

Appendix D

Laplace transform pairs

1.	t^n	$\frac{n!}{s^{n+1}}$
2.	e^{-at}	$\frac{1}{s+a}$
3.	$\frac{1}{a}(1-e^{-at})$	$\frac{1}{s(s+a)}$
4.	te^{-at}	$\frac{1}{(s+a)^2}$
5.	$\frac{1}{b-a}(e^{-at}-e^{-bt})$	$\frac{1}{(s+a)(s+b)}$
6.	$\sin \omega t$	$\frac{\omega}{s^2+\omega^2}$
7.	$\sin(\omega t + \lambda)$	$\frac{\omega \cos \lambda}{s^2+\omega^2} + \frac{s \sin \lambda}{s^2+\omega^2}$
8.	$\cos \omega t$	$\frac{s}{s^2+\omega^2}$
9.	$\cos(\omega t + \lambda)$	$\frac{s \cos \lambda}{s^2+\omega^2} - \frac{\omega \sin \lambda}{s^2+\omega^2}$
10.	$e^{-at} \sin \omega t$	$\frac{\omega}{(s+a)^2+\omega^2}$
11.	$e^{-at} \cos \omega t$	$\frac{s+a}{(s+a)^2+\omega^2}$

12.	$e^{-at} \left(\cos \omega t - \frac{a}{\omega} \sin \omega t \right)$	$\frac{s}{(s+a)^2 + \omega^2}$
13.	$\sinh \omega t$	$\frac{\omega}{s^2 - \omega^2}$
14.	$\cosh \omega t$	$\frac{s}{s^2 - \omega^2}$
15.	c (const.)	$\frac{c}{s}$
16.	$u(t)$ (step)	$\frac{1}{s}$
17.	$u(t-a)$	$\frac{1}{s} e^{-as}$
18.	$\delta(t)$ (impulse)	1
19.	$\delta(t-a)$	e^{-as}
20.	$\rho(t)$ (ramp)	$\frac{1}{s^2}$
21.	$\rho(t-a)$	$\frac{1}{s^2} e^{-as}$
22.	$f(t-a)u(t-a)$	$e^{-as}F(s)$ (shift in t)
23.	$e^{-at}f(t)$	$F(s+a)$ (shift in s)

Operational transforms

In the following,

$$f_0 = f(0^+); \quad f_n = \frac{d^n}{dt^n} f(0^+)$$

24.	$\frac{df}{dt}$	$sF(s) - f_0$
25.	$\frac{d^2f}{dt^2}$	$s^2F(s) - sf_0 - f_1$
26.	$\frac{d^n f}{dt^n}$	$s^n F(s) - s^{n-1}f_0 - s^{n-2}f_1 - \dots - f_{n-1}$

27. $\int f(t)dt$ $\frac{1}{s}F(s) + \frac{1}{s}\left[\int f(t)dt\right]_{t=0}$
28. $\int_0^t f(\tau)d\tau$ $\frac{1}{s}F(s)$
29. $\int_0^t f_1(\tau)f_2(t-\tau)d\tau$ $F_1(s)F_2(s)$ (convolution)