# 3 Tuned Circuits

We have already seen that an ability to select frequencies is crucial to the effective development of radio technology. In early radio, the selection could be adequately achieved by very simple tuned circuits. However, the modern crowded airwaves have made the frequency selection requirement extremely stringent and simple tuned circuits are no longer adequate. Consequently, in the following chapter we introduce some more sophisticated tuned circuits that make higher-fidelity frequency selection possible. Additionally, we study how such circuits can also ensure the optimum transfer of power between the various stages of a radio. Power transfer and filtering are intimately connected since a filter works by transferring power well on its pass frequencies and badly on its blocking frequencies. In order to study tuned circuits we first develop some techniques for the harmonic analysis of circuits, techniques that we will use frequently throughout the rest of this book.

## 3.1 Time-Harmonic Circuits

In order that many users can utilise the available radio frequencies (often called the *radio spectrum*), we will need radio circuits that can isolate slices of frequency and we achieve this through what are known as *tuned circuits*. We will consider circuits in which the radio signals are sinusoidally varying with time at a specific frequency $\omega$ (*time-harmonic* circuits). Consequently, we can assume voltage and current to take the form

$$\mathcal{V}(t) = V_0 + V_1 \cos(\omega t + \phi_v) \text{ and } \mathcal{I}(t) = I_0 + I_1 \cos(\omega t + \phi_i). \tag{3.1}$$

We will assume that all other variations in time (such as modulation) take place over timescales that are very much greater than that of an oscillation period ($T = 2\pi/\omega$). Consequently, we can treat $V_0$, $V_1$, $I_0$ and $I_1$ as constant. In a linear circuit analysis, we can consider the DC ($V_0$ and $I_0$) aspects of the circuit separately from the RF (radio frequency) aspects. In analysing the DC aspects, we simply replace all capacitors by an open circuit and all inductors by a short circuit. To study the RF aspects, we now simply set $I_0$ and $V_0$ to be zero and instead study voltages and currents given by

$$v(t) = V_1 \cos(\omega t + \phi_v) \text{ and } i(t) = I_1 \cos(\omega t + \phi_i) \tag{3.2}$$

(note that we now use lower case letters to signify the RF part of the signal). At this point we will find it simpler to study our circuits in terms of *complex numbers*! A complex

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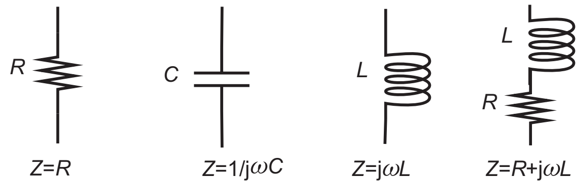


**Fig. 3.1** Impedance of various components.

number is of the form $x+\mathrm{j}y$ where $\mathrm{j}=\sqrt{-1}$ (i.e. $\mathrm{j}^2=-1$), $x$ is known as the real part of the number and $y$ is known as the imaginary part. On noting that $\exp(\mathrm{j}\theta)=cos(\theta)+\mathrm{j}\sin(\theta)$, we can write (3.2) as

$$v(t)=\Re\{V\exp(\mathrm{j}\omega t)\} \text{ and } i(t)=\Re\{I\exp(\mathrm{j}\omega t)\}, \tag{3.3}$$

where $V=V_1\exp(\mathrm{j}\phi_v)$ and $I=I_1\exp(\mathrm{j}\phi_i)$ are complex amplitudes that have absorbed the phases $\phi_v$ and $\phi_i$. (It should be noted that $\Re\{A\}$ is used to signify the real part of complex number $A$ and $\Im\{A\}$ to signify the imaginary part, i.e. $A=\Re\{A\}+\mathrm{j}\Im\{A\}$.)

If we consider $v$ to be the RF voltage drop across a resistor $R$ carrying RF current $i$, then Ohm's law implies that $v=iR$ from which $\Re\{V\exp(\mathrm{j}\omega t)\}=\Re\{I\exp(\mathrm{j}\omega t)\}R$ and $V=IR$. Now consider the voltage drop across an inductor $L$, then $v=L\mathrm{d}i/\mathrm{d}t$ from which $\Re\{V\exp(\mathrm{j}\omega t)\}=L\Re\{\mathrm{j}\omega I\exp(\mathrm{j}\omega t)\}$ and hence $V=\mathrm{j}\omega LI$. Finally consider the voltage drop across a capacitor $C$, then $C\mathrm{d}v/\mathrm{d}t=i$ from which $C\Re\{\mathrm{j}\omega V\exp(\mathrm{j}\omega t)\}=\Re\{I\exp(\mathrm{j}\omega t)\}$ and hence $V=I/\mathrm{j}\omega C$. What we conclude is that both inductors and capacitors behave as if they have an imaginary resistance of the form $\mathrm{j}X$. Quantity $X$ is termed the *reactance* and has ohms as its units. An inductance has positive reactance $\omega L$ and a capacitor has negative reactance $-1/\omega C$. In reality, an inductor, or capacitor, will exhibit some loss that is represented by a resistance ($R_{\mathrm{loss}}$ say) and then it will behave like the complex resistance $Z=R_{\mathrm{loss}}+\mathrm{j}X$ where $X$ is the reactance. A complex resistor is known, in general, as an *impedance*. We can lump several impedances together to form one single impedance and, for impedances $Z_1$ and $Z_2$ joined in series, the combined impedance will simply be their sum (i.e. $Z_1+Z_2$). For impedances in parallel, often denoted by $Z_1 \parallel Z_2$, the combined impedance $Z$ will satisfy $1/Z=1/Z_1+1/Z_2$.

## 3.2 Power Transfer

We have already discussed power in the context of radio waves and for which it was defined as the rate at which energy is transported by the wave. In terms of electrical devices, we will understand *power* to mean the rate of energy transport between these devices. Further, we will understand the term *power loss* to mean the energy that is lost per unit time in such devices. The voltage drop $\mathcal{V}$ across a device is the energy lost when a unit charge travels through it and so the power loss $\mathcal{P}$ will be

$$\mathcal{P}=\mathcal{I}\mathcal{V}, \tag{3.4}$$

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where $\mathcal{I}$ is the current flowing through the device. For a time-harmonic analysis, the average power lost over one cycle of the source will be

$$P_{av} = \frac{1}{2\pi}\int_0^{2\pi} \mathcal{I}(t)\mathcal{V}(t)\mathrm{d}t$$
$$= \frac{1}{2}\Re\{I\bar{V}\} = \frac{V_L\bar{V}_L}{2}\Re\left\{\frac{1}{Z_L}\right\}, \tag{3.5}$$

where $Z$ is the impedance of the device (NB if $z = x + \mathrm{j}y$ then $\bar{z} = x - \mathrm{j}y$ is the *conjugate* of $z$ and $z\bar{z} = x^2 + y^2$).

Now consider a sinusoidally oscillating source and its load, as shown in the circuit of Figure 3.2. The source consists of an ideal voltage source (zero internal impedance) $V_S$ and a source impedance $Z_S$. (A source will always have some internal resistance and this is the reason why the open circuit voltage of a battery is always higher than the voltage when current is being drawn.) We will assume the internal impedance of the source is $Z_S = R_S + \mathrm{j}X_S$ and that the load has impedance is $Z_L = R_L + \mathrm{j}X_L$. We now ask the question as to what sort of load is required for there to be maximum power dissipated in the load resistance $R_S$. The same current $I$ will flow through all components and so the voltage drop around the circuit is zero. As a consequence, we have

$$V_S - R_S I - \mathrm{j}X_S I - \mathrm{j}X_L I - R_L I = 0 \tag{3.6}$$

and the voltage drop $V_S$ across the load will be given by

$$V_L = \frac{V_S(R_L + \mathrm{j}X_L)}{R_S + \mathrm{j}X_S + \mathrm{j}X_L + R_L} \tag{3.7}$$

where $V_L$ is the voltage drop across the load. The power dissipated in the load impedance $Z_L$ will be

$$P_L = \frac{V_L\bar{V}_L}{2}\Re\left\{\frac{1}{Z_L}\right\} = \frac{V_S\bar{V}_S}{2}\frac{R_L}{(R_S + R_L)^2 + (X_S + X_L)^2} \tag{3.8}$$

and will be maximum when $\partial P_L/\partial R_L = \partial P_L/\partial X_L = 0$. This implies that $X_L = -X_L$ and $R_L = R_S$. When these conditions are met, there is said to be a *conjugate match* since $Z_L = \bar{Z}_S$. What is clear from this is that the maximum power that can be transferred to that load is half that which is dissipated in the total circuit (i.e. the load and source dissipate the same amount of power). When the reactances of the source and load cancel

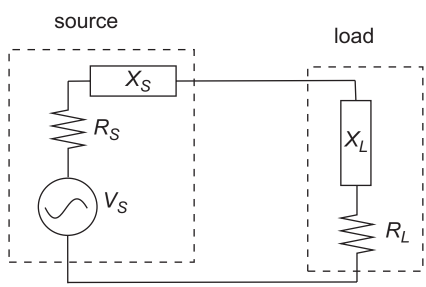


**Fig. 3.2** Model of a sinusoidally oscillating source and load.

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each other, they are said to resonate and the frequency at which this happens is known as the *resonant frequency*.

## 3.3 Basic Tuned Circuits

We will now investigate frequency selective circuits, sometimes known as *tuned circuits*. First consider a series capacitor $C$ and inductor $L$ that connect a resistive (totally real impedance) source $R_S$ to a totally resistive load $R_L$ (see Figure 3.3a). At this point we note an important result from circuit theory. If impedances $Z_1$ and $Z_2$ have a voltage $V$ across their series combination, then the voltage at their junction will be given $VZ_1/((Z_1+Z_2)$, where $Z_1$ is the lower impedance ($Z_1$ and $Z_2$ are said to act as a *voltage divider*). From this result the voltage across the load will be

$$V_L = \frac{R_L V_S}{R_S + \mathrm{j}\omega L + \frac{1}{\mathrm{j}\omega C} + R_L}. \tag{3.9}$$

We have seen from the above considerations that maximum power transfer to the load resistance will occur when the reactances cancel ($\mathrm{j}\omega L + 1/\mathrm{j}\omega C = 0$) and this will occur at a frequency $\omega_0 = 1/\sqrt{LC}$. The series combination of $L$ and $C$ behaves as a short circuit at frequency $\omega_0$ where it is said to have a *series resonance*. Away from the *resonant frequency*, the power transfer will fall away and so the circuit will act as a *band-pass filter* (i.e. it just passes signals at frequencies around the frequency $\omega_0$). We can rewrite (3.9) as

$$V_L = \frac{R_L V_S}{R_S + R_L} \frac{1}{1 + \mathrm{j}Q\left(\frac{\omega}{\omega_0} - \frac{\omega_0}{\omega}\right)}, \tag{3.10}$$

where $Q = L\omega_0/(R_L + R_S)$. The power that is transferred to the load will then be given by

$$P_L = \frac{V_L \bar{V}_L}{2R_L} = \frac{V_S \bar{V}_S R_L}{2(R_L + R_S)^2} \frac{1}{1 + Q^2\left(\frac{\omega}{\omega_0} - \frac{\omega_0}{\omega}\right)^2}. \tag{3.11}$$

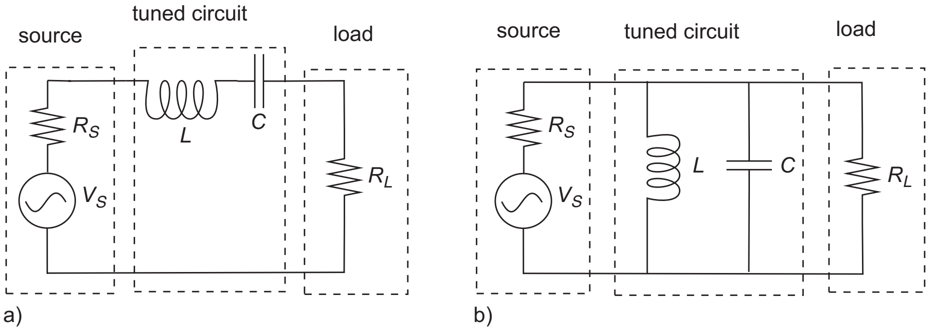


**Fig. 3.3** Series resonant circuit and parallel resonant circuit.

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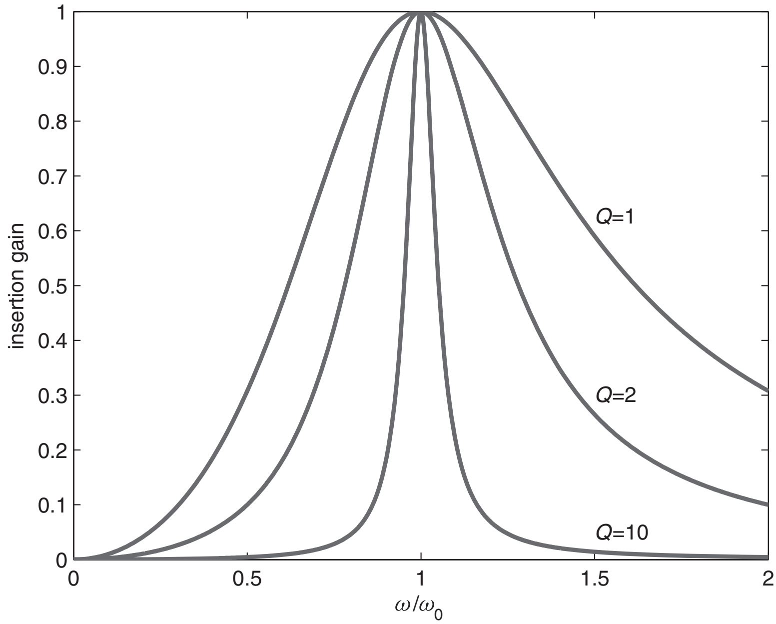


**Fig. 3.4** Insertion gain for the series resonant circuit.

The frequency-selective nature of the above tuned circuit can be seen from the plot of Figure 3.4. This Figure shows the *insertion gain* of the circuit (the power delivered to the load through the tuned circuit when normalised by the power delivered without the circuit) for a variety of $Q$. The *$Q$ factor* is a measure of the *quality* of a tuned circuit and it will be noted that the higher the $Q$, the sharper the filtering effect. For small deviations from resonance, i.e. $\omega = \omega_0 + \delta\omega$, we can approximate the above expression by

$$P_L \approx \frac{V_S \bar{V}_S R_L}{2(R_L + R_S)^2} \frac{1}{1 + 4Q^2 \frac{\delta\omega^2}{\omega_0^2}} \tag{3.12}$$

on noting that $\omega_0/\omega = 1/(1 + \delta\omega/\omega_0) \approx 1 - \delta\omega/\omega_0$ since $1/(1+x) \approx 1 - x$ for small x. The frequency selective nature of a filter is often described in terms of its half-power bandwidth $B$, i.e. the width of the band of frequencies over which the power supplied to the load is more than 50 per cent of the maximum. For the above circuit, the edges of the band will be located at $\omega_0 \pm \omega_0/2Q$ from which $B = \omega_0/Q$. The $Q$ of a circuit is an important quantity as it measures the quality of the circuit when acting as a filter. The *quality factor $Q$* has a broader meaning and, in general, is defined to be

$$Q = 2\pi \frac{\text{maximum energy stored}}{\text{energy lost per cycle}}. \tag{3.13}$$

It is essentially a measure of the storage efficiency of a circuit. (The definition applies to both mechanical and electrical systems.) We will show that this alternative definition still implies that $Q = L\omega_0/(R_L + R_S)$. Both capacitor and inductor are assumed perfect and so

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there is no energy loss in either. Any energy supplied to the circuit will oscillate between these devices or be lost in the resistances. The rate at which energy is lost in a resistor $R$ is given by $R\mathcal{I}^2$ and, integrating over a cycle, we get a loss of $\pi RI\bar{I}/\omega$. Consequently, the energy lost in the circuit over a cycle will be $\pi(R_S + R_L)I\bar{I}/\omega$. Furthermore, the maximum energy stored in the inductor will be $LI\bar{I}/2$. Consequently, as expected, we find from (3.13) that $Q = \omega L/(R_S + R_L)$.

An alternative configuration that acts as a band-pass filter is shown in Figure 3.3b. In this case we have a parallel combination of $C$ and $L$ that shunts the load. At frequency $\omega_0 = 1/\sqrt{LC}$, the parallel combination of $L$ and $C$ will have infinite impedance and they are said to have a *parallel resonance* at this frequency. By a similar analysis to that above, we find that

$$V_L = \frac{R_L V_S}{R_S + R_L} \frac{1}{1 + \mathrm{j}Q\left(\frac{\omega}{\omega_0} - \frac{\omega_0}{\omega}\right)}, \tag{3.14}$$

where $Q = (R_S \parallel R_L)/\omega_0 L$. This is almost identical to the result for the series $LC$ circuit, except for a difference in the way in which $Q$ is defined.

Up until now, we have assumed our inductors to be ideal, i.e. their impedances are purely imaginary. In reality, the windings of an inductor will add an effective series resistance $r_s$ and the inductor impedance will have a small real part. We often describe this resistance through the *unloaded* $Q$ of the inductor, a quantity which is defined to be $Q_U = \omega L/r_s$. It is sometimes more useful to describe the resistance as an effective parallel resistance $r_p$ and, for large $Q_U$, this will be given by $r_p = Q_U \omega L$. Consequently, if $Q_U$ is low, the extra resistance could seriously lower the $Q$ of the total circuit and hence lower its filtering capabilities (bandwidth depends on $Q$). The major aim in the design of inductors is to make their unloaded $Q$ as high as possible, usually by reducing the resistance of the wire as much as possible. This can be done by using wire that is thicker and/or has a high conductivity (silver and gold for example). We can also reduce the resistance by reducing the amount of winding by means of a high-permeability core.

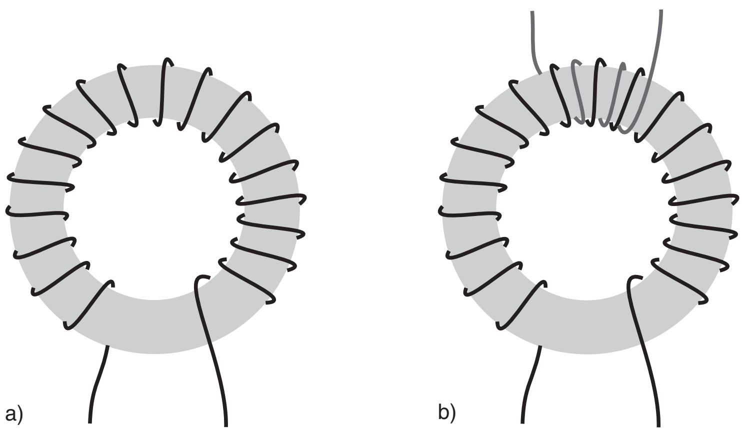


**Fig. 3.5** Toroid inductor and toroid transformer.

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A ferrite or dust iron core is usually to be preferred (see Figure 3.5a) since this will minimise any additional loss in the core.

## 3.4 The Inductive Transformer

It is clear that, to transfer maximum power between the source and load, we need to have a conjugate match. We need to somehow convert the impedance of the load into one that is conjugate to the source impedance. We can do this through an inductive transformer (see Figure 3.6a) of the sort we considered in Chapter 1. Let $V_1$ and $V_2$ be the complex amplitudes of the voltages applied to the primary and secondary windings of the transformer and $I_1$ and $I_2$ be the complex amplitudes of current flowing into these windings. The relations (1.22) and (1.22) of Chapter 1 can then be written as

$$V_2 = \mathrm{j}\omega L_{21} I_1 + \mathrm{j}\omega L_{22} I_2 \tag{3.15}$$

and

$$V_1 = \mathrm{j}\omega L_{11} I_1 + \mathrm{j}\omega L_{12} I_2. \tag{3.16}$$

From Ohm's law we have that $V_2 = -I_2 Z_L$ and so, from (3.15),

$$I_1 = -\frac{\mathrm{j}\omega L_{22} + Z_L}{\mathrm{j}\omega L_{21}} I_2. \tag{3.17}$$

Then, from (3.16) and (3.17),

$$V_1 = (-\mathrm{j}\omega L_{11}\frac{\mathrm{j}\omega L_{22} + Z_L}{\mathrm{j}\omega L_{21}} + \mathrm{j}\omega L_{12}) I_2 = \frac{L_{11}}{L_{21}} V_2 + \frac{1}{Z_L}(\frac{\mathrm{j}\omega L_{11} L_{22}}{L_{21}} - \mathrm{j}\omega L_{12}) V_2. \tag{3.18}$$

It will be noted, from the simple theory of Chapter 1, that $L_{12} = \sqrt{L_{11} L_{22}}$ and $L_{21} = L_{12}$ (a transformer satisfying this condition is said to be *strongly coupled*). Real transformers can have some deviation from strong coupling, but something fairly close to this can be achieved when the coils are wound over each other and, preferably, have a low-loss iron core (a ferrite or iron dust toroid is usually preferred) to increase the field that couples them (see Figure 3.5b). For such transformers, the last term of (3.18) disappears and we

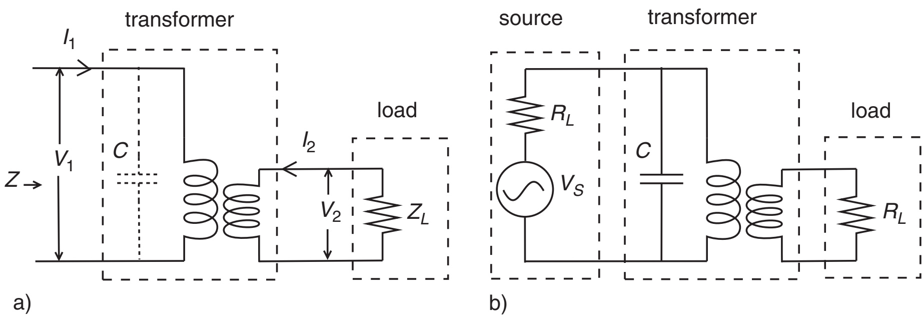


**Fig. 3.6** The inductive transformer.

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have $V_1 = V_2 L_{11}/L_{21}$ and so $V_1 = (N_1/N_2)V_2$, where $N_1$ is the number of turns on the primary of the transformer and $N_2$ is the number of turns on the secondary. Furthermore, if $\omega L_{22}$ is much larger than $|Z_L|$, (3.17) reduces to $I_1 = -(N_2/N_1)I_2$ and we have an *ideal transformer*. A significant increase in $L_{22}$ can be attained by winding the transformer on a high-permeability core and is another reason why RF transformers are often wound on ferrite or iron dust toroids.

Assuming an ideal transformer, and dividing (3.18) by (3.17), we obtain that the impedance $Z$ looking into the input of the transformer is given by

$$Z = \frac{\mathrm{j}\omega L_{11}}{\mathrm{j}\omega L_{22} + Z_L} Z_L. \tag{3.19}$$

Under the assumption that $\omega L_{22}$ is much larger than $|Z_L|$, we then obtain that $Z = (L_{11}/L_{22})Z_L$ (or $Z = (N_1^2/N_2^2)Z_L$ in terms of the turns ratio). If we cannot make $L_{22}$ sufficiently large, we can obtain a similar effect using a shunt capacitor $C$ (shown by broken lines in Figure 3.6a). The impedance looking into the transformer will now become

$$Z = \frac{\mathrm{j}\omega L_{11} Z_L}{\mathrm{j}\omega L_{22} + Z_L - \omega^2 L_{11} C Z_L} \tag{3.20}$$

and will have the desired properties at the resonant frequency $\omega_0$ of $C$ and $L_{11}$ (i.e. when $\omega_0^2 L_{11} C = 1$). We now have a tuned transformer that also behaves as a band-pass filter around the resonant frequency. Consider the tuned transformer with a resistive source of impedance $R_S$ and a resistive load of impedance $R_L$ (see Figure 3.6b). For small deviations $\delta\omega$ about the resonant frequency $\omega_0$, the impedance $Z$ can be approximated by

$$Z \approx \frac{L_{11}}{L_{22}} \frac{R_L}{1 + 2\mathrm{j}\delta\omega C R_L \frac{L_{11}}{L_{22}}}. \tag{3.21}$$

The series combination of $R_S$ and $Z$ will act as a voltage divider and, as a consequence, voltage $V_1$ will be related to voltage $V_S$ through $V_1 = V_S Z/(Z + R_S)$. From this,

$$V_1 = V_S \frac{Z}{Z + R_S} \approx V_S \frac{1}{1 + \frac{R_S L_{22}}{R_L L_{11}} + 2\mathrm{j}\delta\omega C R_S} \tag{3.22}$$

and, on noting that $V_1 = V_2 L_{11}/L_{21}$, the voltage across the load will be

$$V_2 \approx V_S \frac{1/n}{1 + \frac{R_S L_{22}}{R_L L_{11}} + 2\mathrm{j}\delta\omega C R_S}, \tag{3.23}$$

where $n = L_{11}/L_{21} = N_1/N_2$ is known as the *turns ratio* of the transformer. For maximum power transfer between source and load, we need to choose our transformer such that $Z = R_S$ (i.e. $R_S = (L_{11}/L_{22})R_L$) and so the power dissipated in the load will be

$$P_L = \frac{V_2 \bar{V}_2}{2R_L} \approx \frac{V_S \bar{V}_S}{8R_L} \frac{1/n^2}{1 + (\delta\omega C R_S)^2} \tag{3.24}$$

It is clear from this that the half-power bandwidth $B$ will be $2/CR_S$ or $\omega_0/Q$ where $Q = R_S/2\omega_0 L_{11}$.

## 3.5 The L Network

It is possible to match a source and load by means of the simple L network shown in Figure 3.7. Looking into the network from the source there will be an impedance

$$\begin{aligned} Z &= \mathrm{j}X_S + \mathrm{j}X_L \parallel R_L \\ &= \mathrm{j}X_S + \frac{\mathrm{j}X_L R_L}{R_L + jX_L} \\ &= \mathrm{j}X_S + \frac{X_L^2 R_L + \mathrm{j}X_L R_L^2}{X_L^2 + R_L^2}. \end{aligned} \tag{3.25}$$

Obviously, to match the source we need to choose

$$X_S = -\frac{X_L R_L^2}{X_L^2 + R_L^2} \text{ and } \frac{X_L^2 R_L}{X_L^2 + R_L^2} = R_S. \tag{3.26}$$

On defining $Q = |R_L/X_L|$, we can rewrite these equations as

$$X_S = -\frac{X_L Q^2}{1 + Q^2} \text{ and } \frac{R_L}{1 + Q^2} = R_S. \tag{3.27}$$

Rearranging the first of (3.27) we obtain that $Q = \sqrt{R_L/R_S - 1}$ (note that we have assumed $R_L > R_S$ otherwise the network would need to be reversed). Consequently, from $R_S$ and $R_L$ we can calculate $Q$ and then we will have that $X_L = \pm R_L/Q$. The choice of sign is arbitrary, but we will see this choice can be useful. Finally, we can calculate $X_S$ using the relation $X_S = -X_L Q^2/(1 + Q^2)$. If we choose $X_L$ to be negative then this will be realised by a capacitor $C$ and $X_S$ will be realised by an inductor $L$. Obviously, we choose the values of $C$ and $L$ to give the correct $X_L$ and $X_S$ at the desired operating frequency $\omega_0$. This implies that $C = Q/\omega_0 R_L$ and $L = (R_L/\omega_0)Q/(1 + Q^2)$.

As with other $LC$ networks, the L network will also have filtering properties. To see this, consider the voltage $V_L$ across the load in terms of the source voltage $V_S$. Once again we can treat the problem as one of a voltage divider, from which

$$\begin{aligned} V_L &= V_S \frac{\mathrm{j}X_L \parallel R_L}{R_S + \mathrm{j}X_S + \mathrm{j}X_L \parallel R_L} \\ &= V_S \frac{\mathrm{j}X_L R_L}{R_S R_L - X_S X_L + \mathrm{j}(X_S R_L + X_L R_S + X_L R_L)} \end{aligned} \tag{3.28}$$

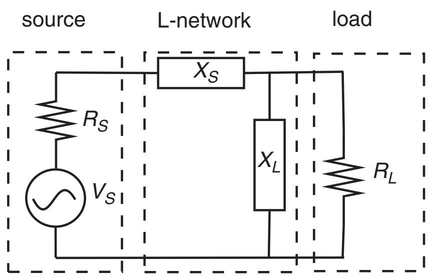


**Fig. 3.7** L-network transformer.

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and, noting that $X_L = -1/\omega C$ and $X_S = L\omega$,

$$V_L = V_S \frac{R_L}{j\omega C(R_S R_L + L/C) - CL\omega^2 R_L + R_S + R_L}. \tag{3.29}$$

We further note that $L/C = R_S R_L$ and $LC = Q^2 R_S/\omega_0^2 R_L$, so that

$$V_L = V_S \frac{R_L}{2jQR_S\frac{\omega}{\omega_0} - Q^2 R_S \frac{\omega^2}{\omega_0^2} + R_S + R_L}. \tag{3.30}$$

The power dissipated in the load will now be given by

$$P_L = \frac{V_S \bar{V}_S}{2R_L}\left(\frac{R_L}{R_L + R_S}\right)^2 \frac{1}{\frac{4Q^2}{(Q^2+2)^2}\frac{\omega^2}{\omega_0^2} + \left(1 - \frac{Q^2}{Q^2+2}\frac{\omega^2}{\omega_0^2}\right)^2} \tag{3.31}$$

on noting that $(R_L + R_S)/R_S = Q^2 + 2$.

Figure 3.8 illustrates the variation of the insertion gain with frequency. For high values of $Q$, the filter has a band-pass characteristic. For small $Q$, however, it behaves more like a *low-pass filter* (i.e. it passes all signals at frequencies below the design frequency $\omega_0$ and blocks those above this frequency).

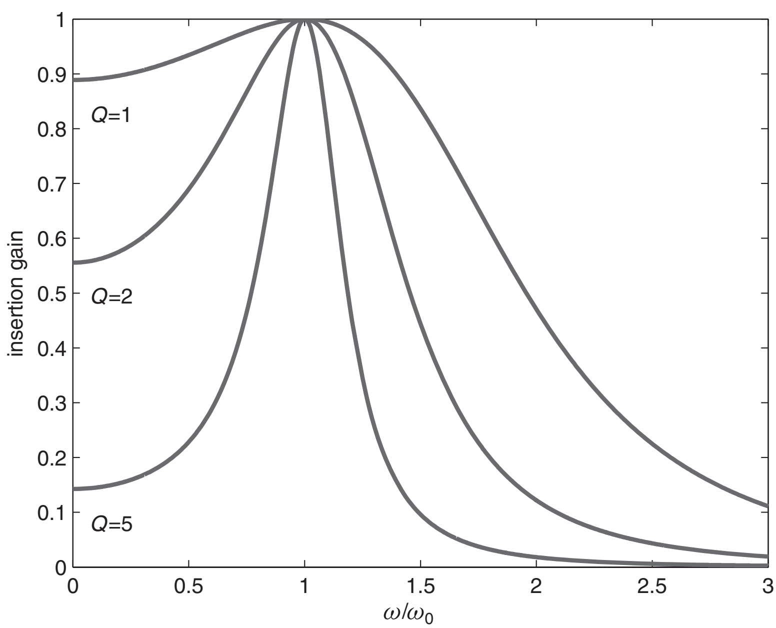


**Fig. 3.8** The filtering characteristics of the L network.

## 3.6 Capacitive Transformer

If a frequency-selective transformer is acceptable, the capacitive transformer of Figure 3.9a is an alternative. This can be regarded as a special case of the general three-element network shown in Figure 3.9b. Looking into the circuit consisting of $Z_1$,$Z_2$ and $Z_3$ we see the impedance

$$Z = Z_1 \parallel (Z_2 + Z_3) = \frac{Z_1(Z_2+Z_3)}{Z_1+Z_2+Z_3}. \tag{3.32}$$

We can analyse the relationship between $V_S$ and $V_L$ by treating the circuit as a pair of voltage dividers. Firstly

$$V_1 = V_S \frac{Z_1 \parallel (Z_2+Z_3)}{R_S + Z_1 \parallel (Z_2+Z_3)} = V_S \frac{Z_1(Z_2+Z_3)}{R_S(Z_1+Z_2+Z_3)+Z_1(Z_2+Z_3)} \tag{3.33}$$

and then

$$V_L = V_1 \frac{Z_3}{Z_2+Z_3} = V_S \frac{Z_1 Z_3}{R_S(Z_1+Z_2+Z_3)+Z_1(Z_2+Z_3)}. \tag{3.34}$$

Rearranging (3.34), we obtain

$$V_L = V_S \frac{Z_3}{Z_2+Z_3} \frac{1}{R_S(\frac{1}{Z_2+Z_3}+\frac{1}{Z_1})+1}. \tag{3.35}$$

In the case of the capacitive transformer, we will have that $Z_1 = \mathrm{j}\omega L$, $Z_2 = 1/\mathrm{j}\omega C_1$ and $Z_3 = (1/\mathrm{j}\omega C_2)//R_L$. If we make the simplification that $R_L \gg 1/\omega C_2$, we then obtain that

$$V_L \approx V_S \frac{C_1}{C_1+C_2} \frac{1}{\mathrm{j}\frac{R_S}{\omega L}(\omega^2 LC - 1)+1}, \tag{3.36}$$

where $C = C_1C_2/(C_1+C_2)$. Maximum power transfer will occur around the frequency $\omega_0 = 1/\sqrt{LC}$ and, for small deviations in frequency $\delta\omega$ around this resonance,

$$V_L \approx V_S \frac{1/n}{1+\mathrm{j}2\frac{R_S}{\omega_0 L}\frac{\delta\omega}{\omega_0}}, \tag{3.37}$$

where $n = (C_1+C_2)/C_1$, i.e. we effectively have a transformer with turns ratio $n$.

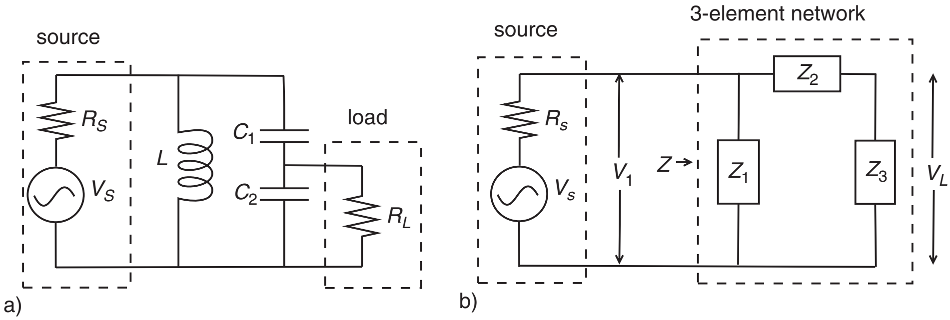


**Fig. 3.9** A general three-element network.

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## 3.7 Filters

It is clear that tuned circuits can act as filters, the $Q$ factor measuring how well they perform this function. We now look at how we can improve this filter function by increasing the complexity of the circuit. Consider the circuits shown in Figure 3.10. These can be viewed as a special case of the general three-element network shown in Figure 3.9b. For the filters of Figure 3.10, we will look at the situation where the reactances of $C$ and $L$ are related to the resistance $R$ of the source and load. We will assume that $L = \alpha R/\omega_0$ and $C = \beta/R\omega_0$ where $\alpha$ and $\beta$ are arbitrary constants and $\omega_0$ is some reference frequency. As a consequence, we can treat the filter as the circuit in Figure 3.9b with $Z_1 = -\mathrm{j}\omega_0 R/\beta\omega$, $Z_2 = \mathrm{j}\alpha R\omega/\omega_0$, $Z_3 = \mathrm{j}\beta\omega_0 R/(\mathrm{j}\beta\omega_0 - \omega\beta^2)$ and $Z_L = Z_S = R$. Then, using (3.34), we can calculate the power dissipated in the load (i.e. $V_L^2/2R_L$) as a function of frequency. Figure 3.11a shows the insertion loss for the case of $\beta = 1$ and $\alpha = 2$, parameter values that define what is known as a *Butterworth filter*. It will be noted that the power falls away very smoothly with a half-power bandwidth $\omega_0$.

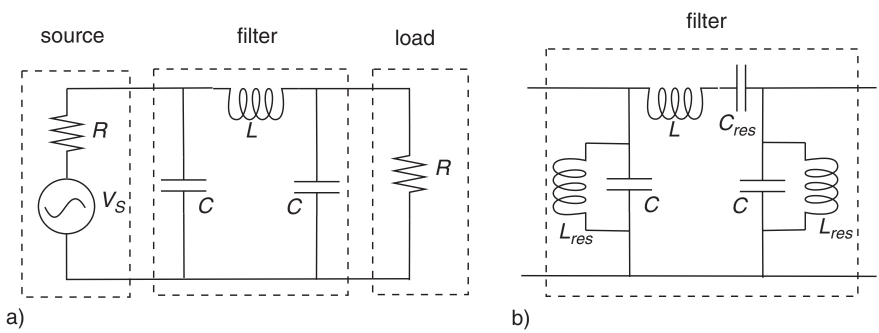


**Fig. 3.10** A three-element low-pass filter and its conversion to a band-pass filter.

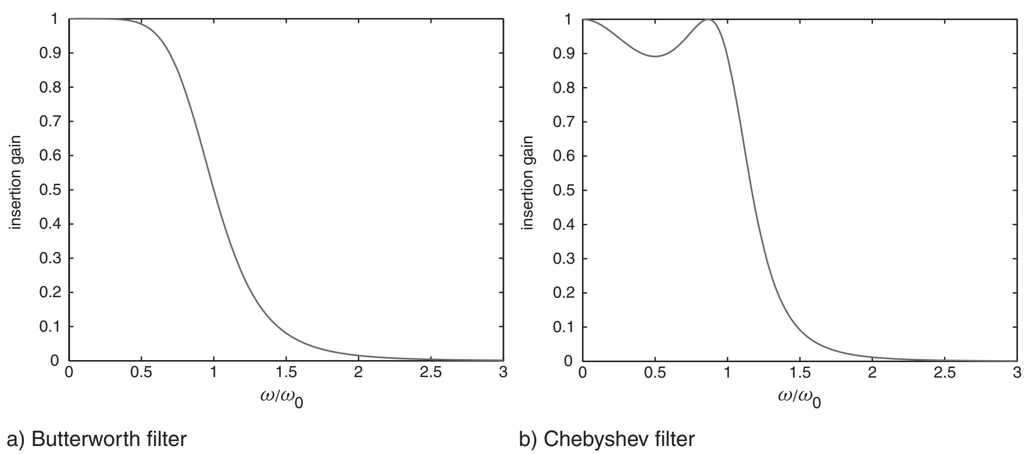


**Fig. 3.11** Performance of a three-element low-pass filter.

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If we can tolerate some *ripple* in the pass band of the filter, we could use the parameters of a *Chebyshev filter*. Such a filter, with a 0.122 ripple, is obtained when we use the parameters $\beta = 1.5963$ and $\alpha = 1.0967$. Figure 3.11b illustrates the performance of this filter and from which it is now clear that there is a much sharper out-of-band response, providing one can tolerate the ripple.

The above low-pass filter can be turned into a band-pass filter by resonating the capacitors and inductors at the centre frequency $\omega_C$ of the desired pass band ($\omega_C = \sqrt{\omega_L \omega_U}$ where $\omega_L$ and $\omega_U$ and the lower and upper half-power edges of the pass band). The capacitors are parallel-resonated with suitable inductors and the inductors are series-resonated with suitable capacitors. At frequency $\omega_C$, the parallel combinations will act as open circuits and the series combination as a short circuit (i.e. the filter acts as a straight through connector between source and load). Consequently, we will have a band-pass filter with maximum power transferred at frequency $\omega_C$ (the half-power pass band will be that of the low-pass filter). Figures 3.12a and 3.12b show the performance of the band-pass filters that are based on the low-pass filters of Figures 3.11a and 3.11b. For this simulation, the centre frequency is twice the bandwidth of the filter ($\omega_C = 2(\omega_U - \omega_L)$) and once again it will be noted that the Chebyshev filter has a much sharper response (at the price of some pass-band ripple).

Better performance could be achieved by cascading several of the above filters, but modern computer techniques allow us to optimise the components in such arrangements and achieve a much better performance than a simple cascade. Figure 3.13a shows the circuit of a five-element low-pass Chebyshev filter with 0.122 pass-band ripple. For this filter $L = 1.2296R/\omega_0$, $C_1 = 1.7058/\omega_0 R$ and $C_2 = 2.5408/\omega_0 R$ ($R$ is the impedance of the filter terminations). The improved performance with the extra elements can be seen in Figure 3.13b and it will be noted that the filter sides are now much sharper, but at the cost of extra ripples. With increasing complexity, however, the losses in realistic components becomes a problem and real filters will exhibit power loss as a result of this

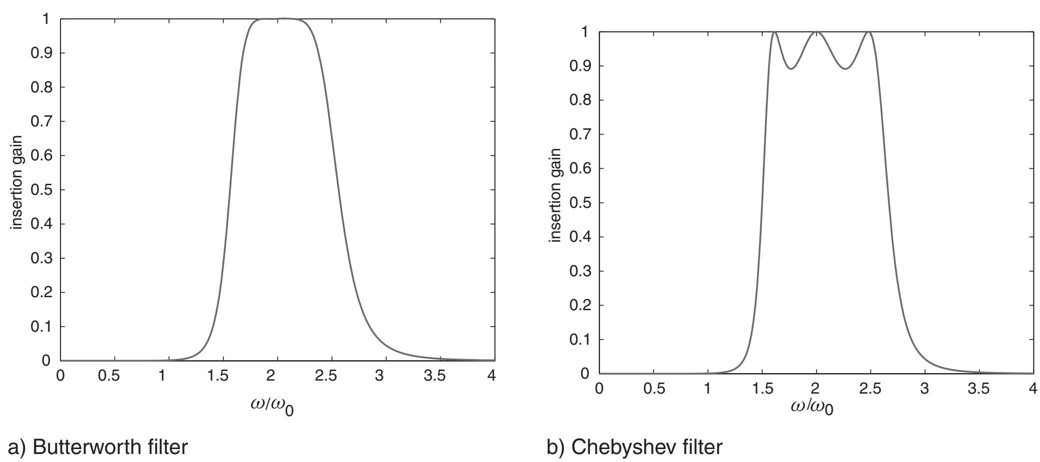


**Fig. 3.12** Performance of six-element band-pass filters.

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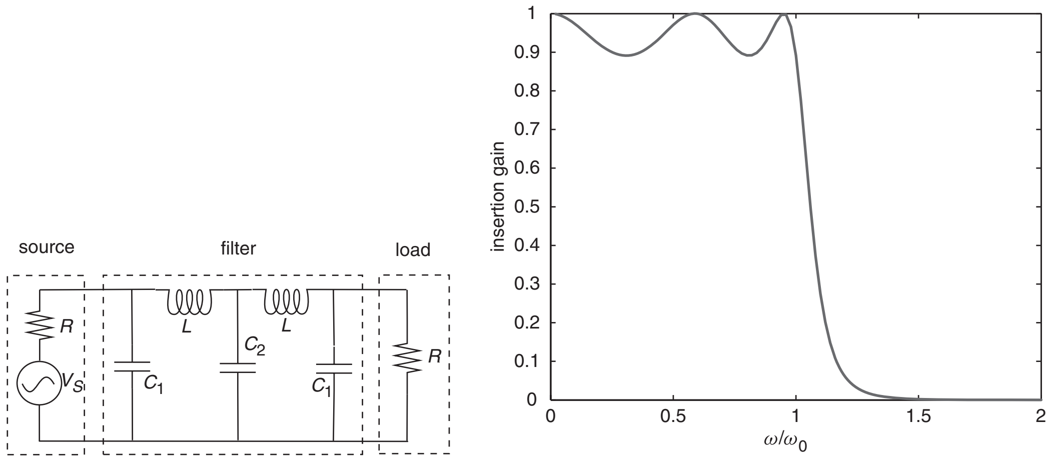


a) filter

b) filter response

**Fig. 3.13** A five-element low-pass Chebyshev filter with 0.122 pass-band ripple.

(i.e. the peak gain is less than 1). The insertion gain $G$ is often expressed in terms of a logarithmic scale in which the value of the gain is given by $10 \log_{10} G$ with units of *decibels* (or dB for short). If there is a loss, this will result in a negative value in the dB scales and the magnitude of this number is often termed the *insertion loss*.

## 3.8 Conclusion

In the current chapter we have developed the techniques that are required for high-fidelity frequency selection, a requirement of modern radio. However, whilst good filtering can help improve the performance of a radio system, the receivers in early radio systems had very low sensitivity and this greatly hindered the development of the technology. The situation changed in 1912 with the development of the first valves that could provide useful amplification and from this point on radio technology greatly accelerated in its progress. In the next chapter we will describe active devices, transistors in particular, and the techniques for their effective use as amplifiers.

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