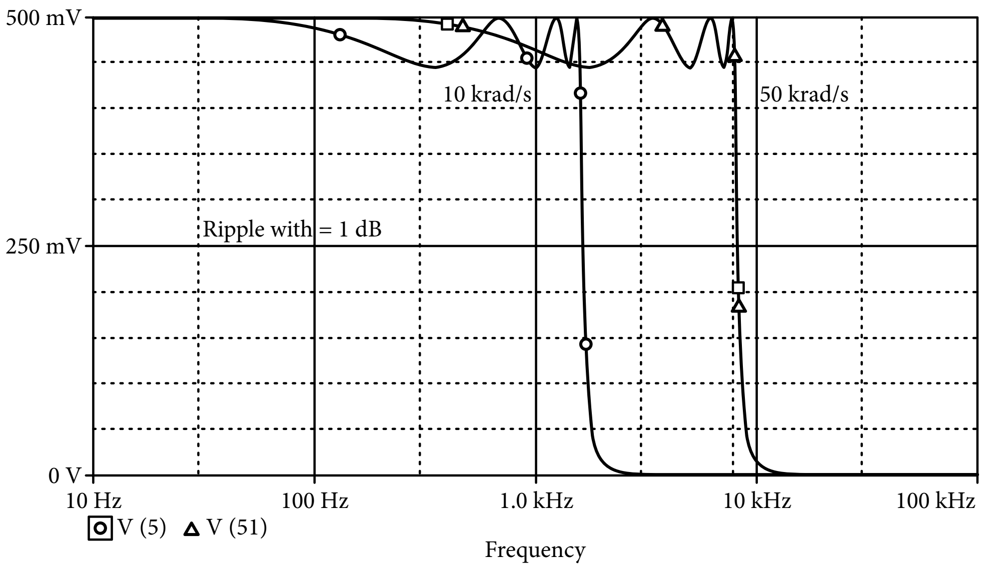
$$R_1 = R_2 = 1\ \Omega,\ C_1 = C_7 = 0.21666\text{ mF},\ C_3 = C_5 = 0.30936\text{ mF},$$
$$L_2 = L_6 = 0.11115\text{ mH},\ L_4 = 0.1735\text{ mH} \tag{5.7b}$$



(a)



(b)

Figure 5.3 (a) Seventh-order passive low pass ladder structure with 1 dB pass band ripple. (b) Transformation from low pass prototype to low pass responses at higher frequencies.

Impedance scaling by a factor of 10^5 will increase the values of the resistors and inductors but decrease the values of the capacitances. Hence, the final element values will be:

$$R_1 = R_2 = 100\text{ k}\Omega,\ C_1 = C_7 = 2.1666\text{ nF},\ C_3 = C_5 = 3.0936\text{ nF},$$
$$L_2 = L_6 = 11.115\text{ H},\ L_4 = 17.35\text{ H} \tag{5.7c}$$

The original passive ladder had a pass band edge frequency ω_p of 1.0 rad/s. After the frequency transformation, the design value of ω_p becomes 10 krad/s. The PSpice simulated value of ω_p from the response shown in Figure 5.3(b) is 9.995 krad/s (1.59 kHz). The ladder was frequency transformed again by a factor of 50 krad/s; the response in this case is also shown in Figure 5.3(b). ω_p is 49.78 krad/s (7.92 kHz).

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5.3 Low Pass to High Pass Transformations

Most of the time, the magnitude function of a filter $|H(j\omega)|$ is sketched in the first quadrant, that is, where the frequency remains positive, though the function $|H(j\omega)|^2$ spreads on to both quadrants. As the magnitude function $H(j\Omega)$ is an even function $|H(j\Omega)| = |H(-j\Omega)|$, it gets reflected on the negative *x*-axis. For example, Figure 5.4 shows a sketch for a maximally flat response for the complete range of frequency, that is, from $-\infty$ to $+\infty$. For the normalized frequency response of an LP filter, pass band extends in the range $|\Omega| \le 1$ and the stop band ranges from Ω_s to ∞ and from $-\Omega_s$ to $-\infty$. To convert the normalized LPP of Figure 5.4 to an HP response, the pass band should range from $\omega = 1$ to ∞ and $\omega = -1$ to $-\infty$, and the stop band from $-\omega_s$ to $+\ \omega_s$, respectively, as shown in Figure 5.5. Study of the two figures suggests the form of frequency transformation from an LPP to an HP *FTD*. Zeros of the LPP at $\Omega = \infty$ and $-\infty$ are to be converted so that they are at $\omega = 0$ for the HP filter. Comparison between Figures 5.4 and 5.5 suggests that the pass band of the LP $(-1 \le \Omega \le +1)$ needs to be converted to a pass band of HP as $(+1 \le \Omega \le -1)$. Such a transformation is obtained by selecting:

$$s = 1/S \text{ or } S = 1/s \tag{5.8}$$

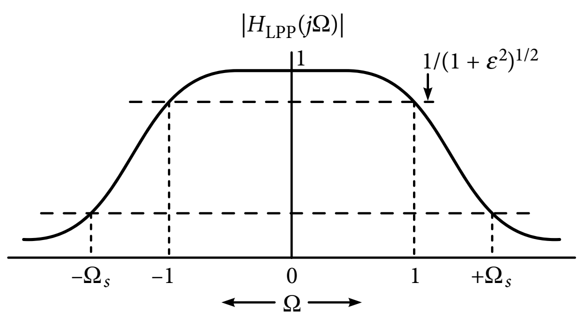

Figure 5.4 Even function response of a maximally flat low pass function.

While working on a transfer function in terms of the complex frequency variable s, it is needed to replace $j\Omega$ by S and $j\omega$ by s. Hence, from equation (5.8),

$$j\omega = 1/j\Omega \to \omega = -(1/\Omega) \text{ or } j\Omega = 1/j\omega = -(j/\omega) \to \Omega = -(1/\omega) \tag{5.9}$$

Let us consider a normalized second-order LPP with quality factor Q and dc gain K, and corresponding frequency de-normalized transfer function, as given by equation (5.10) below:

$$H_{\mathrm{LPP}}(S) = K\frac{1}{S^2 + \left(\dfrac{1}{Q}\right)S + 1} = K\frac{\omega_o^2}{S^2 + \left(\dfrac{\omega_o}{Q}\right)S + \omega_o^2} \tag{5.10a}$$

Application of the transformation equation (5.8) shall lead to the following corresponding HP transfer functions:

$$H_{\mathrm{HP}}(s) = K\frac{s^2}{s^2 + \left(\dfrac{1}{Q}\right)s + 1} = K\frac{s^2}{s^2 + \left(\dfrac{1}{\omega_o Q}\right)s + \dfrac{1}{\omega_o^2}} \tag{5.10b}$$

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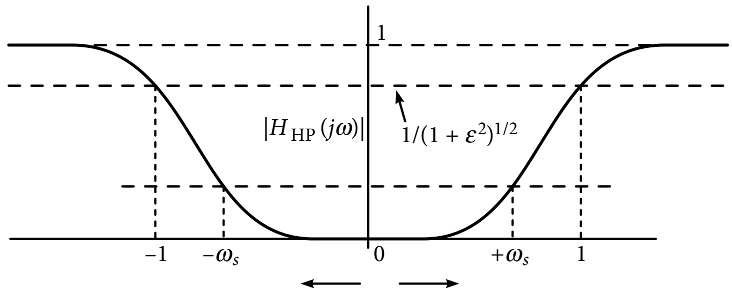

Figure 5.5 Even function response of a maximally flat normalized high pass function.

Obviously, it will retain the same quality factor Q but pass band edge frequency ω_o shall become $(1/\omega_o)$ for the HP filter.

Based on the aforementioned discussion, to obtain a network for a normalized HP filter (HPF) section, the following steps are to be taken.

1. Normalize the specifications of the given HPF by dividing the frequency axis by the pass band edge frequency ω_p so that its pass band is in the normalized frequency range $\omega \geq 1$.
2. Find the normalized stop band edge frequency of the HPF (ω_s/ω_p).
3. Using the transformation relation of equation (5.9) (neglecting the negative sign), obtain the selectivity factor for the LPP (Ω_p/Ω_s).
4. Use any type of magnitude approximation for obtaining the $H_{LPP}(s)$ for the calculated selectivity factor and the given attenuation (or ripples) in pass and stop band.
5. Apply the LP to HP frequency transformation of equation (5.8).

Example 5.2: Design an HPF using LP to HP transformation, with a maximally flat response having the following specifications:

$$\alpha_{\min} = 40 \text{ dB}, \alpha_{\max} = 1\text{dB}, \omega_s = 1000 \text{ rad/s and } \omega_p = 4000 \text{ rad/s}$$

Also determine the attenuation at 1500 rad/s and 750 rad/s.

Solution: In the first step, specifications of the HPF are normalized by dividing the frequency range by ω_p, so the stop band edge normalized frequency becomes 0.25 rad/s.

Next, the selectivity factor of the normalized LPP $= 1/\Omega_s = 1/0.25 = 4$.

Design of the LPP requires calculation of the factor ε and order of the filter n. Application of equations (3.12) and (3.23), respectively, gives:

$$\varepsilon^2 = (10^{0.1\alpha_{\max}} - 1) = (10^{0.1} - 1) = 0.258 \tag{5.11}$$

$$n = \frac{\log\{(10^4 - 1)/(10^{0.1} - 1)\}}{2\log 4} = 3.79 \tag{5.12}$$

Therefore, order of the LPP will be 4. Use of Table 3.1 gives the location of pole for $n = 4$ as:

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$$p_{1,2} = 0.3826836 \pm j0.9238795 \text{ and } p_{3,4} = -0.9238795 \pm j0.3826836$$

The normalized transfer function of the LPP is obtained as shown here:

$$H_{\text{LPP}}(S) = \frac{1}{(S^2 + 0.7653668\ S + 1)(S^2 + 1.847749\ S + 1)} \tag{5.13}$$

Applying LP to HP transformation of equation (5.8) on the transfer function $H_{\text{LPP}}(S)$, the transfer function of the fourth-order normalized HP becomes:

$$H_{\text{HP}}(s) = \frac{s^4}{(s^2 + 0.7653668s + 1)(s^2 + 1.847749s + 1)} \tag{5.14}$$

There are several options to synthesize equation (5.14). In one option, the section is broken into two second-order sections with transfer functions H_1 and H_2 (given in the following equations), which will be cascaded and then frequency and impedance scaling shall be applied.

$$H_1(s) = \frac{s^2}{(s^2 + 0.7653668s + 1)} \tag{5.15}$$

$$H_2(s) = \frac{s^2}{(s^2 + 1.847749s + 1)} \tag{5.16}$$

A single amplifier second-order filter section shown in Figure 5.6 has the following expression in equation (5.17). It is used to realize the transfer functions H_1 and H_2.

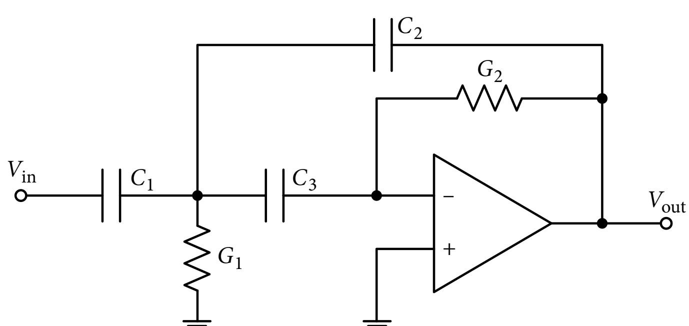

Figure 5.6 A single amplifier, second-order high pass filter.

$$\frac{V_{\text{out}}}{V_{\text{in}}} = -\frac{(C_1 / C_3)s^2}{s^2 + G_2\dfrac{(C_1 + C_2 + C_3)}{C_2C_3}s + \dfrac{G_1G_2}{C_2C_3}} \tag{5.17}$$

To maintain high frequency gain as unity, comparing equations (5.15) and (5.17), and assuming

$C_1 = C_3 = 1$, we get:

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$$G_2 = \left(1/G_1\right), \text{ and with } G_2 \frac{3C}{C^2} = 0.7653 \rightarrow R_2 = 3.919\Omega, \text{ and } R_1 = 0.2551\Omega \tag{5.18a}$$

Frequency normalization is to be done with respect to the 3 dB frequency. To convert the pass band edge frequency of 4000 rad/s to 3 dB frequency, from equation (3.25):

$$\omega_{CB} = \{(10^{0.1\times 3} - 1)/0.2589\}^{1/2\times 4} = 1.183 \tag{5.18b}$$

Hence, the frequency scaling factor will be $4000/1.183 = 3381.2$ rad/s. If all the three capacitors are selected as 0.1 μF, which is a convenient practical value, the impedance scaling factor shall be $10^7/3381 = 2957$. Using this impedance scaling factor, we get $R_{11} = 754\ \Omega$ and $R_{12} = 11.588$ kΩ.

In the same way, comparing equation (5.17) with the transfer function H_2 of equation (5.16), element values are $C_{12} = C_{22} = C_{32} = 0.1$ μF, $R_{12} = 1.797$ kΩ and $R_{22} = 4.803$ kΩ. The cascaded fourth-order HP circuits with the element values used are shown in Figure 5.7(a); their PSpice simulation is shown in Figure 5.7(b).

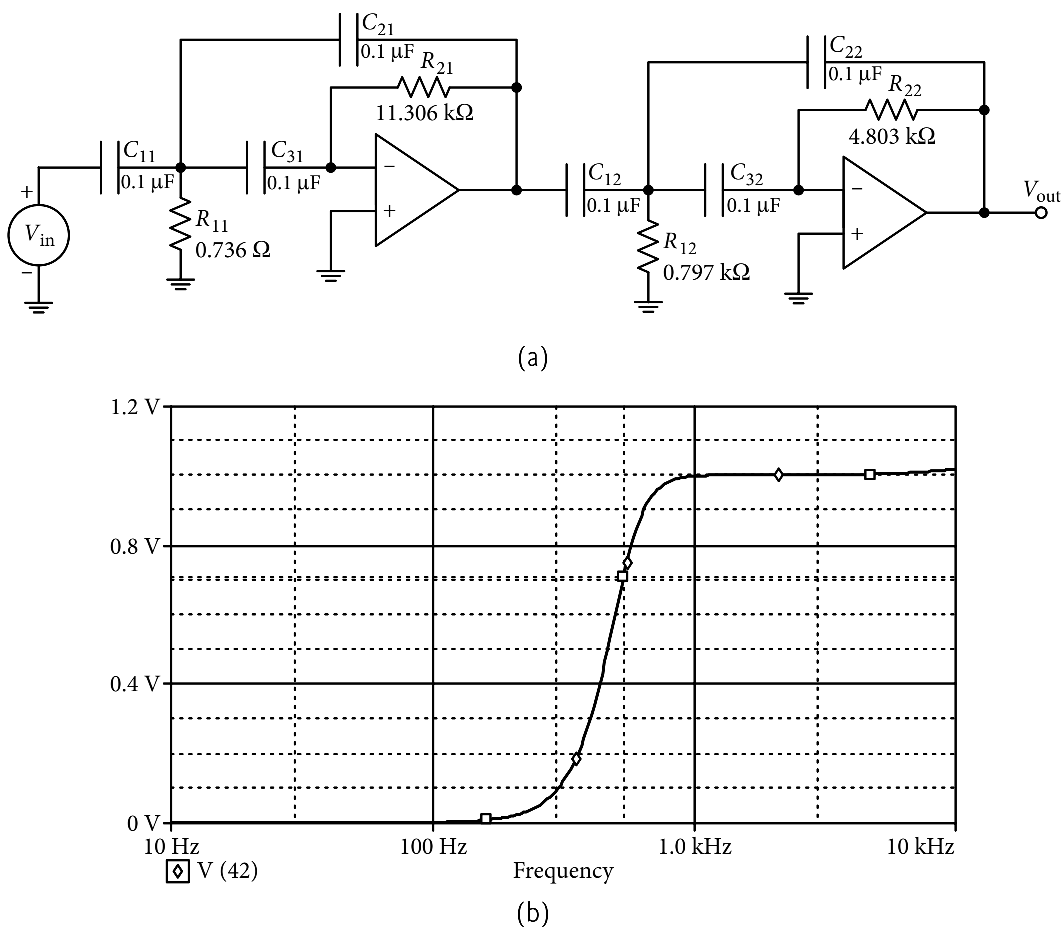

Figure 5.7 (a) Fourth-order high pass maximally flat filter for Example 5.2. (b) Magnitude response of the fourth-order high pass filter of Figure 5.7(a) for Example 5.2.

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Simulated 3 dB frequency is 539.6 Hzor 3391 rad/s, high frequency gain is unity, pass band edge frequency (4000 rad/s) attenuation is 42.6 dB, and stop band edge frequency (1000 rad/s) attenuation is 0.972 dB. Attenuation at 1500 rad/s and 750 rad/s is 28.2 dB and 52.4 dB, respectively. The observed parameters are very close to the design values as specifications are very well satisfied.

Low Pass to High Pass Network transformation: LP to HP frequency transformation can also be applied directly on a network of a LPP. For a network which contains, resistors, inductors, capacitors and active device, frequency trans-formation shall affect only the inductors and capacitors. Use of equation (5.8) shall transform these components as shown in figure 5.8. Impedance of inductance (SL) gets changed to capacitive impedance having capacitor value of (1/L) and correspondingly capacitive impedance (1/SC) changes to inductive impedance with inductance value (1/C). This conversion process is often used instead of dealing in pole/zero reciprocation. While performing network transformation it is important to note that such a conversion shall be done on the elements, when the LP transfer function is normalized to get a normalized HPF with the pass band edge frequency $\Omega p = 1$. Later, frequency scaling shall be performed on the normalized HPF for a desired pass band edge frequency; pole-Q shall remain unchanged from the LPF.

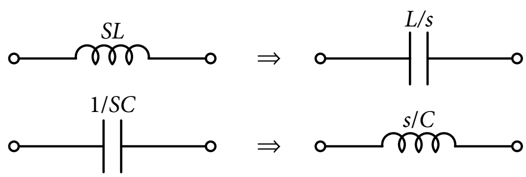

Figure 5.8 Application of an LP to HP frequency transformation on inductive and capacitive impedances.

5.4 Low Pass to Band Pass Transformation

Figure 5.9(a) shows the frequency response of a normalized LP function, approximated in maximally flat form with its 3 dB frequency at $\Omega = 1$ and normalized stop band edge frequency Ω_s. Application of a suitable frequency transformation should give a band pass (BP) response as shown in Figure 5.9(b) converting the 3 dB frequency of the LPF to the lower and upper cut-off frequencies and the stop band frequency gets converted to the two stop band frequencies ω_{s1} and ω_{s2} of the BPF. The pole frequency at $\Omega = 0$ is converted to the normalized center frequency $\omega_o = 1$. The LP response in Figure 5.9(a) was shown only in the first quadrant; whereas for the rational transfer function spread over the whole frequency range of $-\infty$ to $+\infty$, Figure 5.10(a) shows the LP response in inverse Chebyshev form. Its pass band ranges from $\Omega = -1$ to $+1$, which is to be transformed to the pass band of the BPF, extending from frequency ω_1 to ω_2 for positive frequencies. Obviously, the center frequency of the BPF where its magnitude is maximum will lie within the frequency range $\omega_1 \leq \omega \leq \omega_2$; this equals 1.0 for the normalized frequency BPF with $\omega_1 < 1$ and $\omega_2 > 1$. For the LPP, there will be a pole at $\Omega = 0$ and zero at $\Omega = \pm\infty$, whereas for the transformed BP, there will be zeros at $\omega = \pm 1$ and a pole at $\omega = 0$ and $\pm\infty$. For such a conversion, the following function will be sufficient.

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$$\Omega = \frac{1}{(\mathrm{BW})}\frac{(\omega-1)(\omega+1)}{\omega} = \frac{1}{\mathrm{BW}}\frac{(\omega^2-1)}{\omega} \tag{5.19}$$

In equation (5.19), the term BW has been included to normalize and adjust the slope of the function as explained in the following text. For the LPP, its pass band edge frequency $\Omega = 1$. Hence, from equation (5.19), we get:

$$\Omega = 1 = \frac{1}{\mathrm{BW}}\frac{(\omega^2-1)}{\omega} \rightarrow \omega^2 - \omega * \mathrm{BW} - 1 = 0 \tag{5.20}$$

It has been shown that the pass band edge frequency of the LPP ($\Omega = 1$) has been transformed as the pass band edge frequencies of the transformed BPF: $-\omega_1$ and ω_2. As these frequencies $-\omega_1$ and ω_2 should be the solution of equation (5.20), we can express the following.

$$(\omega + \omega_1)(\omega - \omega_2) = 0 \rightarrow \omega^2 - \omega(\omega_2 - \omega_1) - \omega_1\omega_2 = 0 \tag{5.21}$$

It is important to note that the scheme is more useful in passive structures employing both inductors and capacitors. In active-RC circuits it is not preferable as such, because it will convert capacitors as inductors, which shall have to be simulated using active-RC circuits.

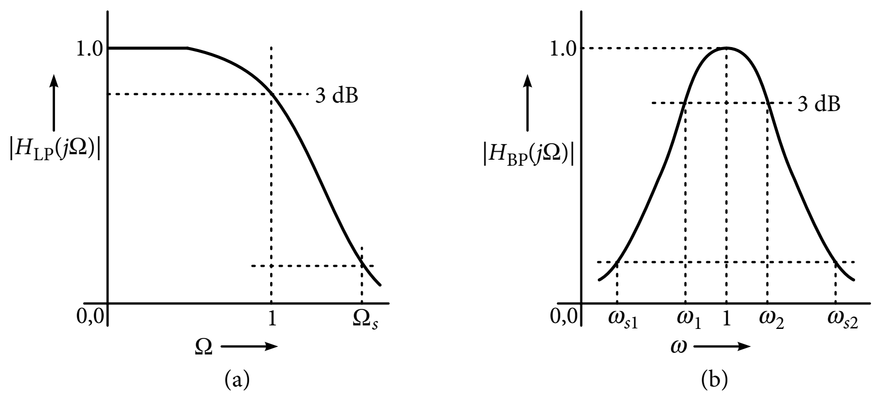

Figure 5.9 Application of a normalized low pass magnitude response transformation to convert it to a normalized band pass response.

Comparing equations (5.20) and (5.21), we get the following expression:

BW = $\omega_2 - \omega_1$, and the product of the normalized pass band edge frequencies,

$$\omega_1 \times \omega_2 = 1 \tag{5.22}$$

Hence, BW = $(\omega_2 - \omega_1)$, introduced in equation (5.19), is the bandwidth of the BPF and its normalized center frequency $\omega = 1$ is the geometric mean of the pass band edge frequencies, ω_1 and ω_2.

Now, multiplying equation (5.19) by j

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$$j\Omega = \frac{j}{\mathrm{BW}}\left(\frac{\omega^2 - 1}{\omega}\right) = -\frac{1}{\mathrm{BW}}\frac{\omega^2 - 1}{j\omega} \tag{5.23}$$

With $S = j\Omega$ and $s = j\omega$, we can write equation (5.23) as follows:

$$S = \frac{1}{\mathrm{BW}}\frac{s^2 + 1}{s} = Q\left(s + 1/s\right) \tag{5.24}$$

Hence, Q, referred to as the *quality factor* is defined as the center frequency ($\omega = 1$) of the BPF divided by the bandwidth (BW).

For a normalized BPF with center frequency as ω_o instead of 1.0, equations (5.19), (5.22) and (5.23) will be modified as follows:

$$\Omega = \frac{1}{\mathrm{BW}}\frac{\omega^2 - \omega_o^2}{\omega} \tag{5.25}$$

$$\omega_1 \times \omega_2 = \omega_o^2 \tag{5.26}$$

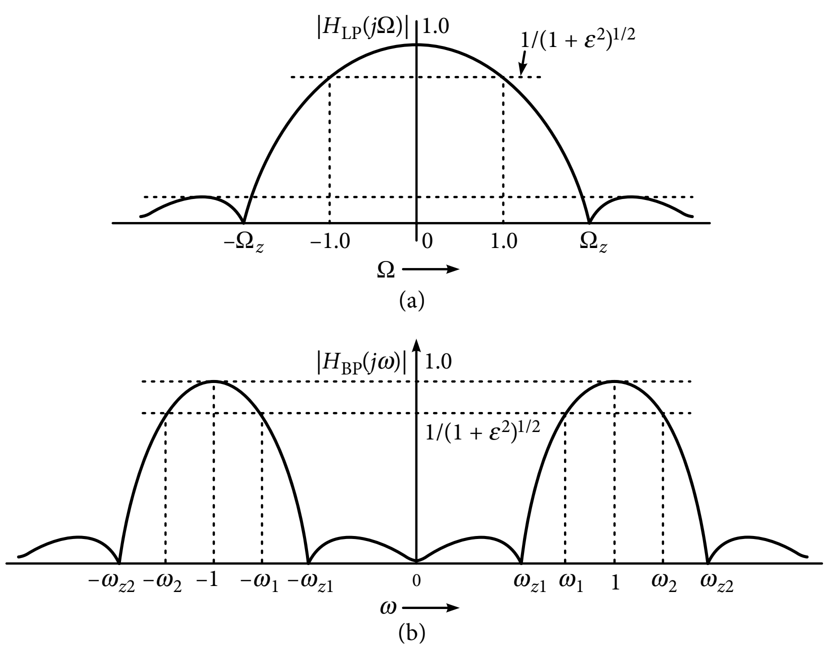

Figure 5.10 (a) A normalized low pass response in inverse Chebyshev approximated form transformed to (b) a normalized band pass response.

$$S = Q\left(\frac{s}{\omega_o} + \frac{\omega_o}{s}\right) \tag{5.27}$$

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With quality factor Q as:

$$Q = \omega_o/(\omega_2 - \omega_1) \tag{5.28}$$

Either equation (5.19) representing the normalized transformation factor or the de-normalized transformation factor of equation (5.25) can be used to convert LPP to BP functions. However, with transformation, the order of the BP function becomes double of the order of the LPP. For example, a third-order LPP became a sixth-order BP, and so on. Hence, the application of the frequency transformation will change the transfer function. Denominator of the transfer function can be expressed as before, either in a polynomial form or in terms of second-order factors. When the denominator is expressed in polynomial form, any direct form of synthesis procedure can be adopted. Alternatively, when the denominator is in the factorized form, a variety of methods employing second-order sections, including a cascade of second-order networks can be used. However, for the factorization, location of poles of the transformed BP function has to be found out as will be discussed here.

A first-order LP function with a real pole at $S = -\Sigma_r$ results in two complex poles in the normalized BP functions as follows:

$$p_{1,2} = -(\Sigma_r/2Q) \pm j\{1 - (\Sigma_r/2Q)^2\}^{1/2} \tag{5.29}$$

While arriving at the result in equation (5.29), it is assumed that $2Q > \Sigma_r$, so that the poles p_1 and p_2 are complex.

With the pole of the first-order LPP being at $-\Sigma_r$, its transfer function $H_{\mathrm{LPP}}(S) = 1/(S + \Sigma_r)$ will be converted to a second-order transfer function for which the transformed BP section will be as follows:

$$H_{\mathrm{BP}}(s) = 1/(s - p_1)\,(s - p_2) = 1/\{s^2 + (\Sigma_r/Q)s + 1\} \tag{5.30}$$

Somewhat more complex is the case when the LPP has complex conjugate poles $(-\Sigma \pm j\Omega)$, which are converted to four poles for the BP function. These four roots appear in the following conjugate complex pair form.

$$\left\{s^2 + \left(\omega_{o1}/q_1\right) + \omega_{o1}^2\right\}\left\{s^2 + \left(\omega_{o2}/q_2\right) + \omega_{o2}^2\right\} \tag{5.31}$$

Here, $\omega_{o1,}$ ω_{o2} are the pole frequencies and q_1, q_2 are the pole quality factors of the two second-order BP sections. Because of the nature of the complex conjugate poles of the LPP function and the transformation factor, equation (5.31) possess the following properties:

$$\omega_{o1} \times \omega_{o2} = 1 \text{ and } q_1 = q_2 = q \tag{5.32}$$

This means that the two normalized pole frequencies are reciprocal of each other and symmetrical about $\omega_o = 1$; the pole quality factor are equal in value.

If the LPP has the following second-order transfer function

$$H_{\mathrm{LPP}}(S) = 1/\left\{S^2 + \left(\Omega_o/Q\right) + \Omega_o^2\right\} \tag{5.33}$$

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Utilization of the properties in equation (5.32) helps in finding the expression for the pole frequencies ω_{o2} and ω_{o1} which have been shown to be:

$$\omega_{o2} = \frac{1}{\omega_{o1}} = \frac{q\Omega_o}{2Q^2} + \frac{1}{2}\left(\frac{\Omega_o^2}{Q^2} - \frac{1}{q^2}\right)^{1/2} \tag{5.34}$$

From equation (5.34), ω_{o1} and ω_{o2} can be found once value of q is known, which can be obtained from the following:

$$q^2 = \frac{Q^2}{\Omega_o}\left[\left(\frac{2Q^2}{\Omega_o} + \frac{\Omega_o}{2}\right) \pm \left\{\left(\frac{2Q^2}{\Omega_o} + \frac{\Omega_o}{2}\right)^2 - 1\right\}^{1/2}\right] \tag{5.35}$$

Restriction in equation (5.35) is that only the plus sign of the square root is taken to obtain the positive value of ω_{o2}.

Example 5.4: An LPP has pole pairs at $S = \Sigma \pm j\Omega = -1.0 \pm j1.0$. Find the location of poles and values of the pole-Q for a transformed BPF.

Solution: For the given pole location, transfer function of the LPP shall be:

$$H_{\mathrm{LPP}}(S) = 1/(S^2 + 2S + 2) \tag{5.36}$$

Hence, the normalized pole frequency and the pole-Q of the LPP are $\sqrt{2}$ and $1/\sqrt{2}$, respectively. Using the LPP to BP transformation factor of equation (5.24), the transfer function of the normalized BPF shall be obtained as follows (for ($Q = 1/\sqrt{2}$)):

$$T_{\mathrm{BP}}(S) = \frac{1}{Q^2\left(\frac{s^2+1}{s}\right)^2 + 2Q\frac{s^2+1}{s} + 2} = \frac{2s^2}{s^4 + 2\sqrt{2}s^3 + 6s^2 + 2\sqrt{2}s + 1} \tag{5.37}$$

To find the location of the four poles of the BPF, equation (5.35) is used to find q (for ($Q = 1/\sqrt{2}$)) as:

$$q^2 = \frac{1}{(2)\sqrt{2}}\left[\left(\frac{2}{2\sqrt{2}} + \frac{\sqrt{2}}{2}\right) \pm \left\{\left(\frac{2}{(2)\sqrt{2}} + \frac{\sqrt{2}}{2}\right)^2 - 1\right\}^{1/2}\right] = 0.5(1 + 1/\sqrt{2}) = 0.85355$$

The equation gives $q = 0.9293$; hence, from equation (5.34):

$$\omega_{o2} = \frac{0.9293\sqrt{2}}{2} \times 2 + \frac{1}{2}\left(2 \times 2 - \frac{1}{0.85355}\right)^{1/2} = 2.155, \text{ and } \omega_{o1} = \frac{1}{2.155} = 0.464 \tag{5.38}$$

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Location of the poles for the BP section are given as:

$$s_{1,2} = -\omega_{o1}\left\{\frac{1}{2q} \pm j\left(1-\frac{1}{4q^2}\right)^{½}\right\}$$

$$= -0.464\left\{\frac{1}{2*0.9293} \pm j\left(1-\frac{1}{4q^2}\right)^{½}\right\} = -0.2496 \pm j0.3901 \quad (5.39a)$$

$$s_{3,4} = -1.159 \pm j1.8121 \quad (5.39b)$$

Obviously, roots of the denominator in equation (5,37) shall yield the pole values as obtained in equation (5.39).

5.4.1 Design steps for transformation to BPF

To design a BPF with the requisite specifications, the following steps are to be taken if an LPP to BP frequency transformation is used.

i. Calculate the pole frequency ω_o of the BPF. If it is not given in direct form, it may be obtained from the pass band edge frequencies as $\omega_o = (\omega_{p1} \times \omega_{p2})^{1/2}$.

ii. Next, the stop band frequencies of the BPF are made geometrically symmetric with respect to the pole frequency obtained in step (i) through altering ω_{s1} or ω_{s2}.

    However, this choice of alteration in either ω_{s1} or ω_{s2} has to be such that one of these becomes more constrained; the stop band becomes narrow, making the design specification a little more severe. For $\omega_{s1} < \left(\omega_o^2 / \omega_{s2}\right)$, ω_{s1} is to be assigned a new value as $\omega_{s1} \geq \left(\omega_o^2 / \omega_{s2}\right)$. Otherwise, a new value assigned to ω_{s2} shall will be calculated from $\omega_{s2} = \left(\omega_o^2 / \omega_{s1}\right)$.

iii. Using the modified stop band of the BPF, selectivity factor of the LPP is calculated as $\Omega_S = (\omega_{s2} - \omega_{s1})/(\omega_{p2} - \omega_{p1})$.

iv. Parameter ω_o becomes modified due to the changed value in ω_{s1} or ω_{s2} as $\omega_o = (\omega_{s1} \times \omega_{s2})^{1/2}$.

v. Change in the value of ω_o creates asymmetry in ω_{p1} and ω_{p2} with respect to it. Hence, either ω_{p1} or ω_{p2} is to be constrained similar to the case for stop band. If $\omega_{p1} < \left(\omega_o^2 / \omega_{p2}\right)$, then ω_{p1} is assigned a new value from $\omega_{p1} \geq \left(\omega_o^2 / \omega_{p2}\right)$. Otherwise, ω_{p2} is assigned a new value from $\omega_{p2} = \left(\omega_o^2 / \omega_{p1}\right)$.

vi. The modified selectivity factor of the LPP is now calculated due to the change in the pass band frequency range.

vii. Out of the two selectivity factors obtained in step (iii) and step (vi), the larger one is selected; transformation parameters ω_o and BW are evaluated corresponding to the steps (i)–(iii) or (iv)–(vi), whichever leads to the larger value of the LPP selectivity, as it leads to the lowest order n for the LPP(S).

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viii. Any method of approximation can be used and the LPP transfer function $H_{LPP}(S)$ is then obtained using the calculated value of the order n and ripple factor ε.

ix. A transformation factor is used to obtain a BP transfer function by replacing S with $\{Q(s^2 + 1)/s\}$ in $H_{LPP}(S)$.

x. The BP is now realized selecting any suitable synthesis process.

Example 5.5: Using the given unsymmetrical frequency specification of a BPF, calculate the selectivity factor for an LPP from which BPF is to be obtained; pass band frequencies, $\omega_{p1} = 5(2\pi)$ krad/s and $\omega_{p2} = 7.2(2\pi)$ krad/s, stop band frequencies, $\omega_{s1} = 4(2\pi)$ krad/s and $\omega_{s2} = 10(2\pi)$ krad/s.

Solution: Center frequency of the BPF, $\omega_o = (\omega_{p1} \times \omega_{p2})^{½} = (10\pi \times 14.4\pi)^{½} = 6 \times (2\pi)$ krad/s. With $\left(\omega_o^2 / \omega_{s2}\right) = 36 \times (2\pi)^2 / 20\pi = 7.2$ krad/s being less than ω_{s1}, the new value of ω_{s2} shall be $\le 36(2\pi)^2/8\pi = 18\pi$ krad/s. As $(\omega_{s1}\omega_{s2})^{1/2} = (9 \times 4)^{1/2}\, 2\pi = 12\pi$, equals the center frequency ω_o, there shall be no change in the pass band edge frequency. Since BW = (7.2–5)2π = 4.4π krad/s, selectivity factor will be = (9–4)/(7.2–5) = 2.27.

Example 5.6: Find the transfer function of a BPF with the following specifications using LP to BP transformation: maximum attenuation of 1 dB between 4 and 9 kHz and minimum attenuation of 40 dBs below 1.5 kHz and beyond 22.5 kHz.

Solution: With the pass band edge frequencies being 4 and 9 kHz, center frequency $f_o = (4 \times 9)^{1/2} = 6$ kHz, and bandwidth = 5 kHz; hence, pole-Q = 1.2.

First, an LPP is to be obtained. Therefore, the specifications of the BPF are to be transformed for the LPP. All the frequencies are normalized with respect to f_o. It gives lower cut-off frequency $\omega_1 = 0.6667$, upper cut-off frequency $\omega_2 = 1.5$; and their product is unity. Normalized lower stop band edge frequency $\omega_{s1} = (1.5/6) = 0.25$ and upper stop band edge frequency $\omega_{s2} = (22.5/6) = 3.75$. Since product of ω_{s1} and ω_{s2} is not unity but less than ω_o^2, a new value has to be given to ω_{s1}, which is equal to $\omega_o^2/\omega_{s2} = 0.26667$ as mentioned in step (ii) of the design process. With the modified stop band, selectivity of the LPP will become:

$$\Omega_S = (\omega_{s2} - \omega_{s1})/(\omega_{p2} - \omega_{p1}) = (3.75 - 0.26667)/(1.5 - 0.6667) = 4.18 \tag{5.40}$$

Required order of the LPP with Chebyshev approximation (from Chapter 4) will be:

$$n = \frac{\ln\left\{4\left(10^{4.0.} - 1\right)/\left(10^{0.1} - 1\right)\right\}^{½}}{\ln\left\{4.18 + \left(4.18^2 - 1\right)^{½}\right\}} = \frac{5.9738}{2.1088} = 2.832 \tag{5.41}$$

Since it is to be rounded to the next integer, $n = 3$. Pole location of the third-order Chebyshev filter obtained from Table 3.4 is as follows:

$$S_1 = -0.4942,\ S_{2-3} = -0.2471 \pm j0.966 \tag{5.42}$$

Normalized transfer function of the third-order LPP will become:

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$$H_{\mathrm{LPP}}(S) = \frac{0.4942}{(S+0.4942)\left(S^2+0.4942S+1\right)} \tag{5.43}$$

$|H_{\mathrm{LPP}}(j\Omega_S = 4.18)|$ from equation (5.43) shows that attenuation is little over 40 dBs, satisfying the requirement.

Numerator of the equation (5.43) is 0.4942 as that will result in $H_{\mathrm{LPP}}(0)$ being unity for the third-order Chebyshev filter. Next, equation (5.27) will be applied on equation (5.43) to get the normalized transfer function of the BPF.

$$H_{\mathrm{BP}}(s) = \frac{0.4942s^3}{1.728s^6+1.4236s^5+6.6772s^4+3.34109s^3+6.6772s^2+1.4236s+1.728} \tag{5.44}$$

The root finder is used to find roots in equation (5.44) which are as follows:

$$s_{1-2} = -0.206 \pm j\,0.979,\ s_{3-4} = -0.142 \pm j\,1.478 \text{ and } s_{5-6} = 0.064 \pm j\,0.67 \tag{5.45}$$

Hence, equation (5.44) is broken into three second-order sections for which factorization of the denominator gives:

$$1.728\,(s^2 + 0.412s + 1)\,(s^2 + 0.284s + 2.204)\,(s^2 + 0.129s + 0.454) \tag{5.46}$$

Equation (5.46) along with the numerator in equation (5.44) can be broken into three second-order sections and realized using the cascade method or equation (5.44) can be used for any direct form of synthesis.

5.4.2 Low pass to band pass network transformation

Similar to the LP to HP transformation case, the LP to BP transformation can also be applied directly to the LPP network. An inductor in the LPP having impedance $Z_p(S) = SL_p$ gets converted to a series combination of an inductor and a capacitor as shown in Figures 5.11(a).

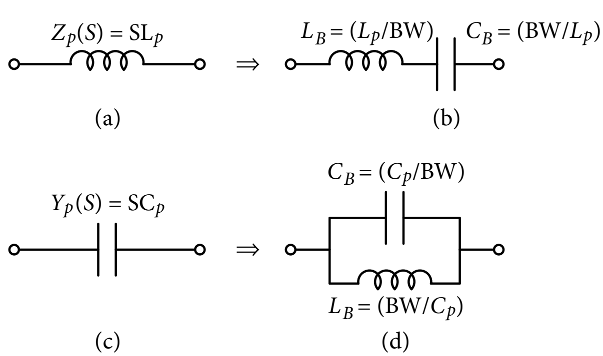

Figure 5.11 Element transformation from low pass prototype to a band pass network.

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$$Z_{\mathrm{BP}}(s) = \frac{1}{\mathrm{BW}}\left(\frac{s^2+1}{s}\right)L_p = \frac{L_p}{\mathrm{BW}}\,\mathrm{s} + \frac{1}{\left(\frac{\mathrm{BW}}{L_p}\right)\mathrm{s}} \tag{5.47}$$

Expressions of the resulting series combination of inductor and capacitor, respectively, are: inductor L_p/BW and capacitor $C_B = \mathrm{BW}/L_p$ as shown in Figure 5.11(b). Likewise, a capacitor in the LPP having an admittance $Y_p(s) = SC_p$, shown in Figure 5.11(c), gets transformed as follows:

$$Y_B = \frac{1}{\mathrm{BW}}\left(\frac{s^2+1}{s}\right)C_p = \frac{sC_p}{\mathrm{BW}} + \frac{1}{\frac{\mathrm{BW}}{C_p}s} \tag{5.48}$$

Equation (5.48) represents a parallel combination of a capacitor (Cp/BW) and an inductor (BW/C_p) as shown in Figure 5.11(d). Resistance being frequency independent, it is not affected by the frequency transformation. Hence, conversion of an LPP network to a BP network can easily be done by using the aforementioned transformations of the inductors and capacitors. It is important to note that the transformed BP network will be a frequency normalized network with center frequency $\omega_o = 1$, which will be applied to passive structures.

Example: 5.7: A third-order Chebyshev approximated normalized LPP ladder structure as shown in Figure 5.12(a) is to be transformed to a BPF through network transformation. Obtain the resulting network and element values of a frequency normalized BP network having normalized BW = 0.1. Also obtain the element values for center frequency $\omega_o = 10^5$ rad/s with an impedance scaling factor of 10^4.

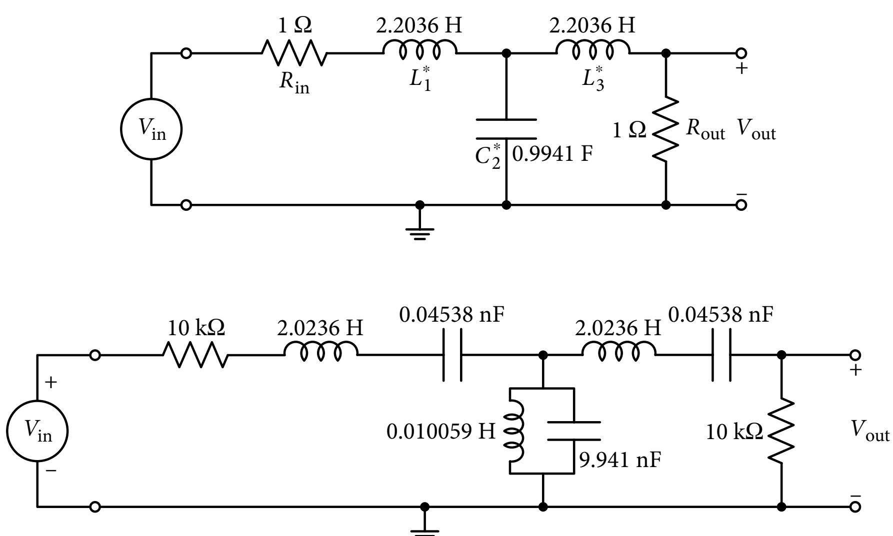

Figure 5.12 (a) Third-order Chebyshev approximated normalized low pass filter. (b) Network transformed de-normalized band pass filter from part (a).

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Solution: For the LPP, which is to have a pass band ripple width of 1 dB, element values are as follows:

$$R_{in} = R_{out} = 1\Omega, L_1^* = L_3^* = 2.0236\text{Ha nd } C_2^* = 0.9941 \text{ F}$$

With BW = 0.1, using equations (5.47) and (5.48), $L_1^* = L_3^*$ changes to a series combination of $l_{1,3} = 20.236$ H and $c_{1,3} = 0.04941$ F, and C_2^* transforms to a parallel combination of $c_2 = 9.941$ F and $l_2 = 0.10059$ H.

Application of frequency translation from $\omega_o = 1$ to 10^5 rad/s and impedance scaling by 10^4 converts elements to the following:

$$L_1 = L_3 = 2.0236 \text{ H}, C_1 = C_3 = 0.04438 \text{ nF}, L_2 = 10.059 \text{ mH}, C_2 = 9.941 \text{ nF},$$
$$\text{and } R_{in} = R_{out} = 10 \text{ k}$$

Figure 5.12(b) shows the transformed BP network with element values. Figure 5.13 shows its PSpice simulated response. The response keeps the nature of variation of output voltage very well with the simulated center frequency being 15.917 kHz; the ripple width in the pass band is 1.15 dB. The lower and upper 3 dB cut-off frequencies are 15.104 kHz and 16.771 kHz, giving BW = 1.667 kHz, and resulting in a pole Q of 9.548.

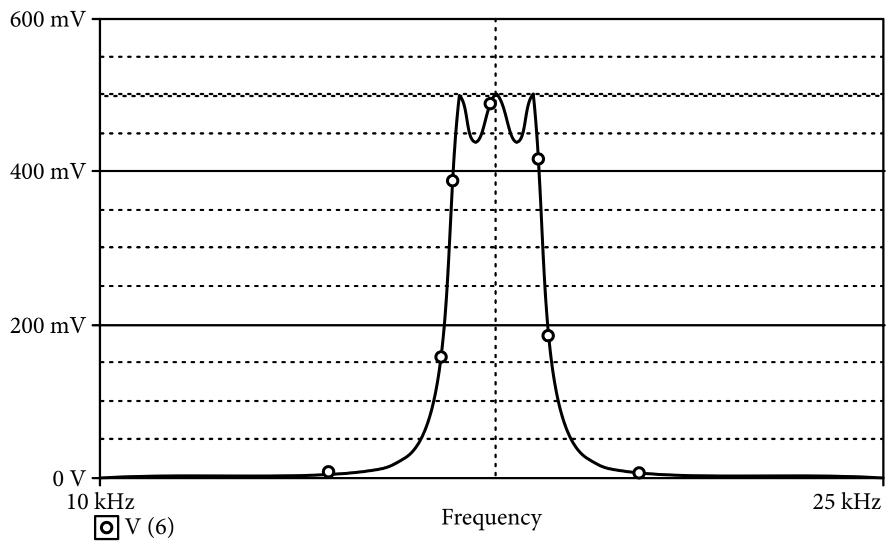

Figure 5.13 Simulated response of the band pass filter of Figure 5.12(b) obtained through network transformation for Example 5.7.

5.5 Low Pass to Band Reject Transformation

Band reject (BR) response being similar in nature to a BP response, it can be transformed from an LPP using a transformation factor similar to the one used for BP transformation:

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$$S = \mathrm{BW}\frac{s}{(s^2+1)} \tag{5.49}$$

Transformation through equation (5.49) can also be explained in terms of two transformations, one from LPP to HPP and one from HPP to a BP transformation as mentioned in the following two steps, resulting in a BR function.

$$S \text{ is replaced by } (1/S'), \text{ then} \tag{5.50}$$

$$S' \text{ is replaced by } \frac{1}{\mathrm{BW}}\left(\frac{s^2+1}{s}\right) \tag{5.51}$$

The BR magnitude response has the following constraints and conditions.

$$\omega_{s1} \times \omega_{s2} = \omega_{p1} \times \omega_{p2} = 1 \tag{5.52}$$

$$\omega_{s2} - \omega_{s1} = \mathrm{BW}\Omega_s' = \mathrm{BW}/\Omega_s \tag{5.53}$$

To realize a BRF, specifications are given in terms of pass band and stop edge frequencies, ω_{p1}, ω_{p2}, ω_{s1} and ω_{s2} and the pass band and stop band attenuations $A_{\max}$ and $A_{\min}$, respectively. Using similar procedure as that for the BP case, selectivity factor of the LPP is found from the expression $\Omega_S = (\omega_{p2} - \omega_{p1})/(\omega_{s2} - \omega_{s1})$. In the same way, frequency specification must be symmetrized with respect to $\omega_o = 1$.

Band reject network transformation: Application of the transformation factor of equation (5.49) with an inductor of the LPP network having admittance $Y_R(S) = 1/(Sl_p)$ becomes an admittance $Y_{\mathrm{BR}}(s) = sC_R + 1/sL_R$; here $C_R = 1/\mathrm{BW}l_p$ and $L_R = \mathrm{BW}l_p$. A capacitor c_p in the LPP with impedance $z_c(S) = 1/(Sc_p)$ gets transformed to an impedance function $Z_{\mathrm{BR}}(s) = sL_R' + 1/(sC_R')$ with $L_R' = 1/(c_p\mathrm{BW})$ and $C_R' = (c_p\mathrm{BW})$. This means that like the BP case, inductances and capacitances of the LPP are transformed to a parallel and series combination, respectively, of an inductor and a capacitor in the BR network. Figure 5.14 shows such a transformation of an inductor and a capacitor.

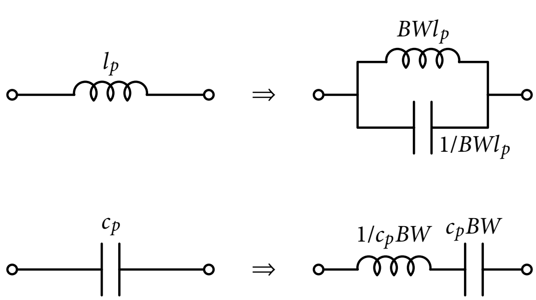

Figure 5.14 Transformation of low pass prototype network elements to normalized network elements of band reject filter.

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Practice Problems

5-1 The circuit shown in Figure P5.1 is a prototype filter at 1 rad/s level. Scale the circuit so that it will have a load resistance value of 1 kΩ and the parallel LC branch will resonate at 10 kHz.

5-2 What will be the value of resistance scaling factor k_R and frequency scaling factor k_ω, for the circuit shown in Figure P5.1, so that the load capacitance will become 10 pF and the inductor will have a value of 5 mH.

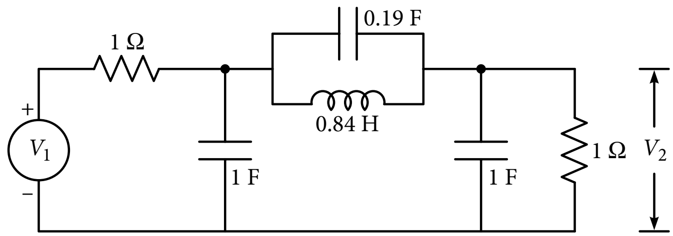

Figure P5.1

5-3 The network shown in Figure P5.2 is to be scaled by increasing the level of impedance by 100 and the level of frequency from 1 rad/s to 10^5 rad/s. Find the element values in the scaled network.

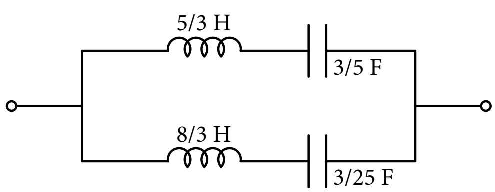

Figure P5.2

5-4 Design an HPF having maximally flat response and the following specifications, using LP to HP transformation:

$\alpha_{min} = 30$ dBs, $\alpha_{max} = 1$ dB, $\omega_s = 1$ krad/s and $\omega_p = 2.6$ krad/s

5-5 Redesign the HPF having the specifications of Problem 5-4 using Chebyshev approximation. Also find the filter attenuation at 1.2 krad/s and test the design.

5-6 (a) Design an HPF with a maximally flat response for which specifications are shown in Figure P5.3 employing LP to HP transformation.

(b) Determine the actual attenuation of the filter at 1800 rad/s and 2200 rad/s.

5-7 An HPF with equal ripples in the pass band is to be designed, employing LP to HP transformation, for which specifications are shown in Figure P5.3.

Design the filter using either a single OA filter circuit of Figure 5.6 or the Sallen–Key section and test the circuit.

Modify the circuit which provides a 10 dB increase in the gain at high frequencies, without employing addition OA.

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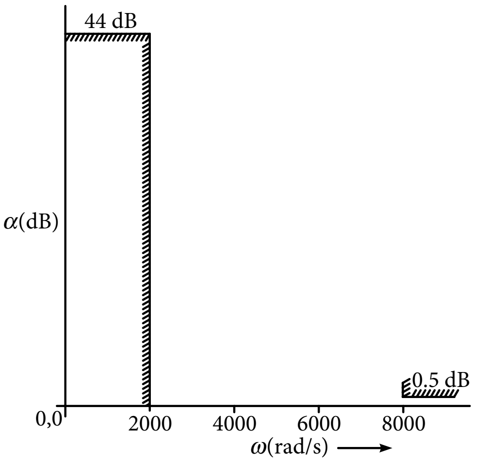

Figure P5.3

5-8 Apply the LP to HP transformation to the following network function $H(s)$, and compare the critical frequencies for both the network functions. What is the inference while comparing the critical frequencies of the LP and the HP functions?

$$H(s)=\frac{2s+1}{s^2+4s+6}$$

5-9 Find the transfer function of the Sallen–Key circuit shown in Figure P5.4. Apply the LP to HP transformation $s \to 1/s$ and obtain the transfer function and structure of the transformed circuit. Apply impedance scaling factor of 10^3 and frequency scaling factor of 10^4 and simulate the circuit.

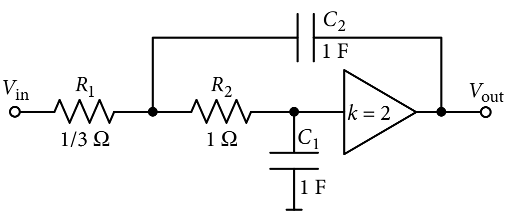

Figure P5.4

5-10 Apply the LP to BP transformation on the LP circuit of Figure P5.5. Find the transfer function of the LP prototype and the transformed network. Determine the value of the pole-Q for the BPF. Use suitable impedance scaling on the BP network such that its center frequency is 10 kHz and test the circuit using PSpice.

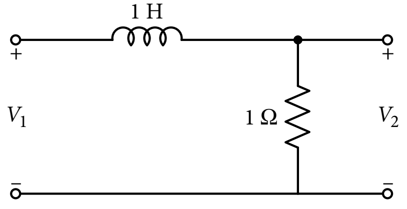

Figure P5.5

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5-11 Design a BPF which satisfies the specification shown in Figure P5.6, with attenuation being 0 dB at $\omega = 2700$ rad/s. Construct the circuit with suitable second- (and first-) order sections; maximally flat approximation is to be usedfor the LP prototype.

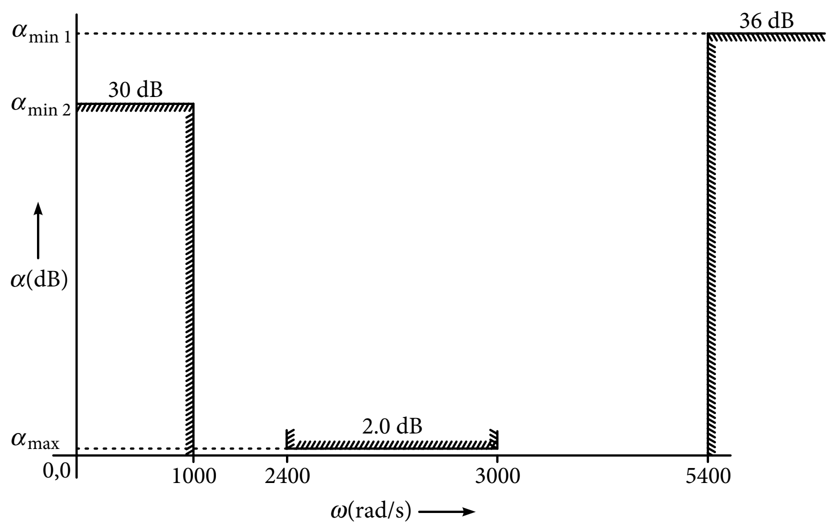

Figure P5.6

5-12 Repeat Problem 5-11 with the equal-ripple approximation used for the LP prototype.

5-13 With a maximally flat response for a BPF, it is desired that the maximum allowable attenuation is 1 dB in the frequency band of 1000 rad/s to 2000 rad/s. Design the BPF with the constraint that only two OAs can be used in the final realization. What shall be the largest obtainable attenuation at a frequency of 6000 rad/s?

5-14 Design a filter with a maximally flat response for which the specifications are: attenuation = 30 dB for $0 \le \omega \le 500$ rad/s and 4000 rad/s $\le \omega \le \infty$, attenuation = 2 dB for 1000 rps $\le \omega \le 2000$ rps. The mid-band gain is to be 0 dB, and only 0.1 μF capacitors can be used in the final realization.

5-15 Redesign the filter in Problem 5-14 with a pass band having equal ripples.

5-16 The LP prototype shown in Figure P5.7 has a 3dB frequency of 1 rad/s.

(a) Apply an LP to BP transform so that the BP filter has $Q = 10$ and center frequency $f_o = 1$ kHz. Verify the response using a computer method.

(b) Convert the LPF to a BRF with band stop width of 0.25 kHz and maximum attenuation at 1 kHz.

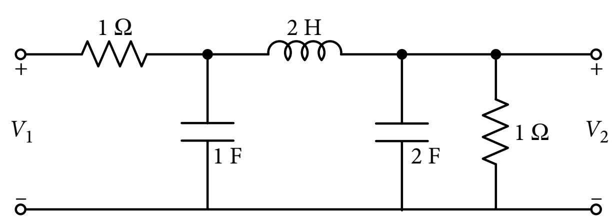

Figure P5.7

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5-17 Third-order Chebyshev approximated normalized LP prototype structure of Figure 5.12(a) is to be transformed to a BS filter through network transformation. Obtain resulting network and element values for normalized bandwidth of 0.1 for the BS filter. Also obtain element values for center frequency of 10^5 rad/s and after using impedance scaling factor of 10^4.

Verify the response for the passive BS filter.

5-18 Repeat Problem 5-13 but employ Chebyshev approximation.

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