

5 Thermal Energy Harvesting

5.1 Introduction

Thermoelectric transducers convert thermal energy into electric energy. Thermal energy is generated as a result of a multitude of phenomena and applications, in some cases intentionally but most of the time as waste heat from a process or reaction, from industrial plants to buildings, heating systems, and automobiles to the human body, which, in turn, provide numerous applications for thermal energy harvesters.

This chapter begins with a description of thermoelectric phenomena and the geometrical structure of a thermoelectric generator (TEG). Next an introduction to the theory of heat transfer is presented in order to provide the theoretical background for the analysis of the performance of thermoelectric generators, and it is followed by theoretical expressions for the efficiency of TEGs. The next section deals with the figure of merit of different thermoelectric materials, and it is followed by a SPICE model of a thermoelectric generator. Finally the chapter ends with selected examples of TEG systems.

5.2 Thermoelectric Phenomena

There are three thermoelectric phenomena that govern the conversion of thermal energy to electrical energy and vice versa: (a) the Seebeck effect, (b) the Peltier effect, and (c) the Thomson effect. In the following, a summary of the three effects is provided.

5.2.1 The Seebeck Effect

According to the Seebeck effect, a temperature gradient between two different metals or semiconductors that are in contact creates a voltage difference between the two components [63]. Given a set of two different metal or semiconductor materials 1 and 2 that are connected forming two junctions as shown in Figure 5.1a, the presence of different temperatures T_H and T_C at the two junctions results in a voltage V_{oc} across the two contacts.

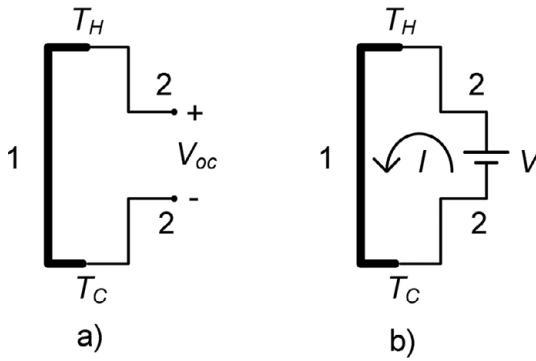


Figure 5.1 Thermoelectric phenomena: (a) the Seebeck effect and (b) the Peltier effect.

The voltage is given by

$$V_{oc} = \alpha_{12} \Delta T, \quad (5.1)$$

where $\Delta T = T_H - T_C$ and the coefficient α_{12} is called the Seebeck coefficient and has units of V/K . The Seebeck coefficient depends on both materials 1 and 2 and can be negative or positive. As we will see in a later section, the Seebeck coefficient is higher when semiconductor materials rather than metals are used to form the junctions, and consequently TEG devices typically are made of semiconducting materials.

5.2.2 The Peltier Effect

The Peltier effect is the inverse of the Seebeck effect [63]. In other words, the application of an external voltage difference V at the junctions of two different metals or semiconductors results in a current I flowing through the junctions, which, in turn, results in one junction absorbing thermal energy and the other junction generating thermal energy. As a result of the current flow, a heat flow rate Q is created. Consequently, a temperature gradient is generated between the junctions (Figure 5.1b).

The Peltier effect is described by the following equation

$$Q = \pi_{12} I, \quad (5.2)$$

where the heat flow rate between the two junctions is measured in W and the current through the circuit in A . The coefficient π_{12} is called the Peltier coefficient and has units of W/A or equivalently V . The Peltier coefficient, like the Seebeck coefficient, is a relative coefficient corresponding to the two materials forming the junctions.

5.2.3 The Thomson Effect

The third thermoelectric effect is the Thomson effect, which occurs in one material (metal or semiconductor) when its edges are subject to a temperature difference and at the same time they are subject to a voltage difference resulting in current flowing through the material [63]. As a result, there is heat Q_T absorbed or dissipated by the material, which depends both on the applied current and the temperature difference. The Thomson effect is described by the equation

$$Q_T = \beta I \Delta T, \quad (5.3)$$

where β is the Thomson coefficient, which is measured in $WI^{-1}K^{-1}$ or equivalently VK^{-1} . The heat rate due to the Thomson effect is smaller than the one due to the Peltier effect; nonetheless, it can become significant when the temperature difference ΔT is large [63].

5.2.4 The Kelvin Relationships

The three thermoelectric coefficients α_{12} , π_{12} , and β are related by the Kelvin relationships [63]

$$\pi_{12} = \alpha_{12} T \quad (5.4)$$

and

$$\frac{d\alpha_{12}}{dT} = \frac{\beta_1 - \beta_2}{T}. \quad (5.5)$$

The first equation relates the Seebeck and Peltier effects and demonstrates the reversible nature of the effects and the fact that the same set of materials is suitable both for electric power generation and for thermal power generation (or refrigeration). The second equation relates the Seebeck effect with the Thompson effect and enables the definition of an absolute Seebeck coefficient for a single material as

$$\alpha = \int \frac{\beta}{T} dT. \quad (5.6)$$

The Seebeck coefficient α_{12} corresponding to the two junctions of the two materials 1 and 2 is proven to be equal to the difference between the absolute Seebeck coefficients of each of the two materials $\alpha_{12} = \alpha_1 - \alpha_2$ [63]. Similarly, once an absolute Seebeck coefficient is defined, (5.4) defines an absolute Peltier coefficient as $\pi_{12} = \pi_1 - \pi_2$. When the magnitude of the Seebeck or Peltier coefficients of the two materials is equal, $\alpha_1 = -\alpha_2$ and $\pi_1 = -\pi_2 = \pi$, then $\alpha_{12} = 2\alpha$ and $\pi_{12} = 2\pi$.

5.3 Thermoelectric Generators

TEGs are typically constructed by forming matrices of pairs of p-type and n-type semiconductor columns called pellets. The pellets are electrically connected in

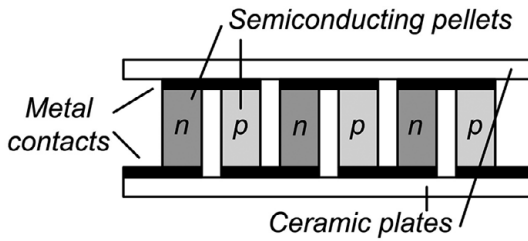


Figure 5.2 Cross-section of a TEG.

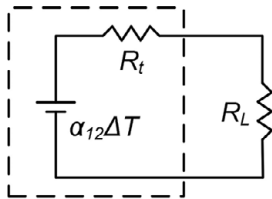


Figure 5.3 Electrical equivalent circuit of a TEG.

series using conducting (for example, copper or aluminum) strips, and are sandwiched between thermally conductive ceramic plates, as shown in Figure 5.2.

The output voltage of TEGs depends on the size and number of pellet pairs and typically ranges from 10 to 50 mV/K [106]. Due to the fact that the output voltage of a TEG in a energy harvesting application scenario takes such small values, a TEG is usually connected to a load consisting of a switched-type voltage converter, such as a boost or fly-back converter in order to produce a desired voltage required by standard circuitry [106].

The electrical equivalent circuit of a TEG consists of a Seebeck voltage source with a value V_{oc} given by (5.1) in series with an electrical resistance R_t representing the heat generated inside the TEG due to thermal losses as the electrical current flows through the pellets. The equivalent circuit is shown in Figure 5.3.

5.4 Heat Transfer Fundamentals

Heat is transferred through three physical mechanisms, *conduction*, *convection*, and *radiation* [107]. Each mechanism is governed by a rate equation, which provides a quantification of the heat flux rate measured in W/m².

Conduction is the transfer of energy between hot and cold particles of a material. The rate equation of conductive heat transfer is Fourier's law. Conductive heat transfer typically occurs in TEGs, and for this reason Fourier's law is described in more detail in the next subsection.

Convection is the transfer of heat due to the bulk, macroscopic motion of fluids. In the case of solids, an important scenario is that of a fluid with temperature T_∞

flowing over a heated solid surface of temperature T_s . In this case, the convection rate equation is

$$q_{cv} = h(T_s - T_\infty). \quad (5.7)$$

The parameter h ($\text{W m}^{-2}\text{K}^{-1}$) is the convection heat transfer coefficient.

Finally, all matter at a temperature higher than zero Kelvin emits thermal radiation whose upper limit is given by the rate equation known as the Stefan–Boltzmann law

$$q_r = \sigma T^4, \quad (5.8)$$

where q_r is the radiated heat flux rate of an ideal blackbody radiator measured in W/m^2 that is at temperature T . The constant of proportionality σ is the Stefan–Boltzmann constant, which is equal to $\sigma = 5.67 \cdot 10^{-8} \text{ W m}^{-2}\text{K}^{-4}$.

5.4.1 Fourier's Law

The conduction heat transfer process is described by Fourier's law expressed by

$$q = -k\nabla T. \quad (5.9)$$

According to Fourier's law, the heat flux rate q measured in W/m^2 is proportional to the temperature gradient ∇T (K m^{-1}). The constant of proportionality k is the thermal conductivity of the material ($\text{W m}^{-1} \text{K}^{-1}$). The minus sign expresses the fact that heat is transferred from points of the material with a higher temperature toward points of lower temperature. A TEG can be approximately modeled as a one-dimensional (1D) problem of heat diffusion from the hot plate toward the cold plate (Figure 5.2). In this case, Fourier's law becomes

$$q = -k \frac{dT}{dx}, \quad (5.10)$$

where x represents the vertical direction between the two plates shown in Figure 5.2. Once the steady-state temperature distribution in the material is defined, the heat flux can be computed using Fourier's law.

In the case of 1D problems, Fourier's law presents an analogy with Ohm's law of electrical circuits [107]. Let us consider an infinitesimal volume $V = AL$, where the heat rate is flowing through the volume surface A and the temperature gradient is taken along the length L . Then we can write Fourier's law as

$$Q = -\frac{kA}{L}(-dT) \Rightarrow Q = K\Delta T \Rightarrow Q\Theta = \Delta T, \quad (5.11)$$

where $K = kA/L$ is defined as the thermal conductance (W K^{-1}) and $\Theta = 1/K$ is the thermal resistance. The analogy with Ohm's law $IR = \Delta V$ is now obvious. This fact is explored in computing the steady state of TEGs using electrical circuit simulators, as we will see in Section 5.6.

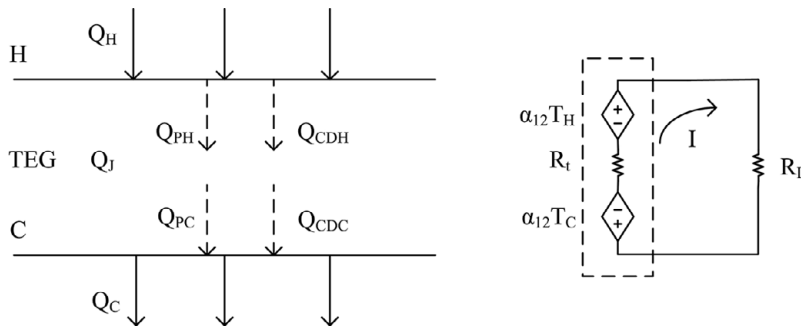


Figure 5.4 Conservation of energy in a TEG.

5.4.2 The First Law of Thermodynamics

In heat transfer problems, one must always apply the first law of thermodynamics, in other words the law of conservation of energy [107]. One may express this law in different formats, for example in terms of energy flux rates or in terms of energy rates (i.e., power). We assume that the heat distribution is uniform over the boundary surfaces of the TEG, and in this case we can convert heat flux rates q to heat rates Q simply by multiplying the former by the area A of the surface of the TEG boundary, i.e., $Q = qA$. The conservation of energy for the TEG system in terms of heat rates is depicted in Figure 5.4, and it takes the form

$$Q_{st} = Q_H - Q_C + Q_J. \quad (5.12)$$

Equation (5.12) states that the stored thermal and mechanical energy rate Q_{st} in the TEG system of a certain volume is equal to the difference between the inflow of energy rate Q_H and outflow of energy rate Q_C at the system boundary surface, plus any thermal energy rate Q_J generated in the system.

When we are dealing with the boundary surfaces of the system, conservation of energy across a boundary is expressed by the fact that the incoming energy rate must be equal to the outgoing energy rate at each boundary. Based on Figure 5.4, the conservation of energy in the hot H boundary becomes

$$Q_H = Q_{PH} + Q_{CDH}, \quad (5.13)$$

where Q_{PH} is the absorbed Peltier heat and Q_{CD} contains contributions from heat conduction and Joule heat generation in the TEG, which we derive using the heat diffusion equation in the next section. The conservation of energy in the cold C junction becomes

$$Q_C = Q_{PC} + Q_{CDC}, \quad (5.14)$$

where Q_{PH} is the emitted Peltier heat and Q_{CD} contains contributions from heat conduction and Joule heat generation in the TEG accordingly. The magnitude of Q_{PH} is different from Q_{PC} because of the different temperatures at the H and C boundaries. Furthermore, Q_{CDH} is different from Q_{CDC} because the heat

conduction rate flows from the H to the C boundary, but the generated Joule heat is emitted from both boundaries equally as we will see in the next sections.

5.4.3 The Heat Diffusion Equation

Once the steady-state temperature distribution in a material is determined, one can compute the heat flow using the rate equations. The heat diffusion equation, or simply the heat equation, provides a means to determine the temperature distribution as a function of time. In Cartesian coordinates, it is given

$$\nabla(k\nabla T) + q_j = \rho c_p \frac{\partial T}{\partial t}, \quad (5.15)$$

where k is the thermal conductivity of the material, ρ is the density (kg m^{-3}), and c_p ($\text{J kg}^{-1} \text{K}^{-1}$) is the specific heat of the material. The product ρc_p is called volumetric heat capacity and measures the ability of the material to store thermal energy [107]. Finally, q_j is the heat density rate measured in Wm^{-3} . In the case of a one-dimensional conduction problem and of a material with a constant thermal conductivity, which represents an approximate model for the TEG, the heat equation becomes

$$\frac{\partial^2 T}{\partial x^2} + \frac{q_j}{k} = \frac{1}{\alpha} \frac{\partial T}{\partial t}, \quad (5.16)$$

where $\alpha = k/(\rho c_p)$ is the thermal diffusivity. The steady-state condition is obtained by setting the partial derivative of the temperature versus time equal to zero, in which case one obtains

$$\frac{\partial^2 T}{\partial x^2} + \frac{q_j}{k} = 0. \quad (5.17)$$

5.5 TEG Efficiency

The conversion efficiency η of a TEG is defined as the ratio of the electrical power delivered to a load R_L connected to the TEG P_L divided by the heat rate absorbed at the hot junction Q_H

$$\eta = \frac{P_L}{Q_H}, \quad (5.18)$$

where Q_H is the heat rate at the hot junction. Assuming a uniform heat distribution along the junction of the TEG, the heat rate is $Q_H = q_h A$, where q_h is the heat flux rate and A is the surface area of the hot junction.

5.5.1 The Carnot Efficiency

Let's consider first the ideal case where only the Peltier and Seebeck phenomena exist. We do not consider heat conduction phenomena and no heat generation

inside the TEG. The first assumption results in the thermal conductivity of the TEG being zero $k = 0$, while the second assumption results in the electrical resistance of the TEG being set to zero $R_t = 0$ in the model of Figure 5.3.

The power delivered to the load is $P_L = I^2 R_L$, where I is the current flowing through the TEG and the load R_L . Using (5.1), one has

$$V_L = \alpha_{12}(T_H - T_C) \quad (5.19)$$

$$I_L = \frac{\alpha_{12}(T_H - T_C)}{R_L} \quad (5.20)$$

and

$$P_L = I_L^2 R_L = \alpha_{12}^2 (T_H - T_C)^2 \frac{1}{R_L}. \quad (5.21)$$

Application of the first law of thermodynamics at the hot junction boundary of the TEG results in the heat rate Q_H absorbed at the hot junction being equal to the Peltier absorbed heat rate, and therefore

$$Q_H = \pi_{12} I_L = \alpha_{12} T_H I_L, \quad (5.22)$$

where (5.2) and (5.4) were used. The TEG efficiency becomes

$$\eta_C = \frac{T_H - T_C}{T_H}. \quad (5.23)$$

This is known as the Carnot efficiency and it represents an upper bound in the efficiency of TEGs. One should highlight that even the ideal Carnot efficiency takes very small values in many application scenarios of energy harvesting, for example a hot junction with temperature $\Delta T = 10$ K above a cold junction at room temperature $T_C = 300$ K gives a Carnot efficiency of 3.33%.

5.5.2 Conversion Efficiency Considering Heat Conduction and Thermal Losses in the TEG

An ideal TEG presents no thermal conductivity and no electrical resistance. All materials, however, present a nonzero thermal conductivity k and a nonzero electrical resistance R_t .

The power delivered to the load in the case of a nonzero electrical resistance R_t as shown in Figure 5.3 becomes

$$V_L = \alpha_{12}(T_H - T_C) \frac{R_L}{R_t + R_L} \quad (5.24)$$

$$I_L = \frac{\alpha_{12}(T_H - T_C)}{R_t + R_L} \quad (5.25)$$

and

$$P_L = I_L^2 R_L = \alpha_{12}^2 (T_H - T_C)^2 \frac{R_L}{(R_t + R_L)^2}. \quad (5.26)$$

The load power becomes maximum when $R_t = R_L$, in which case

$$P_{Lmax} = \frac{\alpha_{12}^2 (T_H - T_C)^2}{4R_t}. \quad (5.27)$$

In order to compute the heat rate at the hot junction, we apply the heat diffusion equation to determine the temperature distribution along the TEG height. The heat diffusion equation in the steady state $\partial T / \partial t = 0$ takes the form

$$\frac{\partial^2 T}{\partial x^2} + \frac{q_j}{k} = 0. \quad (5.28)$$

Considering a uniform heat generation along the TEG, the Joule heat density q_j takes is equal to

$$q_j = \frac{P_J}{V} = \frac{I^2 R_t}{AL}, \quad (5.29)$$

where A is the area of one pellet pair, L its length, and $V = AL$ its volume.

The solution of (5.28) has the form

$$T(x) = -\frac{q_j}{2k}x^2 + C_1x + C_2. \quad (5.30)$$

The temperature variation across the TEG pellet height has a parabolic profile. The constants C_1 and C_2 are determined by the boundary conditions

$$\begin{aligned} T(0) &= T_H \\ T(L) &= T_C, \end{aligned} \quad (5.31)$$

where L is the length (or height) of the pellet. One can easily compute

$$T(x) = -\frac{I^2 R_t}{2ALk}x^2 + \left(-\frac{T_H - T_C}{L} + \frac{I^2 R_t}{2Ak}\right)x + T_H. \quad (5.32)$$

Once we have determined the temperature distribution $T(x)$, application of Fourier's law (5.11) at the hot junction of the TEG gives the conduction heat flux q_{cdh} .

In order to find the heat rate at the hot junction H ($x = 0$) of the TEG as shown in Figure 5.4, first we determine the first derivative of the temperature from (5.32):

$$\frac{dT(x)}{dx} = -\frac{I^2 R_t}{ALk}x - \frac{T_H - T_C}{L} + \frac{I^2 R_t}{2Ak}. \quad (5.33)$$

Then Fourier's law (5.11) gives

$$q_{cdh} = -k \left(\frac{dT}{dx} \right)_{x=0} = k \frac{T_H - T_C}{L} - \frac{I^2 R}{2A}. \quad (5.34)$$

The conduction heat rate Q_{CDH} is computed from the heat flux q_{cdh} using the cross-section A of the pellet pair as $Q_{CDH} = q_{cdh}A$, resulting in

$$Q_{CDH} = \frac{kA}{L}(T_H - T_C) - \frac{I^2 R}{2} = K(T_H - T_C) - \frac{I^2 R}{2}, \quad (5.35)$$

where K is the thermal conductivity of a pellet pair with height L and cross-section A . Q_{CDH} included both the conduction heat rate, leaving the hot junction and a Joule heat term generated in the TEG.

We have seen in (5.13) that the application of the first law of thermodynamics (5.12) at the hot junction boundary gives that the incoming absorbed heat rate Q_H is equal to the Peltier emitted heat rate $Q_{PH} = \alpha_{12}T_H I$ plus the heat conduction and Joule heat generation term Q_{CDH} :

$$Q_H = Q_{PH} + Q_{CDH} = \alpha_{12}T_H I + K(T_H - T_C) - \frac{I^2 R}{2}. \quad (5.36)$$

Following a similar calculation, we can show that the emitted heat rate Q_C at the cold junction boundary of the TEG is

$$Q_C = \alpha_{12}T_C I + K(T_H - T_C) + \frac{I^2 R}{2}. \quad (5.37)$$

The analysis has been made for one pellet pair. A TEG typically has a large number N of pellet pairs that are connected thermally in parallel and electrically in series (Figure 5.2). In this case, the thermal conductance K , the electrical resistance R_t , the Seebeck voltage V_{oc} , and consequently the absorbed Q_H and emitted heat rates Q_C are all multiplied by the number of pellet pairs N .

We have seen in (5.18) that in order to calculate the efficiency of the TEG we only require the absorbed heat rate Q_H at the hot junction. Using (5.18), (5.25) and (5.36) the conversion efficiency of the TEG becomes

$$\eta = \frac{\alpha_{12}^2 (T_H - T_C) R_L}{\alpha_{12}^2 \left[T_H (R_t + R_L) - \frac{\Delta T R_t}{2} \right] + K (R_t + R_L)^2} \quad (5.38)$$

or

$$\eta = \eta_C \frac{Z R_L}{Z \left[(R_t + R_L) - \frac{n_C R_t}{2} \right] + \frac{1}{R_t T_H} (R_t + R_L)^2}, \quad (5.39)$$

where η_C is the Carnot efficiency and $Z = \alpha_{12}^2 / (K R_t)$ is defined as the figure of merit of the thermoelectric material of the TEG. The figure of merit is proportional to the square of the Seebeck coefficient, and inversely proportional to the thermal conductivity and the electrical resistance of the TEG material and has units K^{-1} . Good thermoelectric materials have a large Seebeck coefficient but also low thermal conductivity and low electrical resistance. It is customary to multiply Z with the temperature in order to obtain a unitless parameter.

As we have seen, the condition for maximum delivered electrical power to the load is $R_t = R_L$. However, due to the dependance of both the load power P_L and the absorbed power Q_H on the load resistance R_L , the load value that provides maximum efficiency η is different from the load value that provides maximum delivered power. Taking $\partial \eta / \partial R_L = 0$, one obtains the load value that corresponds to maximum efficiency as

$$R_{Lm} = R_t \sqrt{1 + ZT}, \quad (5.40)$$

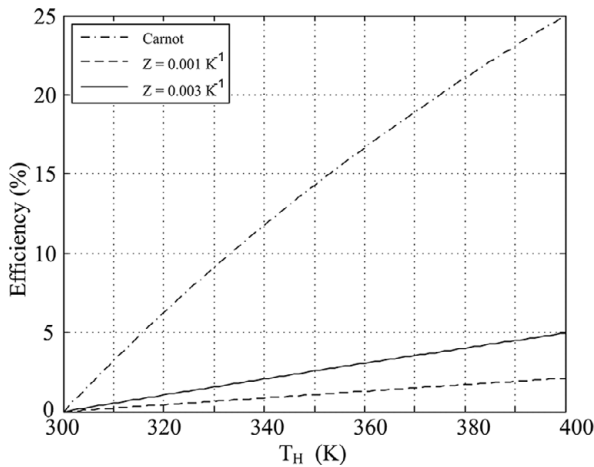


Figure 5.5 TEG conversion efficiency ($T_C = 300$ K).

where $\bar{T} = (T_H + T_C)/2$. The maximum efficiency then becomes

$$\eta = \eta_C \frac{\sqrt{1 + Z\bar{T}} - 1}{\sqrt{1 + Z\bar{T}} + \frac{T_C}{T_H}}. \quad (5.41)$$

The conversion efficiency for different values of the figure of merit versus the hot side temperature is shown in Figure 5.5, when the cold side is at $T_C = 300$ K. The Carnot efficiency is also included for comparison. It should be noted that for a TEG made of a material with figure of merit $Z = 0.003$ K⁻¹, when the hot side is at 10 K above room temperature, the conversion efficiency is just 0.52%.

5.5.3 The Figure of Merit

The figure of merit is proportional to the square of the Seebeck coefficient, inversely proportional to the thermal conductivity and the electrical resistance of the TEG material, and has units K⁻¹. Good thermoelectric materials have a large Seebeck coefficient but also low thermal conductivity and low electrical resistance.

All three parameters depend on the carrier concentration in the material. The dependence is pictured in Figure 5.6 [108]. The results show that highly doped semiconductor materials are more suitable for thermoelectric applications than metals or insulators.

Furthermore, Figure 5.7 [63] shows obtained values of the figure of merit for several materials. The figure of merit Z is normalized to the temperature in order to obtain a unitless parameter. One can see that Bi₂Te₃ has a maximum figure of merit value around room temperature $T = 300$ K, and for this reason it is commonly used in TEG applications. Typical values for the figure of merit range around $2.5 \cdot 10^{-3}$ K⁻¹ to $3 \cdot 10^{-3}$ K⁻¹.

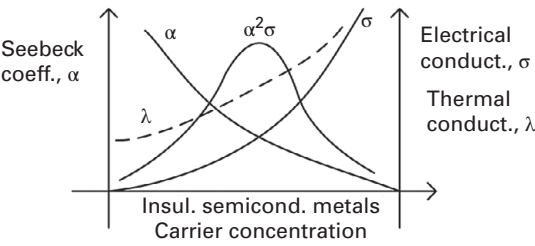


Figure 5.6 Schematic description of Seebeck coefficient, thermal conductivity, and electrical resistance dependence on carrier concentration based on [108].

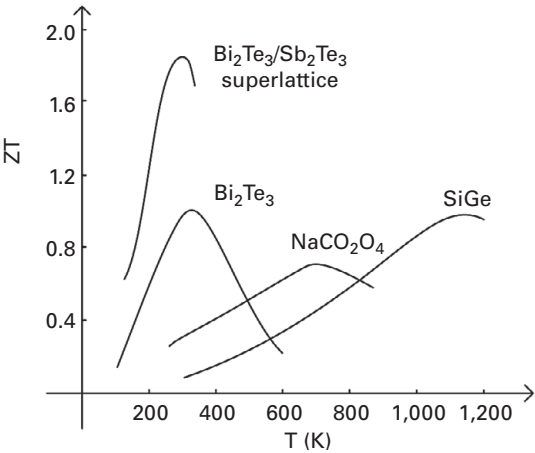


Figure 5.7 Selected normalized figure of merit ZT plots for different thermoelectric materials, reproduced from [63].

5.6 A Thermal and Electrical SPICE Model for the TEG

Fourier’s law for heat conduction presents an analogy with Ohm’s law in electrical circuits. Due to this fact, it is possible to model and analyze thermal problems using electrical simulators such as Simulation Program with Integrated Circuit Emphasis (SPICE). The analogy between the various thermal and electrical quantities is summarized in Table 5.1 [109].

Table 5.1 Analogy between thermal and electrical quantities [109].

Thermal	Unit	Electrical	Unit
Heat rate, Q	W	Current, I	A
Temperature, T	K	Voltage, V	V
Thermal resistance, $\Theta = 1/K$	K W ⁻¹	Resistance, R	Ω
Heat capacity, C	J K ⁻¹	Capacitance, C	F
Absolute zero temperature	0 K	Ground	0 V

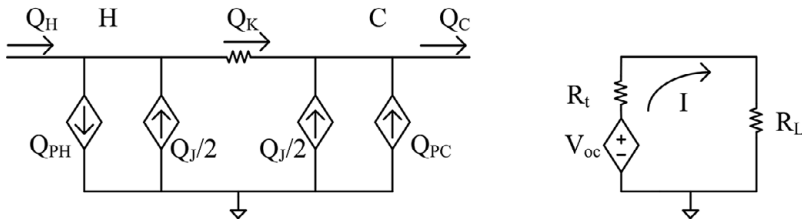


Figure 5.8 SPICE model of a TEG [109].

Using Table 5.1, the energy balance equations at the hot and cold boundaries of the TEG (5.36) and (5.37) can be represented with controlled current sources and a resistor. Thus, it is possible to build a SPICE equivalent model for the TEG [109] shown in Figure 5.8, where

$$\begin{aligned}
 Q_{PH} &= \alpha_{12} T_H I \\
 Q_{PC} &= \alpha_{12} T_C I \\
 Q_J &= I^2 R_t \\
 Q_K &= K(T_H - T_C) \\
 V_{oc} &= \alpha_{12}(T_H - T_C).
 \end{aligned} \tag{5.42}$$

The model can be used to find the steady state of the TEG. The model parameters can be computed from the TEG manufacturer specifications [109]. It is further possible to introduce a thermal capacity C (J/K) in the SPICE model, which allows to study the transient behavior of the TEG [110].

5.7 Thermal Energy Harvester Systems

Seiko presented in 1998 the first watch that was powered by a thermoelectric transducer [111]. The power that is required in order to operate a quartz digital wristwatch is approximately 20–40 μW [63]. There are many application scenarios where electronic circuits generate a large amount of heat. For example, typically in RF and microwave electronics a power amplifier operates with low efficiency in order to maintain an acceptable level of distortion and consequently a significant fraction of the power used to supply the power amplifier is dissipated in heat. Therefore, it is possible to use a TEG in order to convert some of the wasted heat back into electrical power. Such an application scenario has been studied in [112]. A 1.37 W power amplifier with 10 dB of gain was used, operating at 2.45 GHz with a measured power-added efficiency (PAE) of 34%. The PAE is defined as $(P_o^{RF} - P_i^{RF})/P_{dc}$, which means that the amplifier generated approximately 0.88 W of heat.

The amplifier printed circuit board with a commercial TEG placed below is shown in Figure 5.9. The electronics of the TEG board are shown in the figure

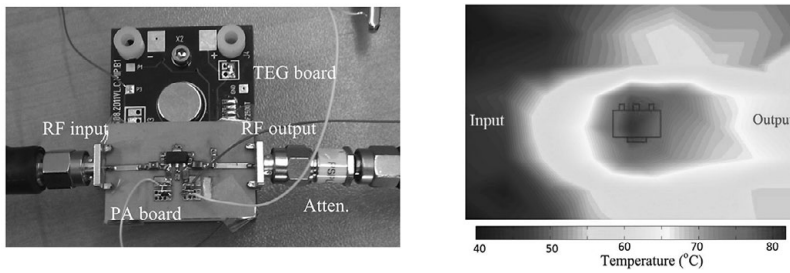


Figure 5.9 Thermal energy harvesting from an RF power amplifier. ©2013 IEEE. Reprinted with permission from [112]

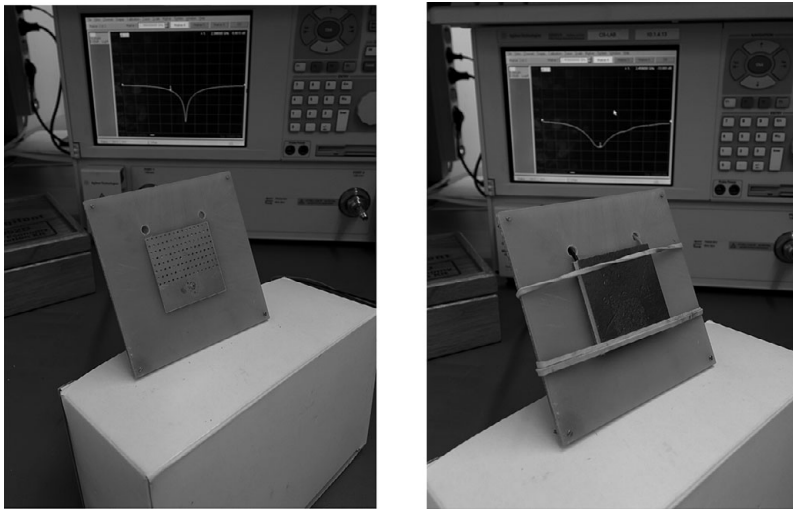


Figure 5.10 Patch antenna integrating a thermoelectric generator [113]. Photo courtesy of Dr. Marco Virili, Qorvo Inc.

whereas the hot side of the TEG was placed in contact with the ground plane of the amplifier board directly below the amplifier packaged integrated circuit. A measured temperature map of the power amplifier printed circuit board when the amplifier was operating is also shown in Figure 5.9. At steady state, the temperature at the hot side of the TEG was measured to be 313.1 K, whereas the temperature at the cold side was 305.9 K. The temperature difference of 7.2 K was maintained with the help of a heat sink placed below the TEG. This temperature gradient corresponds to a Carnot efficiency of 2.3%. A measured output power of 1 mW was obtained from the TEG, which, although represents a very low efficiency for the thermoelectric generator, as an absolute value it is sufficient to power a wireless sensor circuit performing some monitoring function, for example.

The integration of multiple harvesting systems of different technologies is important in order to optimize the energy autonomy of a wireless sensor circuit by

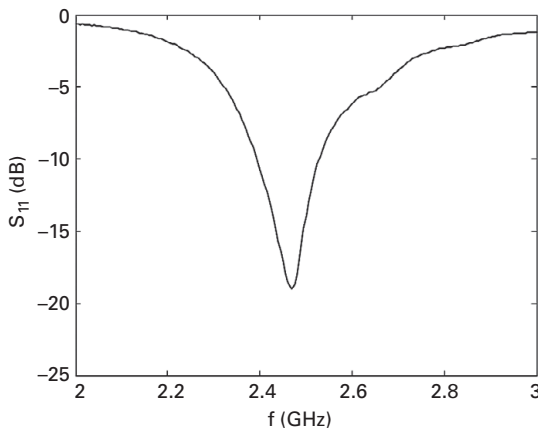


Figure 5.11 S-parameters of the patch antenna with TEG [113].

exploring different sources of power. Due to the typically small size of the sensors, the integration becomes a challenge in order to minimize the used space. In [113], it was investigated whether a TEG can be integrated with an antenna, which can be a communication antenna or an RF energy harvesting antenna. A commercial TEG was placed above a quarter-wave shorted patch antenna implemented in FR4 substrate. The patch antenna with and without the TEG placed on top is shown in Figure 5.10. A shorted patch antenna design was selected in order to provide a low thermal resistance connection between one of the TEG surfaces and the ground plane of the antenna.

The antenna dimensions were retuned with the help of commercial electromagnetic simulator software in order for the antenna to operate in the 2.4 GHz industrial, scientific, and medical (ISM) band. The measured s-parameters of the antenna prototype are shown in Figure 5.11, where we can see that the desired operating bandwidth is obtained. The measured radiation pattern of the antenna with the TEG at 2.45 GHz is shown in Figure 5.12, which showed an obtained gain of approximately 2.3 dB. The presence of the TEG reduced the antenna gain by less than 1 dB.

Following the successful implementation of the patch antenna with the TEG, a shorted patch antenna integrating both a TEG and a solar cell on top was successfully demonstrated in [114]. The antenna prototype is shown in Figure 5.13, whereas its measured performance is shown in Figure 5.14 verifying that with proper design the presence of the TEG and the solar cell has a minimal effect in the operation of the patch antenna. Such systems integrating antennas with TEGs and solar cells are also particularly suitable for smart-fabric interactive-textile systems in a variety of applications such as rescue missions, interventions, and health care [115].

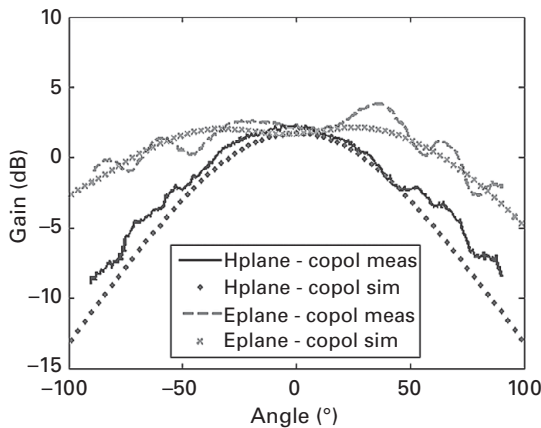


Figure 5.12 Measured gain radiation pattern of the patch antenna with TEG [114].

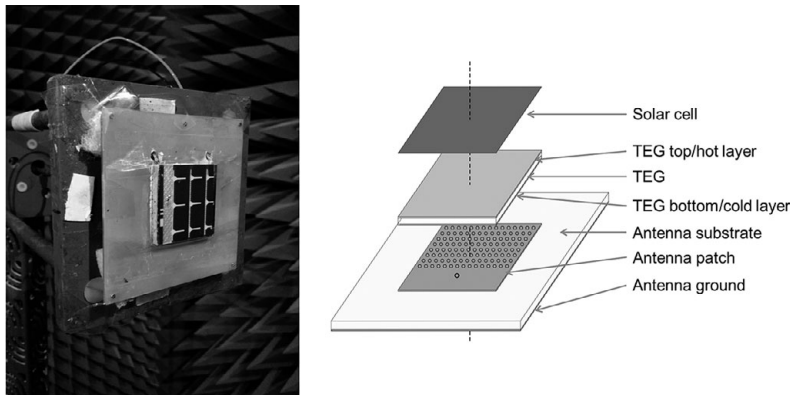


Figure 5.13 Prototype of shorted patch antenna with TEG and solar cell [114]. Antenna photo courtesy of Dr. Marco Virili, Qorvo Inc. Circuit schematic ©2015 IEEE. Reprinted with permission from [114]

5.8 Problems and Questions

1. Describe the three thermoelectric phenomena: (a) the Seebeck effect, (b) the Peltier effect, and (c) the Thomson effect.
2. Describe the three mechanisms of heat transfer.
3. Describe Fourier's law and its analogy to Ohm's law.
4. Describe the conservation of energy in the hot and cold surface boundaries of a TEG.
5. Derive the Carnot efficiency formula assuming the Peltier and Seebeck effects.
6. Calculate the emitted heat from the cold boundary surface of a TEG by solving the heat diffusion equation and applying Fourier's law and the conservation of energy.

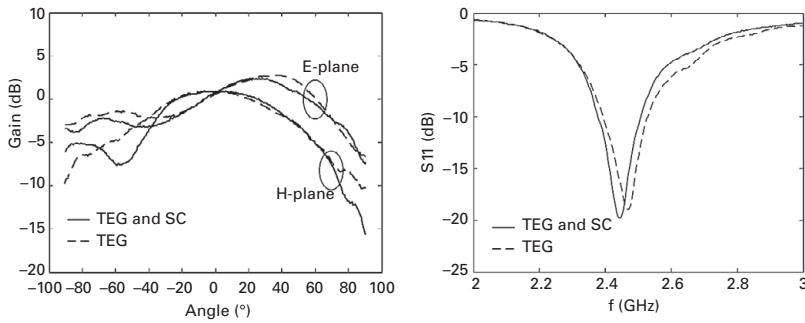


Figure 5.14 Measured performance of the patch antenna with TEG and solar cell [114].

7. Starting from (5.38) or (5.39) for the TEG efficiency, compute the optimum load that maximizes the efficiency and find the maximum efficiency.
8. The hot junction of a TEG is at $T_H = 315$ K and the cold junction at $T_C = 300$ K. The figure of merit of the thermoelectric material of the TEG is $Z = 0.003$ K $^{-1}$. The TEG comprises $N = 100$ pellet pairs, where each pellet pair has electrical resistivity $\rho = 25$ $\mu\Omega$ m, surface $A = 1$ mm 2 , and length $L = 2$ mm. Derive the Carnot efficiency and the TEG efficiency assuming that the TEG is connected to a $R_L = 50$ Ω load. What is the optimum load maximizing the efficiency and what is the optimum efficiency?
9. Derive the optimum TEG efficiency and optimum load when the length of the pellets of the TEG of the previous problem is doubled, i.e., $L = 4$ mm.