CHAPTER

13 Coupled-resonator bandpass filters

We saw in Chapter 4 that the straightforward transformation of a prototype lowpass filter to a bandpass filter yields a circuit with alternating parallel resonant circuits and series resonant circuits as shown in Figure 13.1.

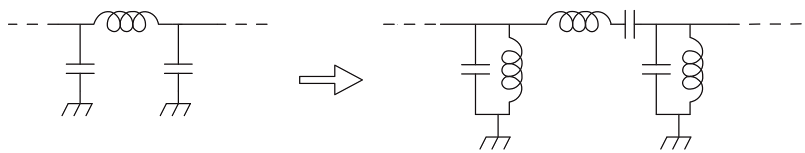


Figure 13.1. Conversion of a lowpass filter to a canonical bandpass filter.

If the prototype lowpass filter has a response $F_{LP}(\omega)$, the corresponding bandpass filter will have the response $F_{BP}(\omega) = F_{LP}(\,|\omega - \omega_C|)$, where ω_C is the center frequency. These canonical bandpass filters work perfectly – when simulated with a network analysis program. But usually they call for impractical component values. The inductors in the shunt branches must be smaller than the inductors in the series branches by a factor on the order of the square of the fractional bandwidth. For a 5% bandwidth filter, the ratio of the inductor values would be of the order of 1:400. For a given center frequency we might be lucky to find a high-Q inductor of any value, let alone high-Q inductors with such different values. Low-Q (resistive) inductors make a filter lossy and change its nominal passband shape. The series and shunt capacitor values have the same ratio. Generally Q is not a problem with capacitors, but very small values are impractical when they become comparable to the stray wiring capacitances.

13.1 Impedance inverters

This component value problem can be solved by transforming canonical bandpass filters into *coupled-resonator bandpass filters*, which can be built with identical or almost identical LC resonant circuits. The coupled-resonator filters have the same filter shapes, based on prototype lowpass designs, such as Butterworth or Chebyshev. Figure 13.2 shows some coupled-resonator fiter designs.

These filters are based on *impedance inverters*. Three examples of impedance inverters are shown in Figure 13.3, a 90° length of transmission line and two lumped LC circuits.

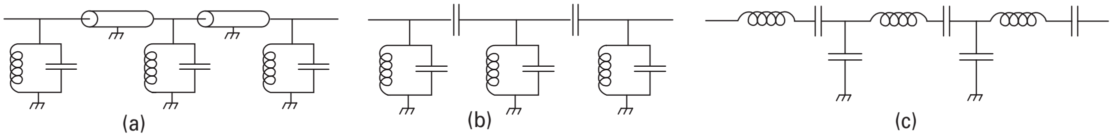


Figure 13.2. Three examples of coupled-resonator bandpass filter circuits.

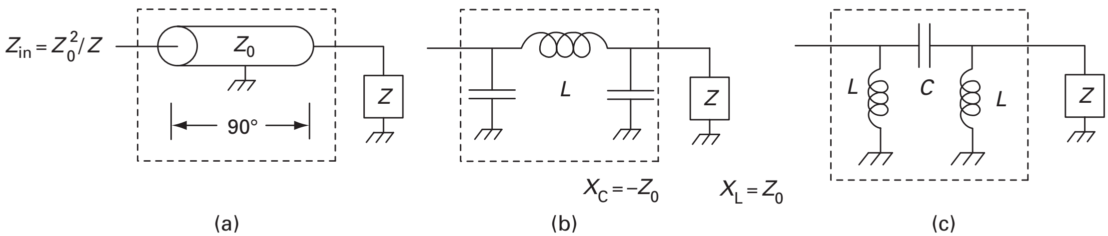


Figure 13.3. Three impedance inverter circuits

In every case, an impedance Z, when seen through the inverter, becomes $Z_0{}^2/Z$ where Z_0 can be called the characteristic impedance of the inverter. For the transmission line inverter, a 90° length of line, Z_0 is just the characteristic impedance of the line. For the LC inverters, both the inductor's reactance, X_L, and the capacitor's reactance, X_C, must be equal to the desired Z_0. Like the 90° cable, the lumped element circuits are perfect inverters only at one frequency but, in practice, they are adequate over a considerable range. An inverted capacitor is an inductor. An inverted inductor is a capacitor. Figure 13.4 shows an inverter (in this example, a 90° transmission line) used to invert a parallel circuit, making an equivalent series circuit.

The mathematics of this inversion is just

$$Z_{\mathrm{in}} = Z_0{}^2 Y = Z_0{}^2\left(\frac{1}{\mathrm{j}\omega L_\mathrm{p}} + \mathrm{j}\omega C_\mathrm{p} + \frac{1}{R_\mathrm{p}}\right) = \frac{1}{\mathrm{j}\omega(L_\mathrm{p}/Z_0{}^2)} + \mathrm{j}\omega(Z_0{}^2 C_\mathrm{p}) + \frac{Z_0{}^2}{R_\mathrm{p}}. \tag{13.1}$$

Let us look at four inverters which include inductors or capacitors with negative values. For these inverters, shown in Figure 13.5, $X_C = Z_0$ or $X_L = Z_0$.

Figure 13.6 verifies the inverter action of the all-capacitor T-section inverter. You can use this kind of reasoning to verify the inverter action of the other circuits:

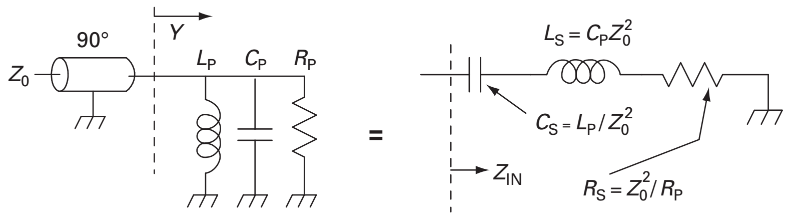


Figure 13.4. Impedance inverter makes a parallel circuit appear as a series circuit.

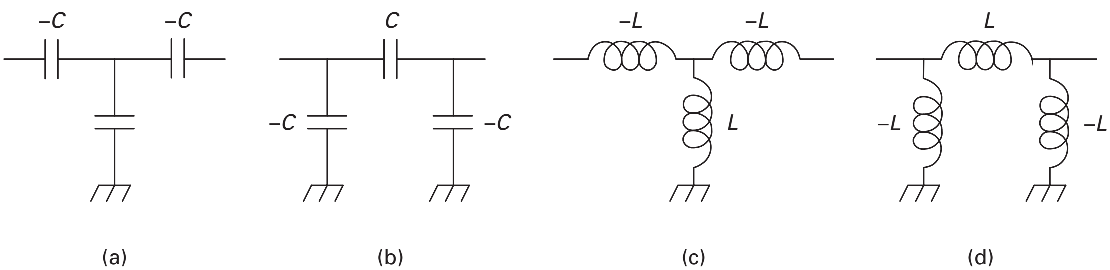


Figure 13.5. Impedance inverters based on negative value components.

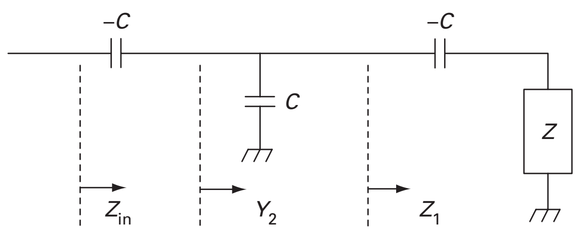


Figure 13.6. Operation of the T-network negative capacitor inverter.

$$Z_1 = \frac{1}{j\omega(-C)} + Z \tag{13.2}$$

$$Y_2 = j\omega C + \frac{1}{Z_1} = \frac{j\omega CZ}{Z - 1/j\omega C} \tag{13.3}$$

$$Z_{in} = \frac{1}{j\omega(-C)} + \frac{1}{Y_2} = \frac{1}{\omega^2 C^2 Z} = \frac{Z_0^2}{Z}. \tag{13.4}$$

Because they contain negative capacitances or negative inductances, the four inverters in Figure 13.5 might seem to be only mathematical curiosities. Not at

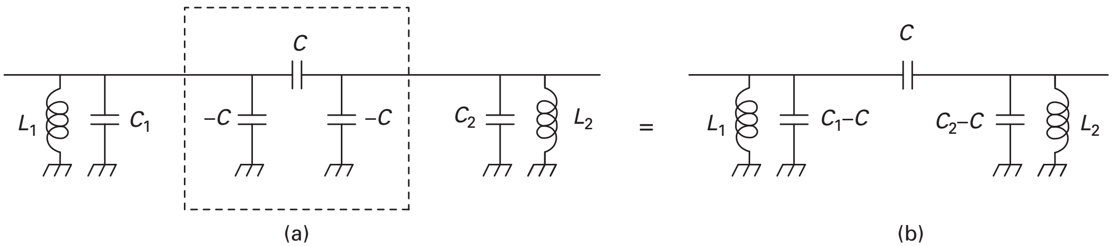


Figure 13.7. Negative capacitors absorbed into adjacent positive capacitors.

all; the negative elements can be absorbed by positive elements in the adjacent circuitry as shown in Figure 13.7, where a π-section capacitor inverter is placed between two parallel LC "tanks."

13.2 Conversion of series resonators to parallel resonators and vice versa

Ladder network filters have alternating series and shunt branches. Let us see how inverter *pairs* are used in ladder filters. Suppose we embed a series capacitor between a pair of inverters at some point along a ladder network.

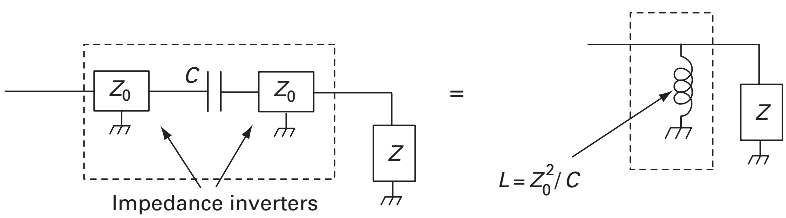


Figure 13.8. A series capacitor between inverters is equivalent to a shunt inductor.

The combination of the capacitor and the inverter pair is equivalent to a shunt inductor, as shown in Figure 13.8.

You can show just as easily that any series impedance, Z_s, together with a pair of bracketing inverters of characteristic impedance Z_0 is equivalent to a shunt admittance $Y_p = Z_0^{-2} Z_s$. Likewise, the combination of any shunt admittance Y and a pair of bracketing inverters is equivalent to a series impedance $Z = Z_0^{\,2} Y$. Figure 13.9 illustrates this, showing how a series resonant series branch in an ordinary bandpass filter can be replaced by a parallel resonant shunt branch imbedded between a pair of inverters.

Likewise, a parallel resonant shunt branch can be realized as a series resonant series branch imbedded between a pair of inverters, as shown in Figure 13.10.

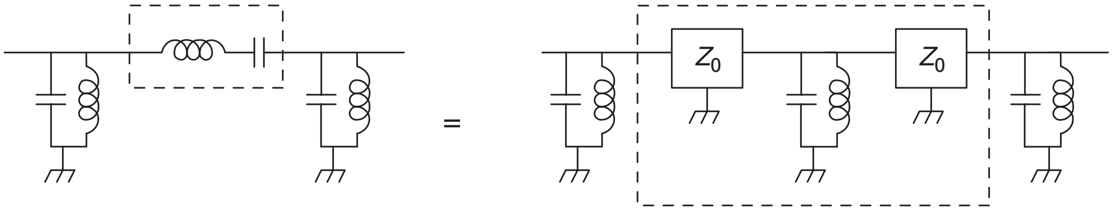


Figure 13.9. A shunt resonator between inverters is equivalent to a series resonator.

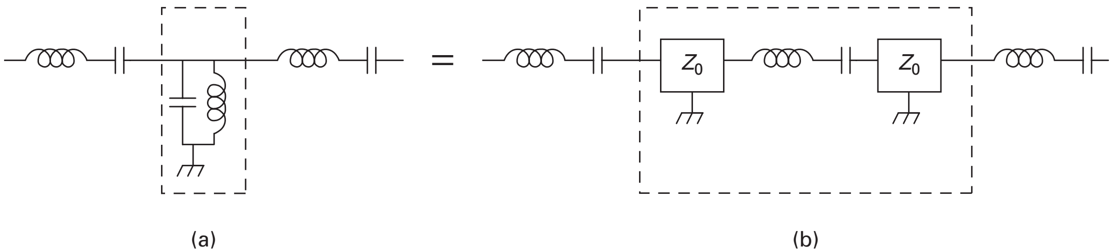


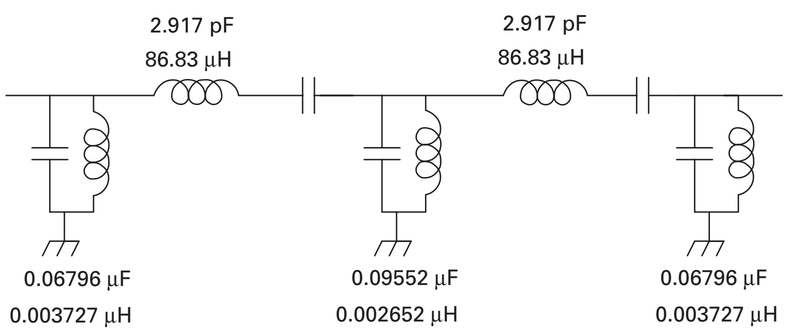
Figure 13.10. A series resonator between inverters is equivalent to a shunt resonator.

13.3 Worked example: a 1% fractional bandwidth filter

Consider a 50-ohm, 1-dB Chebyshev filter with a 10-MHz center frequency and a bandwidth of 100 KHz between the 1-dB points. The filter, which results from the straightforward lowpass to bandpass transformation (Chapter 4) is shown in Figure 13.11 and its response is shown in Figure 13.12.

We might find 86-μH inductors with high Q at 10 MHz but the 3.728 nH and 2.645 nH inductors would be tiny single turns of wire with very poor Q. To get around these component limitations, we will convert this filter into a coupled-resonator filter. Suppose we have in hand some adjustable 0.3 to 0.5 microhenry

Figure 13.11. A straightforward (but impractical) bandpass filter. The calculated response of this filter is shown in Figure 13.12.



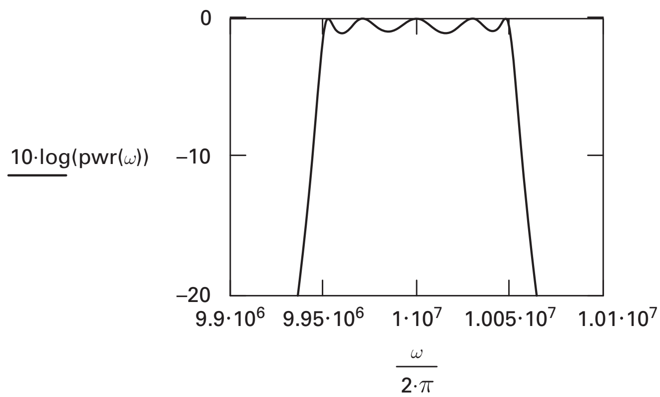


Figure 13.12. Calculated response for filter of Figure 13.11.

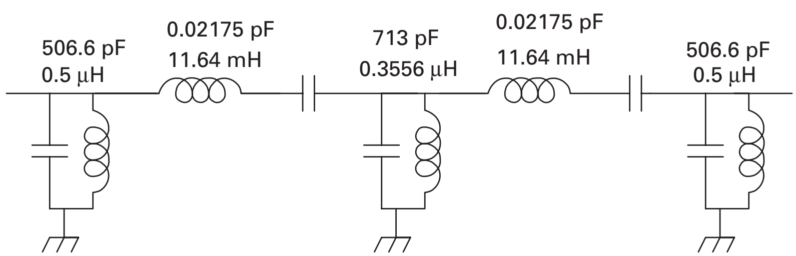


Figure 13.13. Filter of Figure 13.11, scaled from 50 to 6705 ohms.

inductors which, at 10 MHz, have very high Q (we will see later just how much Q is required). Let us first change the working impedance of the filter so that the parallel resonators at the end will use 0.5 μH, which is 134.1 times the original end inductors and implies that the filter will be scaled to $50 \times 134.1 = 6705$ ohms. We multiply the other inductors by 134.1 and divide the capacitors by 134.1 to get the circuit of Figure 13.13.

The parallel resonators now use the desired inductors but the series resonators call for inductors of 11.6 mH, a very large value for which we surely will not find high Q components. Moreover, the series capacitors are only 0.02 pF, a value far too small to be practical. We can solve this problem by using impedance inverters to convert the series resonators into parallel resonators. Let us use the all-capacitor π-section inverters of Figure 13.5(b) and the same parallel resonators we used for the end sections. Figure 13.14 shows how two inverters and the parallel resonator replace each series resonator.

We can calculate the inverter's characteristic impedance, Z_0, as follows:

$$Z_0{}^2 Y = Z;\;\; Z_0{}^2(\mathrm{j}\omega C_\mathrm{p} + 1/\mathrm{j}\omega L_\mathrm{p}) = \mathrm{j}\omega L_\mathrm{S} + 1/\mathrm{j}\omega C_\mathrm{S} \tag{13.5}$$

$$Z_0{}^2 = L_\mathrm{p}/C_\mathrm{S} = 0.5\times10^{-6}/0.02175 \times 10^{-12} = 4796^2. \tag{13.6}$$

Figure 13.14. Inverters transform a 0.5-μH shunt inductor into a 11.644-mH series inductor.

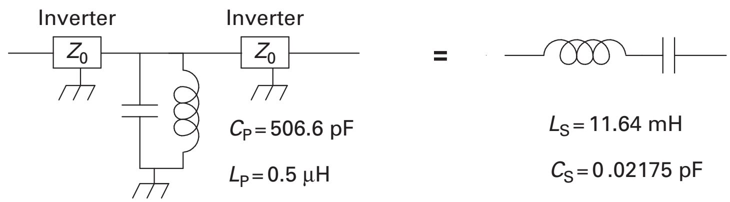
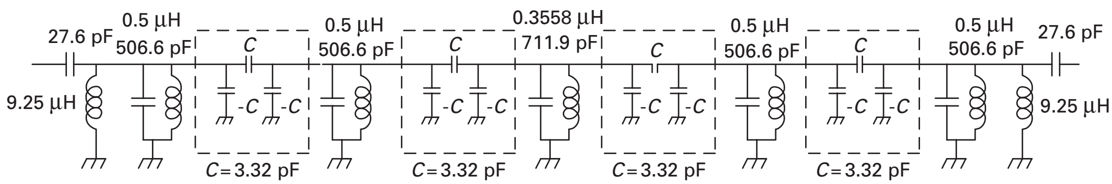


Figure 13.15. Coupled-resonator version of previous bandpass filter.



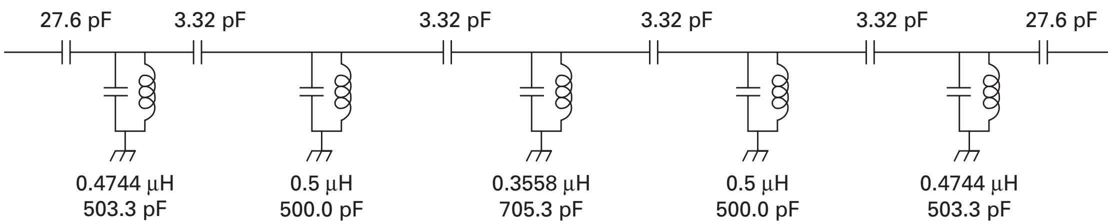


Figure 13.16. Finished coupled-resonator filter.

For this type of inverter, we had seen that $Z_0 = X_C$, so $C = 3.32$ pF. We now have our coupled-resonator filter but since it works at 6705 ohms we will add L-section matching networks at each end to convert it back to 50 ohms. The filter, at this point, is shown in Figure 13.15. All the resonators are now parallel resonators. (In other situations we might use inverters to convert series resonators into equivalent parallel resonators to make an all-series-resonator filter – see Figure 13.1.)

The final clean-up step is to absorb the -3.32 pF capacitors into the resonator capacitors and combine the matching section inductors with the end-section resonator inductors as shown in Figure 13.16.

The response of the finished filter is shown in Figure 13.17 and is almost identical to the response of the prototype filter of Figure 13.11. The difference, a fraction of a dB, occurs because the inverters work perfectly only at the center frequency.

13.4 Tubular bandpass filters

A popular bandpass filter design, the "tubular filter" is produced by many filter manufacturers. Figure 13.18 shows the construction of a three-resonator tubular filter.

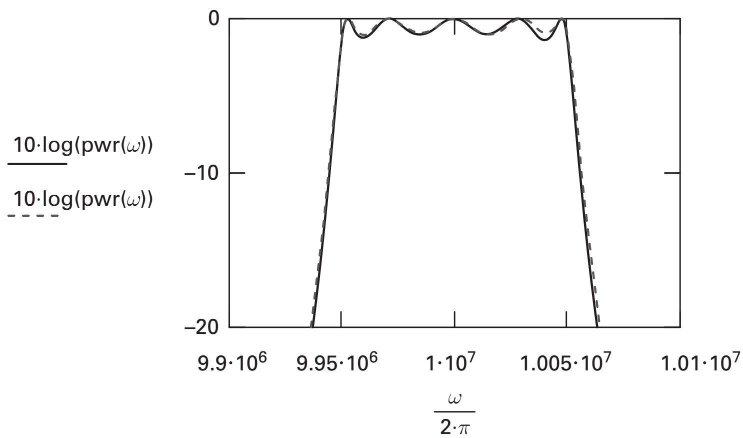


Figure 13.17. Calculated response of the filters of Figure 13.16 (pwr) and Figure 13.11 (pwr).

The only standard electronic components are the coaxial connectors at the ends. There are also (in this example) three inductors (wire coils), four metal cylinders, two dielectric spacers, two (or one long) dielectric sleeves, and a tubular metal body. Figure 13.19 shows how a coupled-resonator filter design, of the type we have discussed, is transformed into the tubular filter design. You can verify that Figure 13.19(d) is the circuit diagram of the tubular filter. The three-capacitor π-sections are formed by the capacitance between the adjacent faces of the metal cylinders and the capacitors are formed between the outside surfaces of the cylinders and the tubular body.

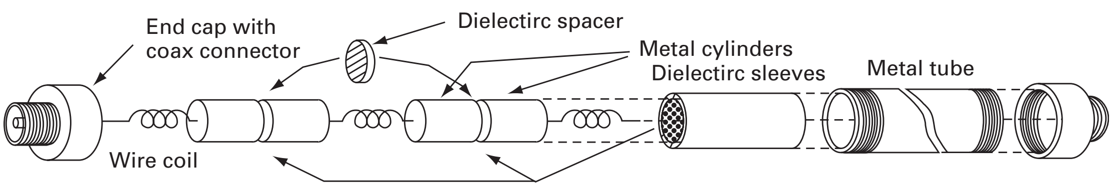


Figure 13.18. Tubular bandpass filter.

Beginning with Figure 13.19(a), we have a standard coupled-resonator bandpass filter using series resonators. In the canonical prototype for this filter, the middle section is a parallel resonator, but this has been replaced by a series resonator sandwiched between two impedance inverters. In (b), the center capacitor has been replaced by two capacitors (each of twice the value of the original capacitor so that, in series, the total series capacitance is the same). The capacitors have been shifted slightly in (c) to identify a T-section capacitor network at each side of the central inductor. Finally, in going from (c) to (d), these T-networks are replaced by equivalent π-networks, to arrive at the circuit of the tubular filter. Any

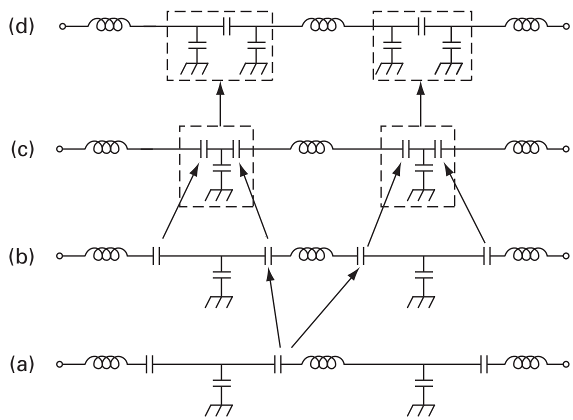
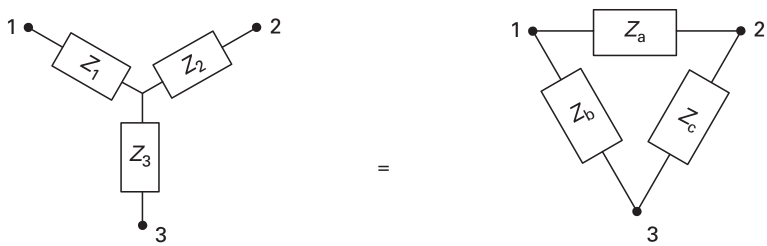


Figure 13.19. Tubular bandpass filter evolution.



$$Z_1 = \frac{Z_a Z_b}{Z_a + Z_b + Z_c} \qquad Z_a = \frac{Z_1 Z_2 + Z_2 Z_3 + Z_3 Z_1}{Z_3}$$

Figure 13.20. Equivalent π-section and T-section networks.

T-network has an equivalent π-network and vice versa (Problem 13.5). These transformations, also known as T−π and π−T are shown in Figure 13.20. Formulas are given for one element in each network; the others follow from symmetry.

13.5 Effects of finite *Q*

These calculated filter responses assume components of infinitely high Q. We can calculate the effects of finite Q by paralleling the (lossless) inductors in our model with resistors equal to Q times the inductor reactances at the center frequency. If, for example, the Q is 500 (quite a high value for a coil), we would parallel the inductors in the filter of Figure 13.15 with resistors of about 15 000 ohms. Reanalyzing the circuit response, we would find that the filter will have a midband insertion loss of 7 dB and that the flat (within 1 dB) passband response becomes rounded. The effect will be somewhat less for a filter with more gradual skirts, e.g., a 0.01 dB Chebyshev or a Butterworth filter. But the real problem is still the small fractional bandwidth. For a filter with small fractional bandwidth to have the ideal shape of Figure 13.17, the resonators must be quartz or ceramic or other resonators with Qs in the thousands. An approximate analysis predicts that the midband loss per section in a bandpass filter will be on the order of

$$\frac{\text{power transmitted}}{\text{power incident}} = \left(1 - \frac{L_0/2}{Q \cdot \text{fractional bandwidth}}\right) \tag{13.7}$$

where L_0 represents the inductor value in the normalized lowpass prototype filter. For our five-section filter we can take L_0 to be about 1.5 henrys. If the inductor Q is 500, the predicted transmission of the five-section filter is $5 \times 10 \log[1-(1.5\ /2)/(500 \cdot (1/100))] = -10$ dB, which is roughly equal to the actual value of −7 dB.

13.6 Tuning procedures

Filters with small fractional bandwidths and sharp skirts are extremely sensitive to component values. In the filter of Figure 13.16, for example, the resonators must be tuned very precisely or the shape will be distorted and the overall transmission will be lowered. (The values of the small coupling capacitors – all that remains of the impedance inverters – are not as critical.) Usually each resonator is adjustable by means of a variable capacitor or variable inductor. All the adjustments interact and, if the filter is totally out of tune, it may be hard to detect any transmission at all. A standard tuning procedure is to monitor the input impedance of the filter while tuning the resonators, one-by-one, beginning at input end. While resonator N is being adjusted, resonator N+1 is short circuited. The tuning of one resonator is done to produce a maximum input impedance while the tuning of the next is done to produce a minimum input impedance. The procedure must sometimes be customized to account for matching sections at the ends.

13.7 Other filter types

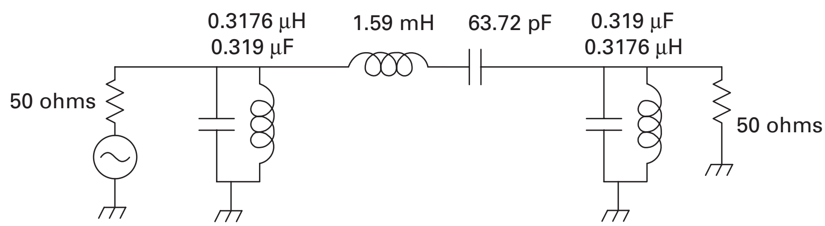
The coupled-resonator technique is used from HF through microwaves. Not all RF bandpass filters, however, use the coupled-resonator technique. The IF bandpass shape in television receivers is usually determined by a SAW (surface acoustic wave) bandpass filter. SAW filters are FIR (finite impulse response) filters, whereas all the LC filters we have discussed are IIR (infinite impulse response) networks. This classification is made according to the behavior of the output voltage following a delta function (infinitely sharp impulse) excitation. Digital filters can be designed to be either FIR or IIR filters.

Problems

Problem 13.1. Use your network analysis program to verify that the filter of Figure 13.16 does indeed give the response shown in Figure 13.17.

Problem 13.2. Verify that the two *LC* circuits in Figure 13.3 are impedance inverters.

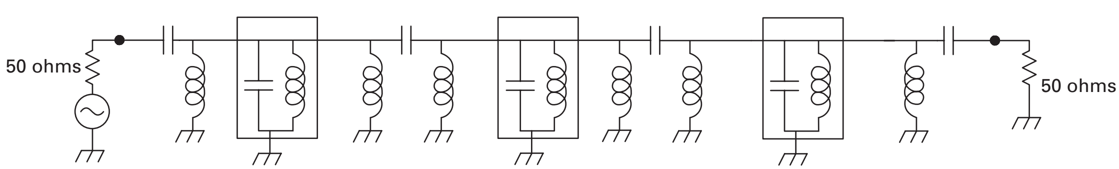
Problem 13.3. The filter shown below was developed in Chapter 4 as an example of the straightforward conversion from a prototype lowpass filter to a bandpass filter. This Butterworth (maximally flat) filter has a bandwidth of 10 kHz and a center frequency of 500 kHz. Suppose you have available some 30 μH inductors with a Q of 100 at 500 kHz. Convert the filter into a coupled-resonator filter that uses these inductors. Use your ladder network analysis program to verify the performance of your filter.



Problem 13.4. A bandpass filter is to have the following specifications:

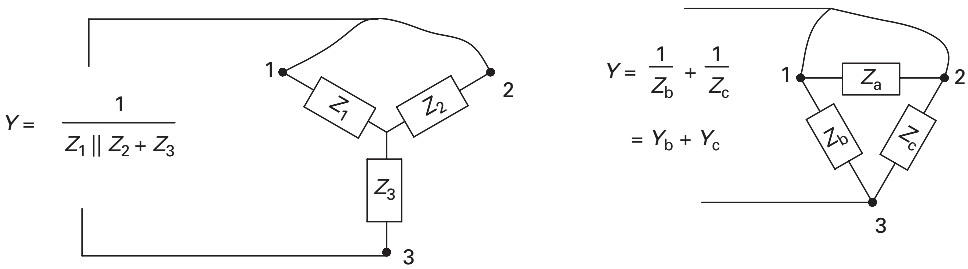
Center frequency: 10 MHz; shape: three-section 1-dB Chebyshev; bandwidth: 3 KHz (between outermost 1-dB points); source and load Impedances: 50 ohms. Since the loaded Q of this filter is very high, $10^6/3000 = 333$, it is important to use very high-Q resonators. Suppose you have located some resonators (cavities, crystals, or whatever) with adequate Q. These resonators are all identical. At 10 MHz they exhibit a parallel resonance, equivalent to a parallel LC circuit. At 10 MHz, they have a susceptance slope of 10^{-6} (1 mho/MHz).

(a) Find the *LC* equivalent circuit for these resonators (in the vicinity of 10 MHz).
(b) Design the filter shown below around these resonators.
(c) Use your ladder network analysis program to verify the frequency response of your design.



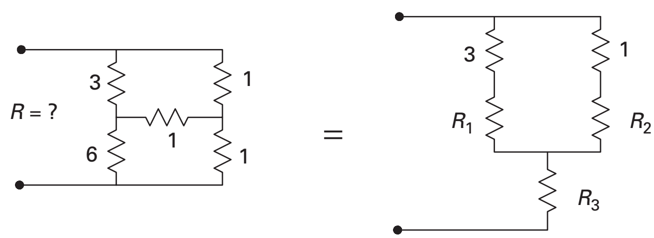
Problem 13.5. Derive expressions for Z_a, Z_b, and Z_c in terms of Z_1, Z_2, and Z_3 for the equivalent T and π networks shown in Figure 13.19. Hint: consider the connections

shown below. The sketched-in wires show that $Y_A + Y_B = (Z_3 + Z_1 \parallel Z_2)^{-1}$. If you write the corresponding $Y_B + Y_C$ and $Y_C + Y_A$ equations, then add the first two and subtract the third, you will have the formula for Y_B. A similar technique yields the expressions for Z_1, Z_2, and Z_3.



(a) (b)

Problem 13.6. The bridge circuit shown below in (a) is the simplest network whose resistance cannot be found immediately by series and parallel reduction. Rather than resorting to loop or node equations, note that the circuit contains two πs and two Ts. Replace a π by its equivalent T or a T by its equivalent π. Now find the resistance of the network by simple reduction. The circuit at the right shows how one of the π's can be replaced by a T.



(a) (b)

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