CHAPTER

17 Small-signal RF amplifiers

In this chapter we discuss the amplifiers used commonly in the front-end and IF stages of receivers and in antenna-mounted preamplifiers. The maximum output power of these amplifiers is typically from 0.01 W to 0.1 W (10–20 dBm). The power amplifiers discussed in Chapter 3 use the full range of the transistor conductance to "push" or "pull" the output voltage to any value from zero to the dc supply voltage(s). Small-signal amplifiers, on the other hand, are class-A amplifiers in which the signal voltages are small, compared with the dc bias voltages. The small ac signals add to dc bias voltages, so the output signal, δV_{out}, produced by an input signal δV_{in} is given by $\delta V_{out} = [\mathrm{d}V_{out}/\mathrm{d}V_{in}]\ \delta V_{in} + 1/2\ [\mathrm{d}^2 V_{out}/\mathrm{d}V_{in}{}^2]\ (\delta V_{in})^2 + \cdots$ The ac voltage gain is, therefore, $\mathrm{d}V_{ou}t/\mathrm{d}V_{in}$, evaluated at the quiescent bias point. When operated over only a small range of δV_{in}, the higher derivatives of V_{out} versus V_{in} make only small contributions, and the amplifier is essentially linear. Key characteristics of these amplifiers are gain, bandwidth, input and output impedances, linearity (those higher derivatives), and internally generated noise.

17.1 Linear two-port networks

Small-signal amplifiers are linear amplifiers; the output signal should be a faithful reproduction of the input signal.[1] A general definition of small-signal amplifiers could be that they are amplifiers built entirely of nominally linear elements (which include resistors, capacitors, inductors, transmission lines, and transistors operated over a small differential range), from which it follows that the overall circuit will also be nominally linear. An amplifier, being an example of a two-port network (or simply a "two-port"), has an input terminal, an output

[1] While small-signal amplifiers are linear almost by definition, an important exception is the limiting amplifier or *limiter*. In these amplifiers, the gain decreases for increasing signal levels. A cascade of limiters can have an output level almost independent of input level. A limiter is used ahead of an FM detector if the particular FM detector is sensitive to amplitude variations as well as frequency variations.

terminal, and a common terminal (ground). The operation of any linear two-port can be described by four variables: the input and output voltages and currents. Any two of these variables can be considered independent variables ("input" or "cause"). The remaining two variables are then dependent variables ("output" or "effect"). If, for example, V_1 and V_2 are chosen as the dependent variables, the two-port is described by the equations:

$$V_1 = Z_{11}I_1 + Z_{12}I_2 \tag{17.1}$$

$$V_2 = Z_{21}I_1 + Z_{22}I_2. \tag{17.2}$$

For this choice of dependent variables, the four coefficients are known as the *Z parameters*. We are implicitly dealing with ac circuit analysis, so these four parameters generally are complex and are functions of frequency. The important point to note is that, for a given frequency, *any* two-port network (amplifier, filter, transmission line, etc.) can be completely described by just four complex numbers. By convention, the current at either terminal is positive when it flows *into* the terminal. Note that the output variables are linear functions of the input variables, since the input variables appear raised only to the first power.[2] By inspection of Equation (17.1) we see that Z_{11} is the network's input impedance when the output current is zero, i.e., when the output is open circuited. The *forward transfer impedance*, Z_{21}, is the open-circuit output voltage divided by the input current – a "transimpedance." If we are given the load impedance, we can use Equations (17.1) and (17.2) to calculate the power gain of an amplifier (Problem 17.1). The *reverse transfer impedance*, Z_{12}, is a measure of reverse feedthrough. If the RF amplifier preceding an unbalanced mixer in a superheterodyne receiver has reverse feedthrough, some power from the local oscillator will get to the antenna and be radiated. But what is more important, reverse feedthrough can cause an amplifier to oscillate for certain combinations of input and output terminations. This two-port formalism provides more than just a top-level description of an amplifier. It is the basis for amplifier circuit analysis and design, since the active devices (transistors) inside the amplifier can themselves be represented as two-port networks, whose parameters are furnished by the manufacturer on data sheets. Another equivalent set of parameters is the *Y*-parameters, defined by

$$I_1 = Y_{11}V_1 + Y_{12}V_2 \tag{17.3}$$

$$I_2 = Y_{21}V_1 + Y_{22}V_2. \tag{17.4}$$

Conversion formulas between parameter sets are easily derived. For example,

[2] The dependence of the output variables on only the first power of the input variables follows from a general definition of linearity: If an input a causes an output A and an input b causes an output B, then an input C_1a+C_2b, where C_1+C_2 are constant*s*, will result in an output C_1A+C_2B.

$$Y_{11} = \left.\frac{I_1}{V_1}\right|_{V_2=0.} = \left[Z_{11} - \frac{Z_{21}Z_{12}}{Z_{22}}\right]^{-1}. \tag{17.5}$$

The widely used *S-parameters*, which are the subject of Chapter 28, form another equivalent four-parameter set, for which the variables are linear combinations of the voltages and currents, and correspond to input and output waves at each port. Two of the parameters, S_{11} and S_{22}, are reflection coefficients, while the other two are transmission coefficients. A characteristic impedance, Z_0, usually 50 ohms, is implicit. Again, conversion formulas are readily derived, for example,

$$S_{11} = \frac{Y_{22} + Z_0^{-1} - Z_0(Y_{11}Y_{22} + Y_{11}Z_0^{-1} - Y_{12}Y_{21})}{Y_{22} + Z_0^{-1} + Z_0(Y_{11}Y_{22} + Y_{11}Z_0^{-1} - Y_{12}Y_{21}}. \tag{17.6}$$

In this introductory chapter, we discuss amplifiers in terms of voltages and currents, in the interest of presenting the basic concepts in terms totally familiar to the reader.

17.2 Amplifier specifications – gain, bandwidth, and impedances

The small-signal gain (forward and reverse), bandwidth, input impedance and output impedances could be called “linear specifications” because they can all be calculated from the amplifier’s Z-parameters or from any of the other equivalent sets of parameters. The gain and bandwidth of an amplifier are ultimately limited by the characteristics of the transistor(s). Transistors have unavoidable built-in reactances: there are at least two capacitors in even the simplest transistor circuit models (approximate equivalent circuits). Elaborate models for microwave transistors can contain a dozen capacitors and inductors. Amplifiers designed for narrowband use (fractional bandwidths of 20% or less) use input and output matching networks to absorb or “cancel” these reactances. At higher frequencies, the shunt capacitive reactances become lower. The matching networks must then necessarily have higher loaded Qs which means that bandwidth decreases. This limitation is fundamental; no matter how complicated the matching network, gain must be traded for bandwidth. Negative feedback around a transistor will lessen the effect of its reactances. But feedback decreases the gain so again there is a tradeoff between gain and bandwidth. In some applications the input and output impedances of an amplifier are critical. For example, if a narrowband filter is placed between two amplifiers, the amplifiers must present the proper impedances to the filter if the intended passband shape is to be realized. The frequency dependences of the input and output impedances of an amplifier are, of course, related to the bandwidth, since the frequency response is normally determined by mismatch (i.e., reflection).

17.2.1 Amplifier stability

An amplifier is required to be stable (not oscillate) in its working environment. A 100-MHz amplifier, for example, will not be satisfactory if it oscillates, even at a very different frequency, say 1 GHz. Oscillation invariably takes the circuit into large-amplitude excursions and the combination of amplification and oscillation is highly nonlinear. An amplifier that remains stable when presented with *any* combination of (passive) source and load impedances (but no external feedback paths) is said to be *unconditionally stable*. Unconditional stability is not always necessary. An IF amplifier in a receiver needs only to be stable in its never-changing working environment. The input RF amplifier in a short-wave radio, however, might be connected to any random arbitrary antenna so it should be unconditionally stable, at least with respect to input impedance. General-purpose commercial modular amplifiers are usually designed to be unconditionally stable. Using these, a system designer can realize a needed transfer function by cascading amplifiers, filters, etc., and know that the combination will be stable. Stability, like gain and input and output impedances, is predictable from the two-port parameters. To find whether an amplifier will be unconditionally stable, it is necessary and sufficient to show that the real parts of the input and output impedances for any frequency are positive for any passive load and source impedances. Suppose that an analysis shows that for some combinations of load impedance and frequency, the real part of the input impedance is negative, but never more negative than −5 ohms. Then adding a series resistor of more than 5 ohms to the input of the amplifier would make it unconditionally stable. Such resistive remedies, however, always decrease gain and increase internally generated noise. The reverse transfer parameter (Z_{12}, Y_{12}, S_{12}, …) plays a key role in stability. For example, a sufficient (but not necessary) condition for unconditional stability is that its reverse transfer parameter be equal to zero. It is important to note, however, "unconditional stability" simply means that the two-port cannot be provoked into oscillation by varying its termination impedances. A multistage amplifier circuit could contain an oscillating internal stage and still have input and output impedances with positive real parts for all frequencies and arbitrary terminations. (Of course a multistage amplifier will be unconditionally stable if every stage is unconditionally stable.)

17.2.2 Overload characteristics

Any amplifier will become nonlinear at high enough signal levels, if only because the output runs up against the "rail" of the dc power supply. But before this occurs, transistor nonlinearity comes into play. A straightforward specification of an amplifier's upper power limit is the *1-dB compression point*. This is the value of the output power at which the gain has dropped by 1 dB, i.e., the point at which the output power is 79.4% of what would be predicted on the basis of low-power gain measurements.

Intermodulation

Small departures from linearity, even when the amplifier is far below compression, become a concern in a receiver when the passband of an RF amplifier contains two or more signals at frequencies f_a, f_b, f_c, … that are much stronger than the desired signal (the signal that will be isolated downstream by a narrow bandpass filter). Nonlinearity can produce mixing products at frequencies of $n_a f_a + n_b f_b + n_c f_c + \cdots$ where $n_i = 0, \pm 1, \pm 2, \ldots$ In receivers, the most troublesome of these products are the third-order products $2f_a - f_b$ and $2f_b - f_a$. (Third-order products will be inevitably produced if the output voltage of an amplifier contains even a small term proportional to the cube of the input voltage.) The special problem with these particular products is that they can fall within the IF passband. To see this, suppose f_2 and f_3 are the frequencies of signals close enough to a desired frequency, f_1, that they will pass through the broadband front-end of a receiver. The local oscillator is tuned to convert f_1 to the center of the IF passband. But the third-order products $2f_3 - f_2$ and/or $2f_2 - f_3$, being very close to f_1, can also fall within the narrow IF passband and interfere with the desired signal. Second-order intermodulation is not so troublesome since the products have frequencies far outside the IF passband.

A standard measurement of intermodulation is the *two-tone test*, which uses two closely spaced signals of equal amplitude, A. On a log-log plot of output power versus input power each of these fundamental signals will fall on a 45° line, with slope = 1. The third-order products, however, will fall on a line with slope = 3 because the power in the third-order products is proportional to the cube of the power in each of the input signals. The *third-order intercept* is the point at which the third-order product would have as much power as each of the fundamental signals. Usually the number given for the intercept point is the *output* signal strength. The second-order intercept is defined the same way. Figure 17.1 shows a third-order intercept point of about +37 dBm. Generally an

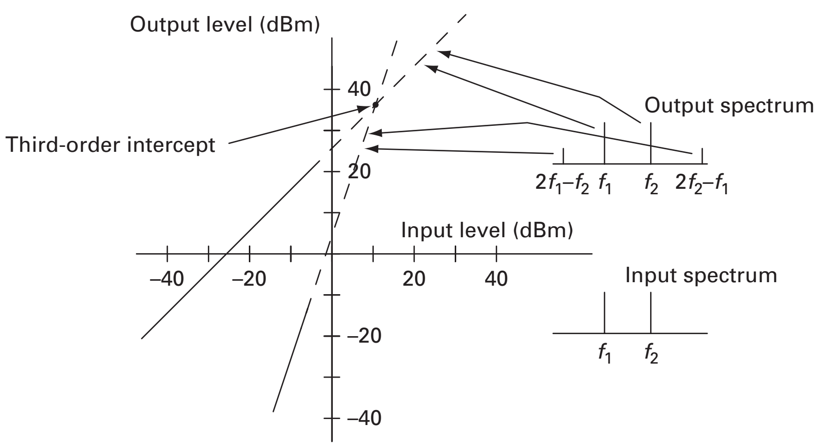

Figure 17.1. Two-tone test to specify amplifier linearity.

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amplifier cannot be driven all the way to the intercept points; they are extrapolations from measurements made at much lower input levels. (The output strengths of the fundamental and the second- or third-order product need only be measured at one input level. Lines with slopes of one, two, or three are then drawn through them to locate the intercepts.)

Dynamic range

Every amplifier adds some noise to the signal. (Later we will discuss amplifier noise in some detail.) Very weak signals will be buried in this noise and lost. The *dynamic range* of an amplifier is therefore determined at the low end by the added noise and at the high end by nonlinearity. In order to handle strong signals, a receiver should keep mixing products small by having as little amplification as possible prior to the narrowest bandpass filter. We will see, on the other hand, that if a receiver begins with a mixer or with a narrowband filter, the loss in these elements adds noise and will render the receiver less sensitive than if the first element after the antenna had been a low-noise amplifier. A trade-off must often be made between sensitivity and dynamic range. Power dissipation is obviously important for battery-operated equipment where milliwatts may count. But to achieve high dynamic range, a small-signal amplifier may have a fairly high-power quiescent point and have to dissipate as much as several watts of power.

17.3 Narrowband amplifier circuits

Amplifiers for frequencies below about 30 MHz look very much like resistance-coupled audio amplifiers. The load resistors are replaced by shunt inductors which cancel the transistor capacitances, which would otherwise tend to be short circuits at RF. These resonant circuits make a narrowband amplifier. Often an even narrower bandpass is desired; the inductors are given smaller values and are shunted with external capacitors (effectively increasing the transistor capacitances). Focusing on one stage of an amplifier (or an amplifier of one stage), the fundamental design decisions are transistor selection and circuit configuration, i.e., common-emitter, common-base, or common-collector. (Here, and usually elsewhere, *emitter, base*, and *collector* can also mean *source, gate, and drain*.) The choice of a transistor will be based on the ability to provide gain at the desired frequency, noise, and perhaps linearity. The orientation of the transistor might be common-emitter when maximum gain is required, common-base when the device is being pushed near its upper frequency limit or when the isolation between input and output is critical, or common-collector when very low output impedance is needed. As far as noise goes, it turns out that the three orientations are equivalent when used in a high-gain multistage amplifier.

17.4 Wideband amplifier circuits

Most wideband amplifiers use feedback. An unbypassed emitter impedance provides series feedback. An impedance between collector and base provides shunt feedback. Commercial modular general-purpose amplifiers use resistive series and shunt feedback. These amplifiers are quite flat up to one or two GHz and have input and output impedances close to 50 ohms over the whole range. Resistive feedback is simple but degrades the noise performance of an amplifier. Wideband low-noise amplifiers often use feedback networks made only of lossless elements, i.e., reactors. The Miller[3] effect multiplies the effective input capacitance in a common-emitter amplifier. This capacitance can be neutralized, at least in a narrowband amplifier. Wideband amplifiers often use the *cascode* circuit in which a common-emitter input stage drives a (low impedance) common base stage. Another good high-frequency circuit, the differential pair, uses an emitter follower stage (high input impedance and low output impedance) to drive a common-base stage.

17.5 Transistor equivalent circuits

An amplifier designer needs a precise electrical description of the transistor(s). For analysis, it is sufficient to have tables of the small-signal parameters of the transistor(s). These tables are usually given in data sheets from the manufacturer; they can be produced using a vector network analyzer. A table of numbers, however, is an awkward representation for design (synthesis) and a common tactic is to represent the transistor by an (approximately) equivalent model circuit of resistors, capacitors, inductors, voltage-controlled voltage generators, voltage-controlled current generators, etc. An exact equivalent circuit for a single frequency can be constructed directly from the small-signal parameters corresponding to that frequency (Problem 17.4). This might be an adequate model for the design of a narrowband amplifier but remember that even an amplifier intended for only a narrow frequency range must be stable at *all* frequencies. For this reason, and also to aid in the design of wideband amplifiers, models are constructed to represent the transistor over a wide frequency range. Normally the topology of an equivalent circuit is based on the construction and physics of the transistor. The element values are determined by least-square fitting programs that make the small-signal parameters of the model agree as closely as possible with the measured small-signal

[3] The collector signal in a common-emitter amplifier has a larger magnitude (due to amplification) than the base signal. It also has the opposite sign. The voltage across the transistor's inherent base-to-collector capacitance is therefore larger than the base voltage. As a result, more current flows in this capacitance than if the collector were grounded. The value of the capacitance is, in effect (Miller effect), multiplied.

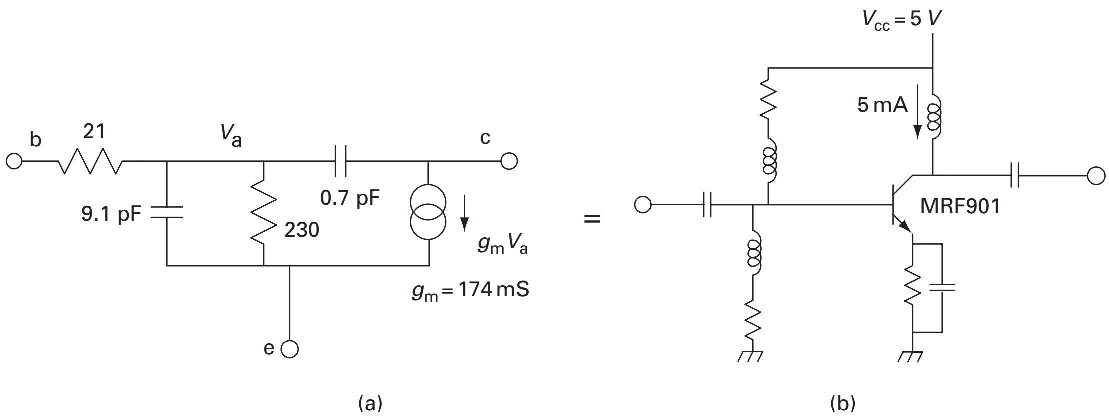

Figure 17.2. Simple equivalent circuit model for the MRF901 bipolar transistor (100 MHz-2 GHz).

parameters of the actual transistor over the desired frequency range. Agreement can always be improved by adding more elements to the model, but an overly-complicated model will block the intuition of the designer. Equivalent circuits have from one to perhaps twenty parameters. Figure 17.2(b) shows a simple "hybrid-π" equivalent circuit model for a common high-frequency transistor. The component values in the model circuit were determined by least-square fitting to the data sheet values over the range from 100 to 2000 MHz. Since the equivalent circuit models a *biased* transistor, it is actually equivalent to the circuit like that of Figure 17.2(a), which includes a power supply and dc biasing components. The biasing components include three resistors to set the dc collector current. A bypass capacitor grounds the emitter at RF frequencies. Blocking capacitors keep the dc bias voltages from interacting with circuitry outside the transistor and vice versa. An RF choke (inductor) provides enough reactance that practically no signal current can flow from the collector to V_{cc} (RF ground). Two more two chokes prevent RF currents from flowing in the base bias resistors. (Usually these resistors have high enough values that these chokes are unnecessary.)

17.6 Amplifier design examples

Designing a small-signal amplifier for microwave frequencies can be difficult, especially when the design must meet specifications for frequency response, gain, stability, input and output impedances, and noise. Computer-aided design is often used. But, to provide an example, let us design a simple amplifier using the transistor model of Figure 17.2. The amplifier is to be driven from a 50-ohm source and is to drive a 50-ohm load. The only other specifications imposed are that the power gain be at least 10 dB at 430 MHz, and that the amplifier not oscillate while connected to the specified source and load. Let us try a very simple common-emitter circuit consisting of the transistor with a matching inductor in series with the collector (rather than going with a two-element

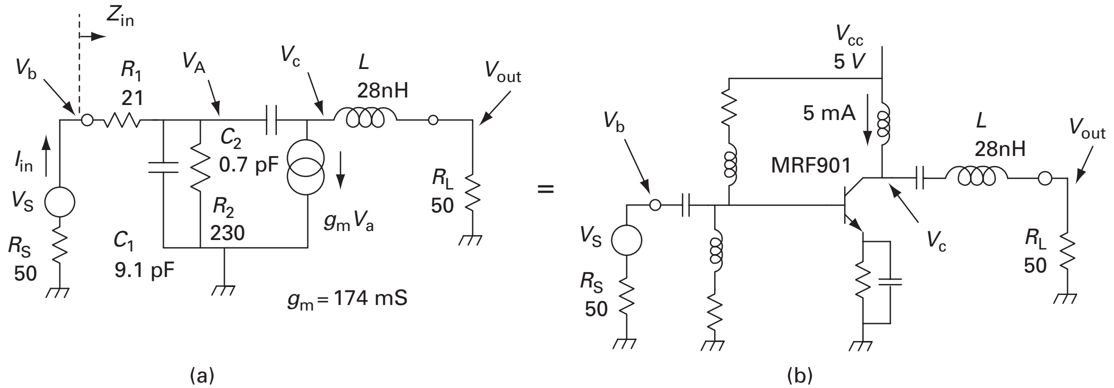

Figure 17.3. Common-emitter amplifier design example: (a) small signal equivalent circuit; (b) full circuit.

matching circuit on each side). The small-signal equivalent circuit of the amplifier is shown in Figure 17.3(a), while the complete amplifier, including bias circuitry, is shown in Figure 17.3(b).

The circuit of Figure 17.3(a) can be analyzed to find V_c in terms of V_b. The output voltage, V_{out}, is then simply $V_c R_L/(R_L + j\omega L)$. Note first that the currents to ground must sum to zero or

$$\frac{V_A - V_b}{R_1} + \frac{V_A}{R_2} + V_A j\omega C_1 + V_A g_m + \frac{V_c}{j\omega L + R_L} = 0. \tag{17.7}$$

Next, consider the current flowing to the left through C_2. This gives us the equation

$$(V_c - V_A) j\omega C_2 = \frac{V_A - V_b}{R_1} + \frac{V_A}{R_2} + V_A j\omega C_1. \tag{17.8}$$

Solving these two equations for V_c as a function of V_b, we find

$$V_c = \frac{-V_b(g_m - j\omega C_2)/R_1}{j\omega C_2(1/R_1 + 1/R_2 + j\omega C_1 + g_m) + (1/R_1 + 1/R_2 + j\omega(C_1 + C_2))/(R_L + j\omega L)}. \tag{17.9}$$

From inspection of Figure 17.3, we see that $V_{out} = V_c R_L/(R_L + j\omega L)$ and that the input current, I_b, is given by $I_b = (V_b - V_A)/R_1$. Since we have already found V_c as a function of V_b, Equation (17.7) (or 17.8) gives us V_A, which lets us calculate I_b and also the input impedance, $Z_{in} = V_b/I_b$. The input power (drive power delivered to the amplifier) is given by $|I_b|^2$ Re(Z_{in}). The power gain of the amplifier is then calculated from

$$\text{Power Gain} = \frac{\text{power out}}{\text{power in}} = \frac{|V_{out}|^2/R_L}{|I_b|^2 \text{Re}(Z_{in})}. \tag{17.10}$$

These expressions are easy to evaluate using a program such as Mathcad or MATLAB. Using Equation (17.9), we find that the power gain at 430 MHz reaches a maximum of 17.25 dB for $L = 28$ nH. This is within 0.25 dB of the

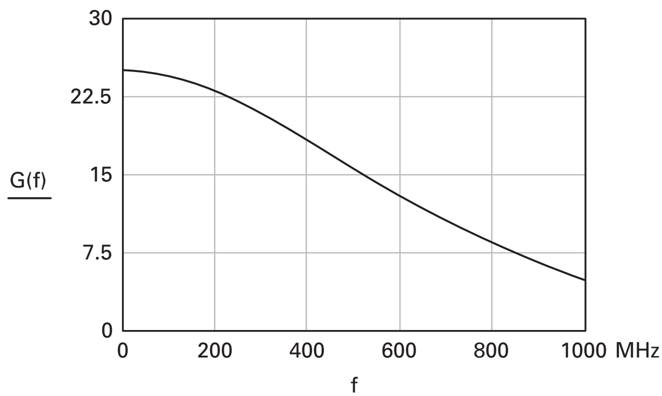

Figure 17.4. Power gain vs. frequency for common-emitter amplifier design example.

maximum power gain available from this transistor (which could be obtained by using a two-element output matching network rather than just a series inductor). Figure 17.4 shows the power gain versus frequency. The input impedance is 13.3–17.0j, from which you can calculate that the output power can be increased by 2.18 dB by the addition of an input matching network to transform the 50-ohm source impedance to 13.3+17.0j.

Analysis of even this simple amplifier requires a fair amount of algebraic effort. Problem 17.5 shows you how to add the hybrid-π transistor model to the circuit analysis program of Problem 1.3. This addition will let you plot the frequency response and find input and output impedances of this amplifier or any arbitrary cascade of transistors and other two-port devices.

We can check the stability of this amplifier when connected to the specified 50-ohm source and load impedances by verifying that any arbitrary set of initial conditions (two capacitor voltages and one inductor current, at $t=0$) results in transient currents that decay exponentially rather than grow exponentially. Because the circuit is linear, we know that there will be transient solutions in which the time dependence goes as $e^{j\omega t}$, where ω is a *characteristic frequency*. If ω has a positive imaginary part, the solution decays exponentially. To find the characteristic frequencies, we set the source voltage, V_s, equal to zero, and assume that V_A and V_B in Equations (17.7) and (17.8) are proportional to $e^{j\omega t}$. Solving these equations results in a third-order polynomial in ω which must be equal to zero. The three roots of the polynomial turn out to be $\omega_1 = j\,2.64E9$, $\omega_2 = j2.16E9$, and $\omega_3 = j18.1E9$. This shows that all three of these transient solutions will decay exponentially, since their imaginary parts of the three frequencies are all positive.[4] For example, $e^{j\omega_1 t} = e^{-2.64E9\ t}$, which is an exponential decay with a time constant of 0.38 nsec. A superposition of these three particular solutions that satisfies an arbitrary initial set of initial conditions will,

[4] Here, the three roots are purely imaginary. In general, they also have real parts, corresponding to decaying *oscillations*. For example, if the load resistance is changed from 50 to 500 ohms, the three roots become j2.44e8 and ±6.89e9 + j1.94e10.

therefore, produce a decaying transient and this circuit will be stable with the 50-ohm source and load impedances.

Let us check stability if we add an input matching network consisting of a series inductor with 17 ohms of reactance to cancel the $-$j17.0 input reactance and a transformer to step the 50-ohm source down to 13.3-ohm input resistance. With the addition of a fourth reactive element, the circuit will now have four characteristic frequencies. Straightforward algebraic manipulations produce a fourth-order polynomial in ω whose four roots are $\pm$4.18E9 +j7.83E8, j2.28E10, and j2.48E10. Since the imaginary parts are positive, the amplifier remains stable with this input matching network and the amplifier's *available gain* (power available from the amplifier divided by power available from the source) becomes equal to the power gain. Chapter 28 discusses how stability is usually evaluated in the context of S parameters.

Figure 17.5(a) shows an amplifier using the same transistor model arranged in a common-base configuration; the base is grounded and the input signal is applied to the emitter. The equivalent small-signal circuit is shown in (b).

You can analyze this circuit with the methods used above for the common-emitter configuration. Write one equation that sets to zero the sum of all the currents away from node "A." Do the same for node "c" (the collector terminal) and solve the two equations simultaneously for V_c as a function of V_e (the emitter voltage).

Figure 17.6 shows a model for a microwave GaAS FET, which can be used in place of the BJT model for the same kind of amplifier analysis. Manufacturers'

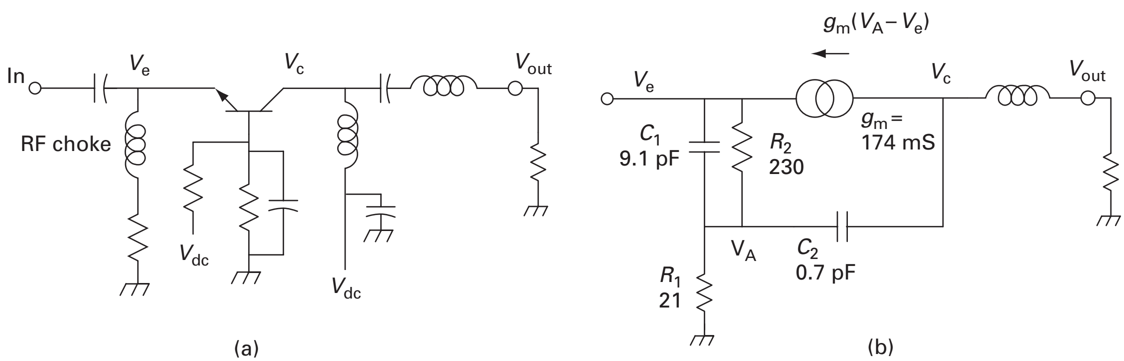

Figure 17.5. A common-base RF amplifier.

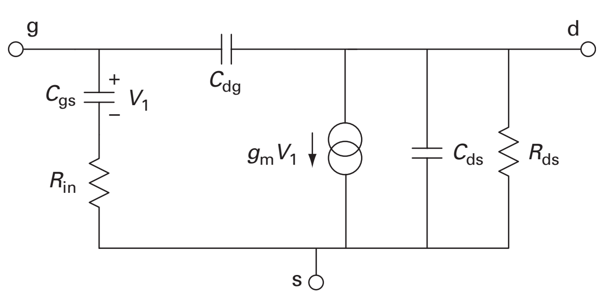

Figure 17.6. High-frequency GaAs FET model.

data sheets often include model circuits along with two-port data. There are many excellent textbooks devoted entirely to the amplifier design.

17.7 Amplifier noise

The output signal from any amplifier will always include some random noise generated within the amplifier itself. Most of the hiss from a radio receiver is due to noise generated by atmospheric electricity. But if the antenna is disconnected, the noise does not entirely disappear. The remainder is being generated within the receiver. Physical mechanisms that cause this noise include thermal noise (discussed below) and shot noise, which is noise due to the randomness in the flow of discrete charges – electrons and holes in transistors. The first stage in most receivers is an RF amplifier, and its noise usually dominates any other receiver-generated noise. This is easy to see; since this stage usually has considerable gain, its output power will be much greater than the noise power contributed by the second stage, so the second stage will hardly change the signal-to-noise ratio. In the same way, noise contributed by the third stage is even less important, and so forth.

Thermal noise

Thermal noise is such a universal phenomenon that it provides the very vocabulary for the definition of terms such as *receiver noise temperature* and *antenna noise temperature*. Let us examine the fundamentals of thermal noise. Any object, being hotter than absolute zero, converts thermal energy (molecular vibrational energy) into electromagnetic radiation: at infrared radiation for ordinary temperatures, but also visible radiation, if the object is red hot. Likewise, a resistor can convert thermal energy into electrical power. If two resistors are connected in parallel, each one delivers a tiny amount of electrical power (in the form of a random voltage waveform) to the other. If they are at equal temperatures, the power flow is equal in both directions.

How much power can a resistor generate by virtue of being hot? Answer: Any resistor can deliver kT watts per hertz, that is, kT is the spectral density of the power that a resistor of R ohms will deliver to a matched load (a load of impedance R+j0). Here k is Boltzmann's constant (1.38×10^{-23} joule/kelvin $= 1.38 \times 10^{-23}$ watt seconds/kelvin) and T is the absolute temperature. It is useful to remember that for $T_0 = 290$ K, which is universally taken as a standard reference temperature, the power available from a resistor, referred to 1 mW, is $= -114$ dBm/MHz, since $10 \log(1.38 \times 10^{-23} \times 10^3 \times 290 \times 10^6) = -114$.

To demonstrate that a resistor should produce this power, kT watts/Hz, an argument appropriate to radio engineering considers an antenna surrounded by a blackbody, i.e., an antenna within a cavity whose walls are at temperature T_1. We will be concerned with the spectral density at a particular spot frequency so

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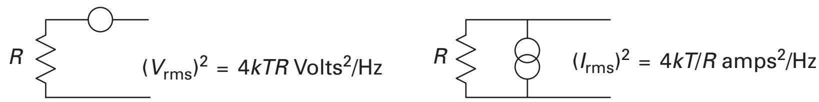


Figure 17.7. Equivalent circuits for a resistor as a noise source.

we can specify that the antenna be resonant, i.e., that it have a purely resistive impedance, R+j 0, at that frequency. Let a transmission line connect the antenna to a resistor R which is outside the blackbody but also at temperature T_1. We know the antenna will intercept blackbody radiation and that power will be transmitted through the line to the external resistor. We also know from thermodynamics that, in this isolated system, it is impossible for the resistor to get hotter than T_1; heat cannot flow from a colder to a hotter object. The only way to resolve this is for the resistor to produce an equal amount of power, which travels back to the antenna and is radiated back into the cavity. We can use some antenna theory to calculate the power. All antennas are directive; when used to receive, they have more effective area to intercept power from some directions than from other directions. But for any antenna, the average area turns out to be $\lambda^2/4\pi$ where λ is the wavelength. Blackbody radiation flux at long wavelengths is given by the Rayleigh–Jeans law, brightness = $2kT/\lambda^2$ watts/m^2/Hz/steradian. This includes power in two polarizations. Since any antenna can respond to only one polarization, we use half the Rayleigh–Jeans brightness to calculate the power the antenna puts on the transmission line:

$$P = \int \frac{B(\theta,\phi)}{2} A(\theta,\phi)\mathrm{d}\Omega = \frac{kT}{\lambda^2}\frac{\lambda^2}{4\pi}4\pi = kT. \tag{17.11}$$

This value, kT, is then also the power a resistor of R ohms will deliver to another resistor of R ohms. It follows that the open-circuit noise voltage from a resistor is therefore $\langle V_n{}^2\rangle = 4kTR$ volt^2/Hz. Figure 17.7 shows the Thévenin and Norton equivalent circuits of a resistor as a noise generator.

17.8 Noise figure

At any given frequency, a figure of merit for a receiver, an amplifier, a mixer, etc., is its *noise figure*, whose definition is as follows:

Noise figure is the ratio of the total output noise power density to the portion of that power density engendered by the resistive part of the source impedance, with the condition that the temperature of the input termination be 290 K.

Noise figure is a function of frequency and of source impedance but (as we will see later) is independent of output termination. Consider Figure 17.8.

The voltage source V_n represents the thermal noise voltage from R_s, the resistive part of the source impedance. The source, V_s, is the actual signal voltage, if any. The internal noise of the amplifier can be considered to result

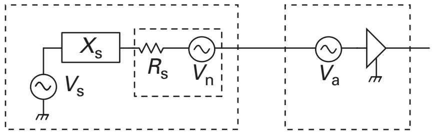


Figure 17.8. Equivalent circuit of an amplifier and signal source.

from another equivalent input noise source, V_a, at the input of the amplifier. With this model, the noise figure, as defined above, can be written in terms of V_n and V_a: $F = ({V_n}^2 + {V_a}^2)/{V_n}^2$. Note that, because the amplifier noise is represented by an equivalent generator at the input side, this expression does not contain G, the gain. Since ${V_n}^2$ is proportional to the source temperature, T_0, it is natural to assign the amplifier an equivalent noise temperature by writing the noise figure as $F = (T_0 + T_a)/T_0$. This amplifier noise temperature is just $T_a = (F - 1)$ $T_0 = (F - 1) \times 290$ K. Conversely, the noise figure is given by $F = (290 + T_a)/290$. An amplifier can have a noise temperature less than its physical temperature. The dish-mounted amplifiers used for home satellite reception have typical noise temperatures of 30 K. (Refrigeration, however, can help; FET amplifiers for radio astronomy are often physically cooled to about 10 K and produce noise temperatures of only a few kelvins.)

So far we have not mentioned the signal voltage, V_s. Equation (17.12) shows that the noise figure also specifies the ratio of the input signal-to-noise ratio to the output signal-to-noise ratio:

$$\frac{\text{Input SNR}}{\text{Output SNR}} = \frac{V_s^2/V_n^2}{V_s^2/(V_n^2 + V_a^2)} = (V_n^2 + V_a^2)/V_n^2 = \text{F}. \qquad (17.12)$$

Cascaded amplifiers

The noise from an amplifier of only modest gain will not totally dominate the noise added downstream, so it is useful to know how noise figures add. Suppose amplifier 1, with noise figure F_1 and gain G_1, is followed by amplifier 2 with F_2 and gain G_2. Suppose further that they are matched at their interface so that $G = G_1 G_2$ and that the output impedance of amplifier 1 is equal to the source impedance corresponding to the specified F_2. Figure 17.9 shows how to compute the overall noise figure.

The noise figure of the cascade is $F_{12} = F_1 + (F_2 - 1)/G_1$. It is interesting to calculate the noise figure of an infinite cascade of identical amplifiers as it is a lower limit to the noise we would get from any shorter cascade. You can verify that T_{infinity}, the equivalent noise temperature of the infinite chain, is given by $T_a/(1 - 1/G)$ where T_a and G refer to the individual identical amplifiers.

Finally, let us look at the overall noise figure of an amplifier preceded by an attenuator, as shown in Figure 17.10. Suppose the gain of the attenuator is G_{attn}. (The gain of an attenuator is less than unity; the gain of a 6-db attenuator, for example, is 1/4.) Referred to the amplifier input, the noise power engendered by

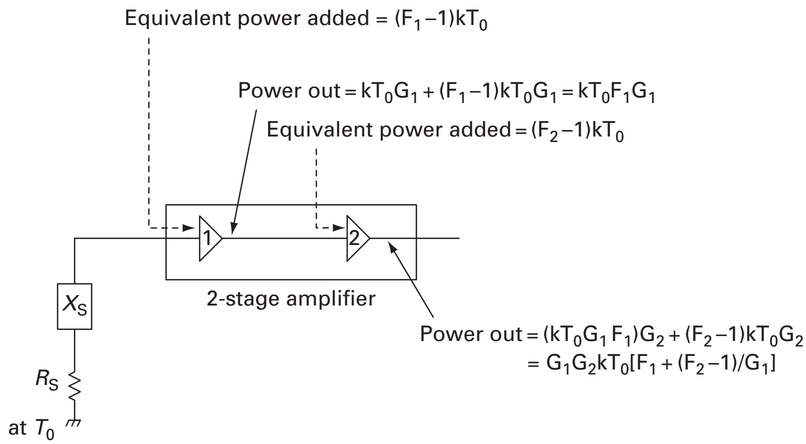


Figure 17.9. Overall noise figure of cascaded amplifiers.

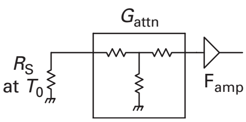


Figure 17.10. An amplifier preceded by an attenuator.

the source resistance is T_0G_{attn}. Since the amplifier still sees its standard source impedance, its total noise, referred to the input, is still $(F_{\text{amp}})T_0$.

The overall noise figure is therefore

$$F_{\text{tot}} = \frac{F_{\text{amp}}T_0}{G_{\text{attn}}T_0} = \frac{F_{\text{amp}}}{G_{\text{attn}}} \tag{17.13}$$

so, if an amplifier is preceded by an M-dB attenuator ($G_{\text{attn}} = 10^{-M/10}$), the noise figure of the combination is M dB higher than the noise figure of the amplifier alone. We could just as well have derived this result by using the relation for cascaded devices. The noise figure of the attenuator, from the definition of noise figure, is $F_{\text{attn}} = kT_0/(G_{\text{attn}}kT_0) = 1/G_{\text{attn}}$. The noise figure of the cascade becomes $F_{\text{tot}} = 1/G_{\text{attn}} + (F-1)/G_{\text{attn}} = F/G_{\text{attn}}$ as before.

17.9 Other noise parameters

In what we have considered so far, the noise produced by a device, a transistor, amplifier, etc. is specified by a single parameter, its noise figure. But the noise figure depends on the source impedance from which the device is fed, which makes this parameter something less than a complete noise description of the device. We will see in Chapter 24 that a total of *four* noise parameters are sufficient to describe a device. The noise figure for any given source impedance can then be calculated from these four parameters which are R_{opt}, X_{opt}, $F_{\min}$, and R_n. The (complex) impedance $Z_{\text{opt}} = R_{\text{opt}} + X_{\text{opt}}$ is the source impedance that yields the minimum noise figure, $F_{\min}$. The "noise resistance," R_n, is a parameter that determines how fast the noise figure increases as the source impedance departs from Z_{opt}. We will see in Chapter 24 that the noise figure for an arbitrary source impedance is given by

$$F = F_{\min} + (R_n/G_s)|Y_{\text{source}} - Y_{\text{opt}}|^2. \tag{17.14}$$

(Here Y_{opt} is just $1/Z_{opt}$ and G_s is the real part of the source admittance.) We will also see that noise figure is somewhat deficient as a figure of merit. A piece of wire has $F=1$ but is not a valuable amplifier since it has no gain. With a given transistor, circuit A might produce a lower noise figure than circuit B, but circuit A may have less gain. We will see that $T_{infinity}$, defined above, is the proper figure of merit.

17.10 Noise figure measurement

A straightforward determination of an amplifier's noise figure is possible if one knows its gain and has a spectrum analyzer suitable for measuring noise power density. Consider the common situation where we have an amplifier to be used in a 50-ohm environment. We connect a 50-ohm load to its input and use the spectrum analyzer to measure the output power density, S_{out}(watts/Hz), at the frequency of interest. We know that the portion of this power density engendered by the input load is kTG, where G is the amplifier gain. The noise figure is therefore given by $F=S_{out}/(kTG)$. This assumes we have done the measurement at $T=290\,\text{K}$. If T was not 290, you can verify that $F=[S_{out}-Gk(\mathrm{T}-290)]/(290Gk)$. For low-noise amplifiers, a comparison method is used. This method requires a cold load and a hot load, i.e., two input loads at different temperatures, T_{hot} and T_{cold}. The amplifier is connected to a bandpass filter (whose shape is not critical) and then to a power meter (which needs to have only relative, not absolute accuracy). The ratio of power meter readings, hot to cold, is called the Y-factor. The noise temperature of the amplifier is then given by:

$$T_a = (T_{hot} - YT_{cold})/(Y-1). \tag{17.15}$$

Problems

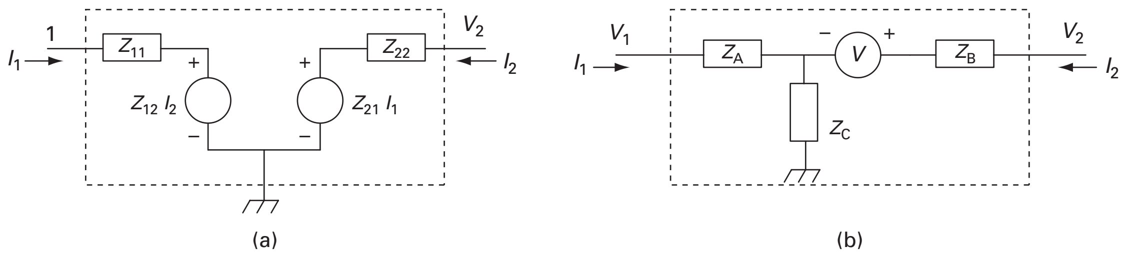
Problem 17.1. Derive an expression for the power gain (output power/input power) for the two-port network described by Equations (17.1) and (17.2) when the load impedance is Z_L.

Problem 17.2. For a general two-port network, derive expressions for the Y parameters in terms of the Z parameters.

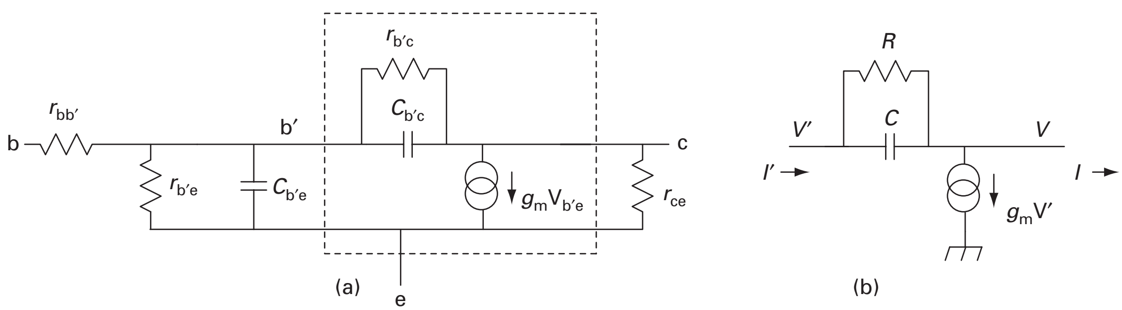
Problem 17.3. (a) A certain amplifier with 20 dB of gain has a third-order intercept of 30 dBm (one watt at the output). If the input consists of 0 dBm (0.001watt) signal at 100 MHz and another 0 dBm signal at 101 MHz, what will be the output power of the third-order products at 102 MHz and 99 MHz?

(b) Same as (a) except that the input signal at 100 MHz increases in power to 10 dBm (0.1 watt) while the input signal at 101 MHz remains at 0 dBm.

Problem 17.4. The Z-parameter description of a two-port corresponds in a one-to-one fashion to the equivalent circuit shown below in (a). Another circuit is shown in (b). Find expressions for Z_A, Z_B , Z_C and V in terms of the Z parameters to make the two circuits equivalent.



Problem 17.5. The figure below at (a) shows a small-signal hybrid-π model for a common-emitter BJT transistor. (This model contains one more component than the model of Figure 17.2.)



The components form a simple ladder network, except for the portion inside the dashed line box. Show, for this box, the circuit in (b), that the relations between the input voltage and current and the output voltage and current are given by

$$V' = \frac{V}{1 - g_m Z} + \frac{IZ}{1 - g_m Z}$$

and

$$I' = \frac{V g_m}{1 - g_m Z} + \frac{I}{1 - g_m Z}.$$

Use these equations to make this box a new circuit element in the analysis program of Problem 1.3. The program will then be able to analyze BJT common-emitter amplifiers. (Note that the other two resistors and the other capacitor in this hybrid-π model can be included in a circuit as if they were external components.)

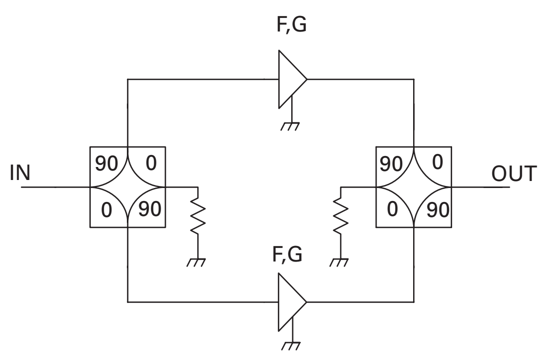
Example answer: For the MATLAB example given in Problem 1.3, add the element, "HY_PI" by inserting the following sequence of statements in the "elseif chain":

```
elseif strcmp(component,HY_PI)==1
ckt_index=ckt_index+1; gm=ckt{ckt_index}; %gm
ckt_index=ckt_index+1; R=ckt{ckt_index};%R
ckt_index=ckt_index+1; C=ckt{ckt_index}; %C
Z=R/(1+1j*w*R*C);
Iold=I; I=(I+gm*V)/(1-gm*Z);V=(V+Iold*Z)/(1-gm*Z);
```

Problem 17.6. Show that the noise figure of an infinite cascade of identical amplifiers is given by $F_\infty = (F - 1/G)/(1 - 1/G)$. Assume that the amplifiers have a standard input and output impedance such as 50 ohms, that G is the gain corresponding to this impedance, that F is the noise figure corresponding to a source of this impedance, and that F_∞ is also to be with respect to this standard impedance. Hint: use the formula for a cascade of two amplifiers and the standard "infinite-chain-of-anything" technique – adding another link does not change the answer.

(This problem is not just academic. With only a few amplifiers the gain will be high enough to make the noise figure very close to F_∞ which is the best possible combination of the given amplifiers.)

Problem 17.7. Consider the balanced amplifier circuit shown below. The 3-dB, 90° hybrids are ideal. The amplifiers are identical and all impedances are matched. The individual amplifiers have power gain G and noise figure F_0. The hybrids are perfect, i.e., they have no internal loss and are perfectly matched.



a. Show that the overall noise figure of this circuit is equal to the noise figure of the individual amplifiers.
b. If one amplifier dies, i.e., provides zero output, what is the overall noise figure?

Problem 17.8. We derived the overall noise figure, $F_{tot} = F\,(G_{attn})^{-1}$, for an amplifier preceded by an attenuator when the physical temperature of the attenuator is T_0, the standard 290 K reference temperature. Assuming now that the attenuator is at some different physical temperature, T_1, show that the overall noise figure is given by $(G_{attn})^{-1}$ $[G_{attn} + (T_1/T_0)(1 - G_{attn}) + \mathrm{F} - 1]$.

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