CHAPTER

5 Frequency converters

A common operation in RF electronics is frequency translation, whereby all the signals in a given frequency band are shifted to a higher frequency band or to a lower frequency band. Every spectral component is shifted by the same amount. Cable television boxes, for example, shift the selected cable channel to a low VHF channel (normally channel 3 or 4). Nearly every receiver (radio, television, radar, cell phone, …) uses the *superheterodyne* principle, in which the desired channel is first shifted to an intermediate frequency or "IF" band. Most of the amplification and bandpass filtering is then done in the fixed IF band, with the advantage that nothing in this major portion of the receiver needs to be retuned when a different station or channel is selected. The same principle can be used in frequency-agile transmitters; it is often easier to shift an already modulated signal than to generate it from scratch at an arbitrary frequency. Frequency translation is also called *conversion* and is even more commonly called *mixing*.[1]

5.1 Voltage multiplier as a mixer

A mixer takes the input signal or band of signals (segment of spectrum), which is to be shifted, and combines it with a reference signal whose frequency is equal to the desired shift in frequency. In a radio receiver, the reference or "L.O." signal is a sine-wave, generated within the receiver by a *local oscillator*.[2] Mixers, in order to produce new frequencies, must necessarily be nonlinear since linear circuits can change only the amplitudes and phases of a set of superposed sine waves. Multiplication is the nonlinear operation used in mixers

[1] In audio work "mixing" means addition, a linear superposition that produces no new frequencies. In RF work, however, mixing means multiplication; an RF mixer either directly or indirectly forms the product of the input signal voltage and a sinusoidal "local oscillator" (L.O.) voltage. Multiplication produces new frequencies.

[2] The earliest radio receivers employed no frequency conversion, so they had no "local" oscillator; the only oscillator was remote – at the transmitter location.

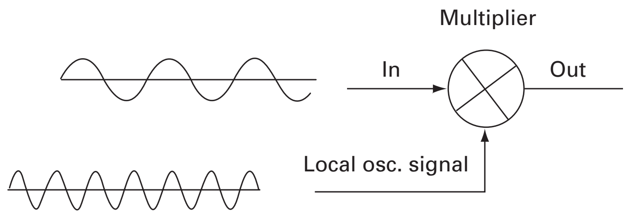

Figure 5.1. A voltage multiplier used as a frequency converter (mixer).

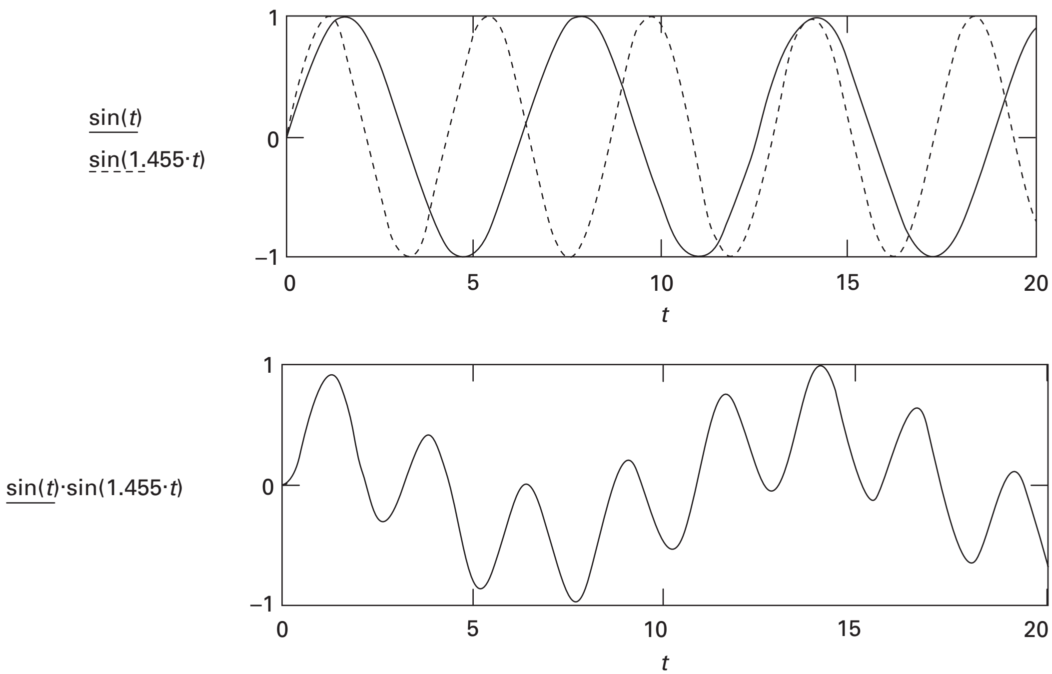

Figure 5.2. Multiplier input and output waveforms.

to produce new signals at the shifted frequencies. Figure 5.1 shows a voltage multiplier with its signal and L.O. inputs. A circumscribed X is the standard symbol for a mixer. The input signal port of the mixer is usually labeled "RF," while the other two ports are labeled "LO" and "OUT" (or "IF").

The output voltage from the multiplier is the product (or proportional to the product) of the two input voltages. In Figure 5.2 a sine-wave input signal is multiplied by an L.O. that is 1.455 times higher in frequency. These multiplicands are shown in the top graph. The bottom graph shows their product which can be seen to contain frequencies both higher and lower than the original frequencies.

The familiar "sin(a)sin(b)" trigonometric identity shows that, in this simple case, the multiplier output consists of just two frequencies: an up-shifted signal at $\omega_L + \omega_R$ and a down-shifted signal at $\omega_L - \omega_R$:

$$\sin(\omega_R t)\sin(\omega_L t) = {}^1\!/_2[\cos(\omega_R - \omega_L)t - \cos(\omega_R + \omega_L)t]. \tag{5.1}$$

If we replace the single RF signal by $V_1\sin(\omega_{R1}t)+V_2\sin(\omega_{R2}t)$, a signal with two spectral components, you can confirm that the output will be

$$\begin{aligned}&{}^1\!/\!_2V_1\cos([\omega_{R1}-\omega_L]t)+{}^1\!/\!_2V_2\cos([\omega_{R2}-\omega_L]t)\\&-{}^1\!/\!_2V_1\cos([\omega_{R1}+\omega_L]t)-{}^1\!/\!_2V_2\cos([\omega_{R2}+\omega_L]t).\end{aligned}\tag{5.2}$$

Just as this linear combination of two signals is faithfully copied into both an up-shifted band and a down-shifted band, any linear combination, i.e., any spectral distribution of signals, will be faithfully copied into these shifted output bands. With respect to signals applied to the RF port, you can see that the mixer is a linear device; all the components are translated (both up and down) in frequency, but their relative amplitudes are left unchanged and there is no interaction between them. Usually one wants only the up-shifted band or only the down-shifted band; the other is eliminated with an appropriate bandpass filter. Ideal analog (or digital) multipliers are being used more commonly as mixers in RF electronics as their speeds increase with improving technology.

5.2 Switching mixers

If the L.O. waveform is square, rather than a sinusoidal, the mixer output will contain not only the fundamental up-shifted and down-shifted outputs but also components at offsets corresponding to the third, fifth, and all other odd harmonics of the L.O. frequency, i.e., at offsets corresponding to all the frequencies in the Fourier decomposition of the square wave. You can confirm this by simply multiplying the Fourier series for the square-wave L.O. by the superposition of signals in the input at the RF port. These new components are usually very easy to filter out so there is no disadvantage in using a square-wave L.O. In fact, there is an advantage. Consider an L.O. signal that is a square wave with values ±1. In this case, since the multiplier multiplies the input signal only by either +1 or − 1, it can be replaced by an electronic SPDT switch that connects the output alternately to the input signal and the negative of the input signal. This equivalence is shown in Figure 5.3.

The phase inversion needed for the bottom side of the switch can easily be done with a center-tapped transformer and the switching can be done with two

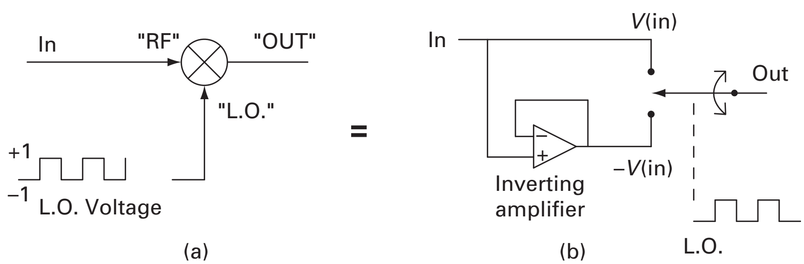

Figure 5.3. Switching mixer operation.

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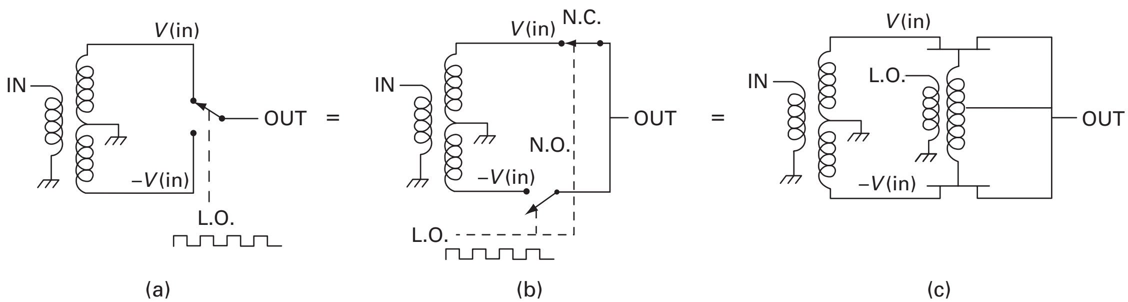
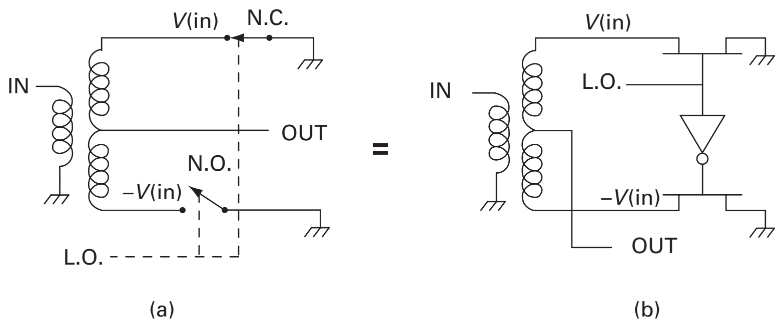


Figure 5.4. Active switching mixer using transistors.

Figure 5.5. Alternate active switching mixer.



transistors, one for the high side and one for the low side. In the circuit of Figure 5.4 the switches are FETs. (Mixers based on transistors are called *active mixers*.) A second center-tapped transformer provides the L.O. phase inversion so that one FET is turned on while the other is turned off.

We could just as well have taken the signal from the center tap and used the FETs to ground one end of the secondary and then the other. With this arrangement, shown in Figure 5.5, it is easier to provide the drive signals to the transistors, since they are not floating.

Diodes are commonly used as the switching elements for the arrangement shown in Figure 5.5. This results in the *passive* switching mixer circuit shown in Figure 5.6.

Voltage from the L.O. transformer alternately drives the top diode pair and the bottom pair into conduction. The L.O. signal is made large enough that the conducting diodes have very low impedance (small depletion region) and the non-conducting diodes have a very large impedance (wide depletion region). The end of the input transformer connected between the turned-on diode pair is effectively connected to ground through the secondary of the L.O. transformer. Note that current uses both sides of the L.O. transformer on the way to ground, so no net flux is created in that transformer and it has zero impedance for this current. This circuit is usually drawn in the form shown in Figure 5.6(b) and is referred to as a *diode ring mixer*.

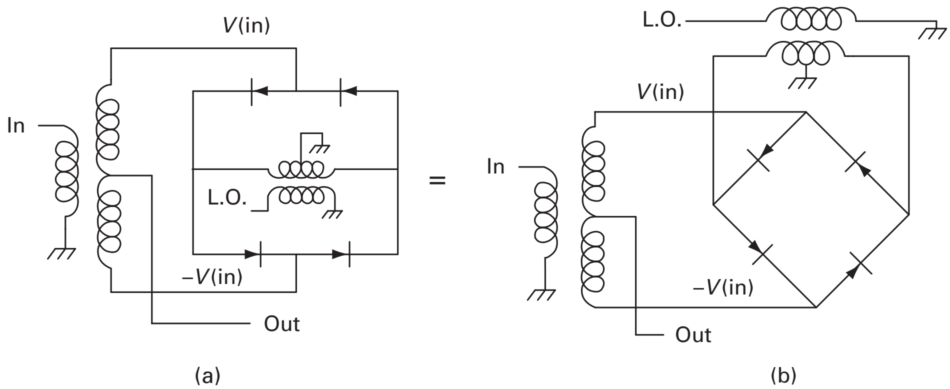
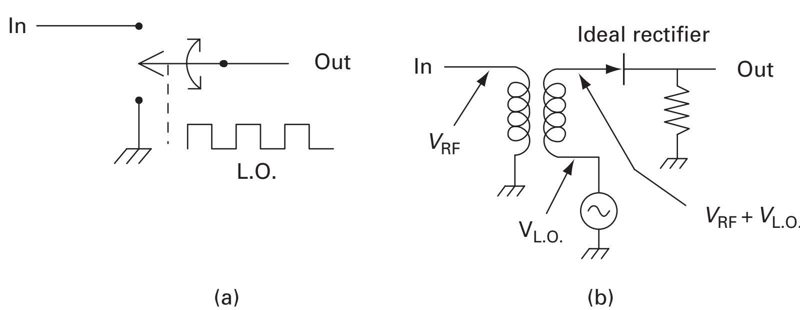


Figure 5.6. Diode ring mixer.

Figure 5.7. Unbalanced switching mixers.



All the switching mixers shown above are "double-balanced" which means that no L.O. frequency energy appears at the RF or IF ports and no RF, except for the mixing products, appears at the OUT port. A balanced mixer is desirable, for example, when it is the first element in a receiver. An unbalanced mixer would allow L.O. energy to feed back into the antenna and the radiation could cause interference to other receivers (and could also reveal the position of the receiver). An unbalanced switching mixer is shown in Figure 5.7(a). It multiplies the signal by a square wave that goes from +1 to 0 (rather than +1 to − 1) which is just a $+\frac{1}{2}$ to $-\frac{1}{2}$ square wave together with a bias of $\frac{1}{2}$. The square-wave term produces the up-shifted and down-shifted bands as before, but the bias term allows one-quarter of the RF input power to get through, unshifted in frequency, to the output.

A simple version of this mixer, using an ideal rectifier, is shown in Figure 5.7 (b). The L.O. and RF voltages are added here, by means of a transformer, and their sum is rectified. The voltage at the output is equal to the sum voltage when the sum is positive and is equal to zero when the sum is negative. If the L.O. voltage is large compared to the RF voltage, the rectifier effectively conducts when the L.O. voltage is positive and disconnects when the L.O. voltage is negative, allowing the resistor to pull down the output voltage to zero. Thus, the RF signal is switched to the output at the L.O. rate. Note, however, that this mixer is totally unbalanced; both the L.O. and RF signals appear at the mixer, together with the sum and difference frequencies.

Switching mixer loss

Let us consider the loss in the switching mixers in Figures 5.4, 5.5, and 5.6. Refer to Figure 5.4(a) and assume that the OUT port is terminated by a load resistor R_L whose value equals R_S, the source impedance of a sine-wave signal source connected to the IN port. Note that the source has no way of knowing that the load resistor is being reversed on half of every cycle of the L.O. The source just sees a matched load and therefore delivers its maximum power. Some of the power on the load is at the desired sum or difference frequency. The ratio of this desired power to the power available from the source is known as the conversion gain. For the diode ring mixer, the ratio is less than unity, i.e., a conversion loss. We can easily calculate this loss. The reversing switch presents the load with a voltage that is half the source voltage (since R_S and R_L form a voltage divider), multiplied by a dimensionless ±1 square wave. Fourier analysis shows that the square wave is made of a sine wave at the square-wave frequency plus a sine wave at every odd multiple of this frequency. The amplitude of the fundamental sine wave is $4/\pi$. The amplitudes of the higher harmonics fall off as $1/n$. Evaluating the product of this square wave and the source voltage we have

$$\begin{aligned} V_{\mathrm{OUT}} &= \tfrac{1}{2}V_S\cos(\omega_R t)\cdot[4/\pi)(\cos(\omega_L t)+3^{-1}\cos(3\omega_L t)+5^{-1}\cos(5\omega_L t)+\ldots] \\ &= \tfrac{1}{2}V_S(4/\pi)\cos(\omega_R t)\cos(\omega_L t)+\ldots \\ &= \tfrac{1}{2}V_S(4/\pi)[(1/2)\cos(\omega_R-\omega_L)t+(1/2)\cos(\omega_R+\omega_L)t]+\ldots \end{aligned} \tag{5.3}$$

We see that the amplitude of the desired sum or difference frequency component is $\tfrac{1}{2}V_S(4/\pi)\cdot(1/2)$. The amplitude available from the source is $\tfrac{1}{2}V_S$, so the conversion gain is the ratio of the squares of these amplitudes:

$$\text{Conversion gain} = \left(\frac{1}{2}\frac{4}{\pi}\frac{1}{2}\right)^2 \Bigg/ \left(\frac{1}{2}\frac{4}{\pi}\frac{1}{2}\right)^2\left(\frac{1}{2}\right)\left(\frac{1}{2}\right)^2 = \left(\frac{4}{\pi}\frac{1}{2}\right)^2 = 0.4052 \tag{5.4}$$

or, in dB, 10 log (0.4052) = −3.92 dB. In practice, the loss is typically greater than this by a dB or so, due to loss in the diodes and in the transformers.

5.3 A simple nonlinear device as a mixer

Finally, let us consider a mixer that uses a single nonlinear device, but not as a switch. Figure 5.8 shows a single-diode mixer. The first op-amp is used to sum the RF and L.O. voltages. The sum is applied to the diode. The input of the second op-amp is a virtual ground so the full sum voltage is applied across the

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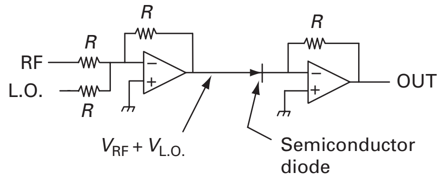

Figure 5.8. Hypothetical single-diode mixer circuit.

diode. With its feedback resistor, the second op-amp acts as a current-to-voltage converter; it produces a voltage proportional to the current in the diode. The current, a nonlinear (exponential) function of the applied voltage, will contain mixing products at frequencies $N\omega_{RF} \pm M\omega_{L.O.}$ where N and M are simple integers. Note that this circuit is essentially the same as that of Figure 5.7(b), except that here we are considering low-level signals, where the diode cannot be treated as an ideal rectifier.

This op-amp circuit is intended to emphasize that the diode's nonlinearity operates on the *sum* of the RF and L.O. Commonly used circuits use passive components and the summing is not always obvious. Diodes are exponential devices; the current vs. applied voltage is given by

$$I = I_s(\exp(V/V_{th}) - 1), \tag{5.5}$$

where $V_{th} = V_{thermal} = kT/e$ (Boltzmann's constant × absolute temperature / electron charge) = 26 mV. The term I_s is a temperature-dependent "saturation current." In a small-signal situation, i.e., when $V \ll 26$ mV, we can expand the exponential to find the output of the above mixer:

$$V_{out} = I_s R[V/V_{th} + (V/V_{th})^2/2! + (V/V_{th})^3/3! + \ldots \tag{5.6}$$

Since $V = V_{RF} + V_{L.O.}$, the first term will give feedthrough (no balance) at both the RF and L.O. frequencies. The second term (the square law term) will produce the desired up-shifted and down-shifted sidebands since the square of $V_{L.O.} + V_{RF}$ contains the cross-product, $2V_{RF}\,V_{L.O.}$. This term also produces bias terms and double frequency components. The third-order term will give outputs at the third harmonics of the RF and L.O. frequencies and at $2\omega_{RF}+\omega_{L.O.}$, $2\omega_{RF} - \omega_{L.O.}$, $2\omega_{L.O.}+\omega_{RF}$, and $2\omega_{L.O.} - \omega_{RF}$. Normally these products are far removed from the desired output band and can be filtered out. If the input voltage is small enough, we do not have to continue the expansion. For larger signals, however, the next term (fourth-order) gives undesirable products within the desired output band. To see how this happens, consider an input signal with two components, $A_1\cos(\omega_1 t)$ and $A_2\cos(\omega_2 t)$. One of the fourth-order output terms will be, except for a constant,

$$\cos(\omega_L t)[A_1 \cos(\omega_1 t) + A_2 \cos(\omega_2 t)]^3. \tag{5.7}$$

You can expand this expression to show that it contains components with frequencies $\omega_L+2\omega_1+\omega_2$ and $\omega_L - 2\omega_1 - \omega_2$. When ω_1 and ω_2 are close to each other, $2\omega_1+\omega_2$ and $2\omega_1-\omega_2$ are nearby and can lie within the desired output band. In a radio receiver, this means that two strong signals will create an objectionable mixing product at a nearby, i.e., inband, frequency which will impede the reception of a weak signal at that frequency.

Simple mixers can also be made with a transistor. A bipolar transistor, if driven by the sum of the RF and L.O. voltages, will have a collector current containing the same set of frequency components as the diode mixer discussed above. Sometimes a dual-gate FET is used as a mixer; the L.O. voltage is applied to one gate and the RF voltage is applied to the other. This provides some isolation between L.O. and RF (which is provided automatically in a balanced mixer such as the diode ring mixer).

We will see later that multiplication, the basis of mixing, is also the operation needed to modulate the amplitude of a *carrier* sine wave, i.e., to produce amplitude modulation (AM). Multiplication, mixing, and AM modulation are all the same basic operation.

Problems

Problem 5.1. Sometimes two multipliers, two phase shifters, and an adder are used to build a mixer that has only one output band (a so-called single-sideband mixer). The design for an *upper* sideband mixer, for example, follows directly from the identity:

$$\cos([\omega_{\mathrm{RF}} + \omega_{\mathrm{L.O.}}]t) = \cos(\omega_{\mathrm{RF}}t)\cos(\omega_{\mathrm{L.O.}}t) - \sin(\omega_{\mathrm{RF}}t)\sin(\omega_{\mathrm{L.O.}}t).$$

Draw a block diagram for a circuit that carries out this operation.

Problem 5.2. The diode ring switching mixer also works when the L.O. and RF ports are interchanged. Explain the operation in this case.

Problem 5.3. Show that the diode ring switching mixer will work if the L.O. frequency is one-third of the nominal L.O. frequency. This is sometimes done for convenience if this $\frac{1}{3}\omega_{\mathrm{L.O.}}$ frequency is readily available. Find the conversion gain (loss) for this situation. Why would this scheme not work if the L.O. frequency is half the nominal L.O. frequency?

Problem 5.4. Consider a situation where two signals of the same frequency but with a phase difference, θ, are separately mixed to a new frequency. Suppose identical mixers are used and that they are driven with the same L.O. signal. Show that the phase difference of the shifted signals is still θ.

Problem 5.5. In RF engineering, considerable use is made of the trigonometry identities $\cos(a+b)=\cos(a)\cos(b)-\sin(a)\sin(b)$ and $\sin(a+b)=\sin(a)\cos(b)+\cos(a)\sin(b)$. Prove these identities, either using geometric constructions or using the identity $e^{jx}=\cos(x)+j\sin(x)$.