CHAPTER

24 Amplifier and oscillator noise analysis

A fundamental limitation in amplifiers and oscillators is the internally generated random noise whose physical causes are shot noise in semiconductors and thermal noise (Johnson noise) in resistors. Amplifier noise is a practical concern whenever this noise level is comparable to the signal being amplified. Oscillator noise, characterized as phase or frequency fluctuations, adds noise to FM systems, limits how closely carriers may be spaced in frequency division multiplexing (FDM), and limits the length of signal averaging times in coherent communication systems. In this chapter we discuss the way transistor noise is characterized, treating the transistor itself as a two-port network, and how these characteristics are taken into account to design low-noise circuits.

24.1 Amplifier noise analysis

In Chapter 17 we quoted a formula showing how the noise figure of any active two-port device depends on the impedance of the signal source:

$$F = F_{\mathrm{min}} + (R_n/G_{\mathrm{s}})|Y_{\mathrm{s}} - Y_{\mathrm{opt}}|^2. \tag{24.1}$$

Here F_{min} is the minimum noise figure; $Y_{\mathrm{s}} = G_{\mathrm{s}} + \mathrm{jB_s}$ is the source admittance, $Y_{\mathrm{opt}} = G_{\mathrm{opt}} + \mathrm{jB_{opt}}$ is the source admittance that produces F_{min}, and R_n is a real parameter known as the *noise resistance*. Equation (24.1) applies as well to audio and instrumentation amplifiers as to radio and microwave amplifiers, as long as the four noise parameters of the device, F_{min}, R_{n}, G_{opt}, and B_{opt}, are known for the frequency in question. While Equation (24.1) requires some effort to derive, its use is simple, at least with RF (narrowband) amplifiers.

24.1.1 Noise matching

To design a low-noise RF amplifier, we get out the data sheet for the transistor, find the value of Y_{opt} for the desired frequency, and design an input network to convert the impedance of the intended source (probably 50 ohms) into $1/Y_{\mathrm{opt}}$.

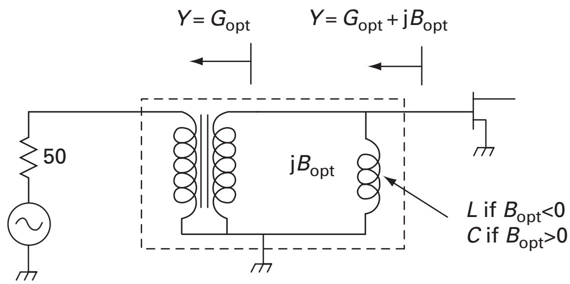


Figure 24.1. Input network to convert a 50-ohm source impedance to $G_{OPT} + jB_{OPT}$.

Figure 24.1 shows how a transformer and a reactor can form the matching network. (Of course there are many equivalent networks.)

With this input network, the transistor is said to be "noise matched," because the noise figure of the overall amplifier will be F_{min}. The input network that provides noise matching generally does not provide impedance matching, i.e., it does not convert the source impedance into the complex conjugate of the amplifier's input impedance. The resulting power mismatch means we will have less signal at the amplifier output than with a conjugate impedance match. But converting the source impedance into $1/Y_{opt}$, rather than into $1/Y^*_{in}$, will reduce the transistor noise more than the mismatch reduces the signal power. The net effect is to maximize the signal-to-noise ratio, i.e., provide the minimum possible noise figure.

(Note: In some cases simultaneous noise matching and impedance matching can be achieved or at least approached. If the device has a nonzero reverse transfer coefficient, the amplifier's input impedance is not fully determined by the input network. Depending on the transistor characteristics, a designer might find an output network to make the input impedance of the amplifier approach Z_0 while using an input network that provides noise matching. The use of feedback can provide additional degrees of freedom with which to pursue simultaneous matching.)

24.1.2 Equivalent circuits for noisy two-port networks

Equation 24.1 was derived by Rothe and Dahlke [4] who pointed out that a noisy two-port network (transistor, amplifier, etc.) can be modeled, for small signals, as an ordinary passive linear two-port with two external frequency-dependent noise sources. These noise sources can be series voltage generators at the input and output, shunt current generators at the input and output, a current source at one end and a voltage source at the other end, or, finally, both a voltage source and a current source at the same end. Figure 24.2 shows these equivalent circuits.

Note that when a current generator and a voltage generator appear on the same side they can be interchanged without changing their parameters, so Figures 24.2(e) and (f) (or g and h) are actually the same circuit.

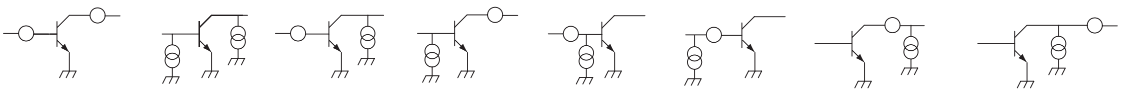


Figure 24.2. Equivalent representations of a noisy transistor as a noiseless transistor with two external noise generators.

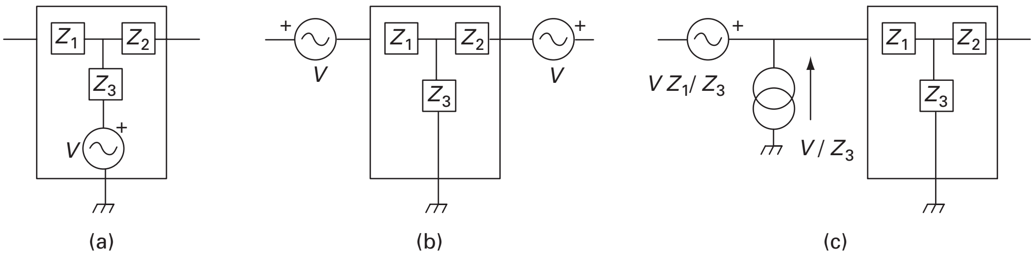


Figure 24.3. (a) An arbitrary two-port network with an internal voltage source; (b) and (c) networks equivalent to (a), but with external sources.

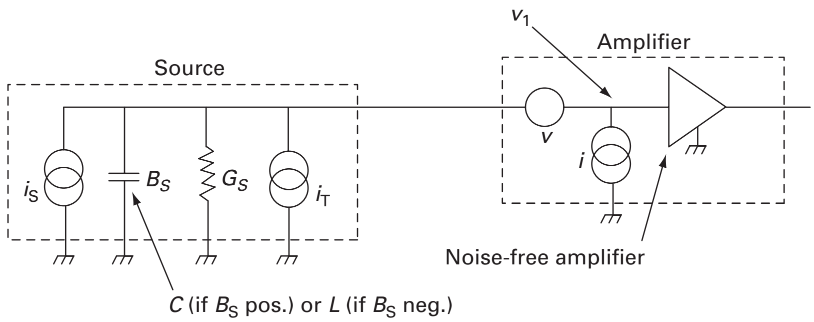
Note: Replacing internal sources with equivalent external sources is a sort of generalization of Thévenin's theorem and Norton's theorem, from one-port networks to two-port networks. Figure 24.3(a) shows an arbitrary two-port network with an internal voltage generator. Both (b) and (c) are equivalent to (a). In this example, the network is passive, but it can just as well be active, containing dependent voltage and current sources. When the internal source is a sine-wave source, the external generators are specified by complex amplitudes. When the internal source is a noise source, the external sources are specified by their rms values and the correlation between them. For example, if the external sources are a voltage source and a current source, the correlation is the complex number $\langle iv^* \rangle$, the time average of the product of the complex amplitude i and the complex conjugate of the complex amplitude v. When a two-port has multiple internal noise sources,[1] the resulting external sources can be consolidated into two external sources.

24.1.3 Noise figure of the equivalent circuit

Figure 24.2(e) (or 24.2f) is the easiest equivalent circuit to analyze for noise figure. Since both noise sources are placed at the input, the transfer characteristics (Z, Y, or S parameters) of the two-port will not come into play. In Figure 24.4, the circuit to be analyzed, the amplifier block can represent a transistor, a tube, or a complete amplifier. The two external noise sources, v and i, are equivalent to the device's internal sources of noise, whatever their number and physical nature. The other current source, i_T, represents the thermal

[1] A circuit model for a transistor usually contains several noise sources.

Figure 24.4. Signal source and noisy amplifier – equivalent circuit.



noise of the source, i.e., $\langle i_T \rangle^2 = 4kTG_s$. Since there are four noise parameters to be determined, we need more than just $\langle i^2 \rangle$ and $\langle v^2 \rangle$. The other two parameters come from the fact that i and v are generally not independent but are correlated. The part of i that is correlated with v defines the "correlation admittance," Y_c, through the relation $i_c = Y_c\, v$. Thus

$$\langle iv^* \rangle = \langle (i_c + i_u) v^* \rangle = \langle i_c v^* \rangle = \langle Y_c |v|^2 \rangle = Y_c \langle |v|^2 \rangle (G_c + jB_c) \langle |v|^2 \rangle. \quad (24.2)$$

To find an expression for the noise figure, we must find the voltage v_1 at the input of the device and use the definition of noise figure to find the ratio of the average of $\langle |v_1|^2 \rangle$ to the part of $\langle |v_1|^2 \rangle$ attributable to the source. The result is independent of the input impedance of the device since, using superposition, we can find v_1, considering the noise sources one at a time. They all have the same effective source impedance, $1/Y_s$. For convenience, then, we can take the device to have an infinite input impedance. It follows immediately that

$$v_1 = (i + i_T)/Y_s + v. \quad (24.3)$$

To get the equivalent input *power* we use the average squared magnitude of v_1:

$$\langle |v_1|^2 \rangle = (\langle |i|^2 \rangle + \langle |i_T|^2 \rangle / |Y_s|^2 + \langle |v|^2 \rangle + (\langle iv^* \rangle / Y_s) + (\langle i^* v \rangle / Y_s^*). \quad (24.4)$$

Here we have used the fact that there is no correlation between i and i_T or between i_T and v. The noise figure is given by this total power divided by the part due to the noise current of the source:

$$F = \frac{(\langle |i|^2 \rangle + \langle |i_T|^2 \rangle)/|Y_s|^2 + \langle |v|^2 \rangle + (\langle iv^* \rangle / Y_s + \langle i^* v \rangle / Y_s{}^*)}{\langle |i_T|^2 \rangle / |Y_s|^2} \quad (24.5)$$

or

$$F = 1 + \langle |i|^2 \rangle / \langle |i_T|^2 \rangle + (\langle |v|^2 \rangle / \langle |i_T|^2 \rangle) |Y_s|^2 (1 + Y_c/Y_s + Y_c^*/Y_s^*). \quad (24.6)$$

The noise current of the source is given by $\langle |i_T|^2 \rangle = 4kT_0 G_s$. It is convenient now to introduce the terms "noise conductance," g_n, and "noise resistance," R_n, which are defined by

$$\langle|i|^2\rangle = 4kT_0 g_n \quad \text{and} \quad \langle|v|^2\rangle = 4kT_0 R_n. \tag{24.7}$$

(The lower-case g_n follows the original notation of Rothe and Dahlke who reserved G_n for the part of i that is correlated with v.) In terms of these parameters we have

$$F = 1 + (g_n/G_s) + (R_n/G_s)|Y_s|^2(1 + Y_c/Y_s + Y_c^*/Y_s^*). \tag{24.8}$$

Replacing Y_s by $G_s + jB_s$ and Y_c by $G_c + jB_c$, this becomes

$$F = 1 + (g_n/G_s) + (R_n/G_s)(G_s^2 + B_s^2) + 2(R_n/G_s)(G_c G_s + B_c B_s). \tag{24.9}$$

Differentiating with respect to B_s and setting the derivative to zero, we find the optimum source susceptance,

$$B_{opt} = -B_c. \tag{24.10}$$

Differentiating next with respect to G_s and setting that derivative equal to zero gives the optimum source conductance:

$$G_{opt} = \sqrt{g_n/R_n - B_c^2}. \tag{24.11}$$

After a few more steps we find

$$F = 1 + 2R_n(G_c + G_{opt}) + (R_n/G_{s)})([G_s - G_{opt}]^2 + [B_s - B_{opt}]^2), \tag{24.12}$$

which is the desired relation, the Rothe–Dahlke formula.The first term on the right is just T_{min}. Summarizing these relations between the equivalent external noise generators and the standard noise parameters we have

$$R_n = \frac{\langle|v|^2\rangle}{4kT_0} \tag{24.13a}$$

$$B_{opt} = \frac{-\mathrm{Im}\langle iv^*\rangle}{\langle v^2\rangle} \tag{24.13b}$$

$$G_{opt} = \sqrt{\frac{\langle|i|^2\rangle}{\langle|v|^2\rangle} - \left(\frac{\mathrm{Im}\langle iv^*\rangle}{\langle|v|^2\rangle}\right)^2} \tag{24.13c}$$

$$F_{min} = 1 + \frac{2\langle|v|^2\rangle}{4kT_0}\left(\frac{\mathrm{Re}\langle iv^*\rangle}{\langle|v|^2\rangle} + G_{opt}\right). \tag{24.13d}$$

The inverse relations are

$$\langle|v|^2\rangle = 4kT_0 R_n \tag{24.14a}$$

$$\langle|i|^2\rangle = 4kT_0({G_{opt}}^2 + {B_{opt}}^2) \tag{24.14b}$$

$$\mathrm{Im}\langle iv^*\rangle = -B_{\mathrm{opt}}\langle|v|^2\rangle \tag{24.14c}$$

$$\mathrm{Re}\langle iv^*\rangle = 4kT_0R_n\left(\frac{F_{\min}-1}{2R_n} - G_{\mathrm{opt}}\right). \tag{24.14d}$$

24.1.4 Devices in parallel

Noise matching allows us to realize the best noise figure, $F_{\min}$, from a transistor. We saw earlier that multiple devices in series (an amplifier cascade) result in an unavoidable increase in noise figure. What about amplifiers in parallel? Identical transistors might be literally paralleled, emitter-to-emitter, base-to-base, and collector-to-collector. Or amplifiers might have their inputs directly connected and their outputs combined with a hybrid coupler. The result for two devices in parallel (see Problem 24.4) is that the resulting amplifier, compared to the individual amplifiers, has half as much noise resistance, twice as much noise conductance, and twice as much correlation admittance. This leaves $F_{\min}$ unchanged. It does, however, double Y_{opt}, making it easier to noise match to a low-impedance source, and can be a useful technique, for example, at very low frequencies where transformer matching is impractical.

24.1.5 Noise measure

We pointed out earlier that an amplifier with a good noise figure is of little value if its gain is low (a piece of wire has a perfect noise figure, unity, but has no gain). The application of negative feedback may lower the noise figure of a given transistor but at the same time it will reduce the gain. To recover the lost gain we can cascade two or more amplifiers. We saw in Chapter 17 the expression for F_∞, the noise figure of a cascade of identical amplifiers:

$$F_\infty - 1 = \frac{F-1}{1-1/G}. \tag{24.15}$$

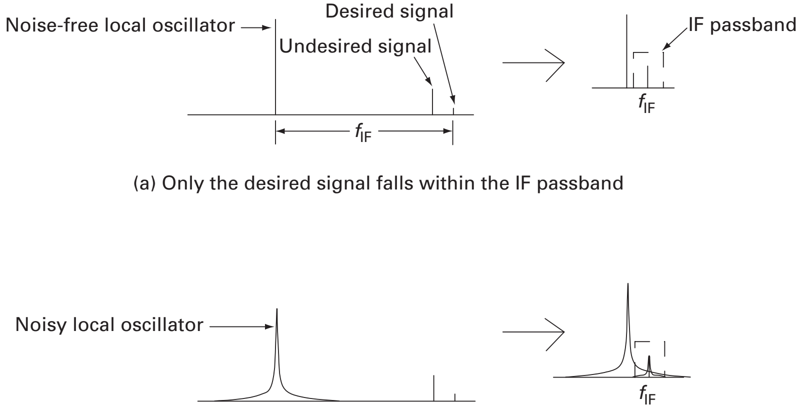
This quantity, called *noise measure*, is a better figure of merit than noise figure in that a low-gain amplifier scores low, even if its noise figure is good. The optimum input network should, therefore, minimize F_∞ rather than F. But the gain, G, depends also on the output network. Haus and Adler [1] showed that the *minimum noise measure* is a fundamental invariant when the gain, G, is taken to be the “available gain” of the device (the maximum gain that can be obtained by varying the output match while leaving the input untouched). Minimum noise measure, like minimum noise figure, is therefore determined by the input network. Haus and Adler proved that the minimum noise measure of a device is left unchanged when the device is embedded in an arbitrary network of lossless reactances. It had been noticed earlier that the noise figure of a device could sometimes be lowered by using feedback. Haus and Adlers’ results showed that any improvement in noise figure must be accompanied by a decrease in gain such that the minimum noise measure remains constant. A corollary is that changing the orientation of the device (common-emitter, common-collector, common-base) does not change the minimum noise measure. Circuit techniques, then, cannot produce a breakthrough low-noise amplifier. Any standard configuration can have the best possible noise performance. Circuit techniques, however, can improve bandwidth and stability. If we know the characteristics of a given device, we can calculate the minimum noise measure (best possible noise performance) at every frequency. There is, however, no theory available that tells us how well we can do over a given frequency band. R. N. Fano showed how well impedance matching can be done over a given band but no one has done the same for noise matching.

24.2 Oscillator noise

The frequency stability of an oscillator is determined by changes in ambient temperature and humidity, mechanical vibration, component aging, power supply variations, load variations, and finally, random noise generated by the circuit components – primarily the active element (transistor, tube, op-amp, etc). Except for noise, these effects are systematic, i.e., nonrandom, and can, in principle, be eliminated by using constant-temperature chambers, thermal compensation schemes, and shock mounting. But even when the systematic variations have been reduced to an acceptable level, random variations remain and determine the power spectrum of an oscillator (called "line shape" in spectroscopy). An ideal oscillator would have a power spectrum that is a delta function, i.e., an infinitely narrow unmodulated carrier. Noise broadens the spectrum of any real oscillator. In some applications the concentration of power is important; a specification might state, for example, that at least 90% of the power must be contained in a band within 0.01 Hz of the nominal frequency. More often it is only necessary that the spectral density beyond some distance from the center frequency be very low. In such a case it might be specified that "the noise power in a 1-Hz bandwidth centered 1 kHz from the carrier must be at least 60 dB below the total power." Spectral energy outside the narrow band containing most of the power is referred to as "sideband noise."

In a superheterodyne receiver the noise sideband power of the local oscillator can mix with an incoming signal that is nominally outside the IF passband to produce noise that falls within the passband, as shown in Figure 24.5. An undesired signal that is translated to a frequency outside the IF passband will normally be rejected by the IF bandpass filter. But a noisy L.O. will add noise sidebands to this signal that extend into the bandpass. If the undesired signal is near the bandpass and if it is much stronger than the desired signal, this noise can override the desired signal.

Figure 24.5. Translation of phase noise into the passband of a receiver.



(a) Only the desired signal falls within the IF passband

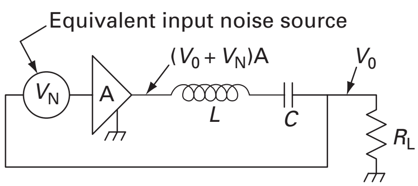
(b) Oscillator noise sidebands on the undesired signal fall within the IF passband

A similar situation occurs in MTI (moving target indicator) radar where the Doppler effect is used to distinguish moving objects from background "clutter" (the ground and stationary objects). Moving targets can be identified as spectral components shifted away from the frequency of the strong clutter echo. But, in as much as the transmitter's master oscillator or the receiver's local oscillator is noisy, moving targets with small velocities will be buried in the broadened clutter return. Good subclutter visibility therefore requires low-noise oscillators.

24.2.1 Power spectrum of a linear oscillator

In the simplest analysis, an oscillator is considered to be a linear system with just enough positive feedback to sustain oscillation. We have already used this approach to design a feedback oscillator. Here we will present a simple analysis [2, 3] that predicts the shape of the spectral line. This shape does not depend on the circuit configuration so here we will consider the simple series-mode oscillator presented in Chapter 11. This circuit, shown in Figure 24.6, uses a noninverting voltage amplifier having voltage gain, A (independent of frequency), infinite input impedance, and zero output impedance.

Figure 24.6. Prototype oscillator for phase noise analysis.



We will put an equivalent generator at the input to account for the noise produced by the amplifier. As long as the amplifier is linear, we need only consider the amplifier's noise spectrum in the vicinity of the oscillation frequency. The feedback voltage is taken from the load resistor forming the bottom leg of a voltage divider whose top leg is the series LC circuit. By inspection of Figure 24.6, we can write the loop equation:

$$V_0 = \frac{(V_0 + V_N)AR}{R + \mathrm{j}X} \tag{24.16}$$

where A is the voltage gain of the amplifier and $X = \omega L - 1/(\omega C)$. Note that V_0 and V_N are spectral densities, i.e., their units are volts $/\sqrt{Hz}$. Solving for V_0 we have

$$V_0 = \frac{V_N AR}{R(1-A) + jX}. \tag{24.17}$$

Expanding X around the resonant frequency, we let $\omega = \omega_0 + \omega'$, where $\omega_0{}^{-2} = LC$. The reactance, X, then simplifies to $X \approx 2\omega' L$ and Equation (24.17) becomes

$$V_0(\omega') = \frac{-V_N A}{(A-1) - 2\mathrm{j}\omega' L/R}. \tag{24.18}$$

The value of A is determined by the condition $\int |V_0{}^2(\omega')|/R \, \mathrm{d}\omega' = P$, where P is the total output power:

$$P = \frac{1}{R}\int V_0^2(\omega')\mathrm{d}\omega' = \frac{1}{R}\int_{-\infty}^{\infty} \frac{A^2 V_N^2 \mathrm{d}\omega'}{(A-1)^2 + 4\omega'^2 L^2/R^2} = \frac{\pi A^2 V_N^2}{2(A-1)L}. \tag{24.19}$$

Solving for $(A-1)$ we find

$$\frac{A-1}{A^2} = \frac{\pi V_N^2}{2PL}. \tag{24.20}$$

Note that $A-1$ can be very small; the voltage gain is only slightly greater than unity. Using Equation (24.20) and the fact that A is approximately unity, Equation (24.18) gives the power spectrum:

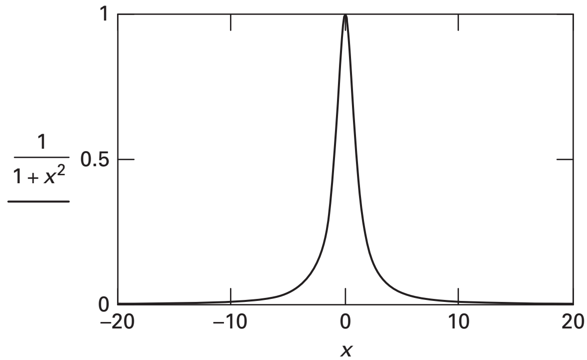
$$S(\omega) = \frac{|V_0(\omega')|^2}{R} = \frac{4P^2Q^2R^2}{\pi^2 V_N^2 \omega_0^2} \frac{1}{1 + (\omega'/\Delta)^2} \tag{24.21}$$

where Δ, the half-width of the line, is given by

$$\Delta = \frac{\pi V_N^2 \omega_0^2}{4PQ^2R}. \tag{24.22}$$

The $1/(1+x^2)$ "Lorenztian" line shape of Equation (24.21) gives the bell-shaped curve shown in Figure 24.7.

Figure 24.7. Lorentzian line shape.



24.2.2 Sideband shape

As explained earlier, it is often important that the far-out sideband noise be low compared to the carrier power. If we assume $\omega' \gg \Delta$ then Equations (24.21) and (24.22) reduce to

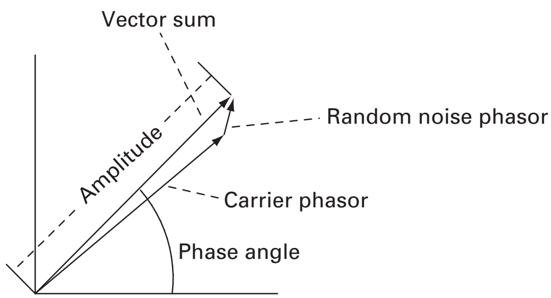
$$\frac{S(\omega')}{P} = \frac{\omega_0^2 V_N^2}{\omega'^2 4Q^2 RP}\ \mathrm{Hz}^{-1}, \tag{24.23}$$

and we see that the far-out sideband noise falls off as $1/\omega'^2$.

Phase noise

So far, the oscillator noise has the same character as if it had been produced by amplifying white noise and passing it through a very narrow filter. The oscillator output voltage is equivalent to the sum of a noiseless phasor, i.e., the carrier, and a random noise phasor, as shown in Figure 24.8.

Figure 24.8. Vector diagram showing oscillator noise.



When the noise component is parallel to the carrier, the vector sum has the correct phase but an altered amplitude. When the noise is perpendicular to the carrier, the result is mostly just a phase error. The signal can be passed through an amplitude limiter to remove the amplitude fluctuations. This leaves only the perpendicular noise components, i.e., the phase noise. The parallel and perpendicular noise signals

https://doi.org/10.1017/CBO9780511626951.025 Published online by Cambridge University Press

are uncorrelated, so if the amplitude variations are removed, the total noise power is cut in half. The symbo $\mathscr{L}(\omega')$ is often used to represent the phase noise power in a 1-Hz bandwidth at an offset of ω' from the nominal carrier frequency:

$$\mathscr{L}(\omega') = \frac{\omega_0^2 V_N^2}{\omega'^2 8Q^2 RP}\ \mathrm{Hz}^{-1}. \tag{24.24}$$

We can express the noise power $V_N{}^2(\omega')/R$ as FkT where F is the noise figure of the amplifier, k is Boltzmann's constant, and T is the reference temperature, 290 K:

$$\mathscr{L}(\omega') = \omega_0^2 \frac{FkT}{\omega'^2 8Q^2 P}\ \mathrm{Hz}^{-1}. \tag{24.25}$$

This result shows that best performance is obtained by using a low-noise device, by running it at high power, and by maximizing the loaded Q (by using a high-Q resonator with light loading).

24.3 Effect of nonlinearity

The analysis given above actually gives a lower limit for the noise. Very close to the carrier, oscillators depart from the predicted Lorenztian line shape and, as the offset frequency approaches the carrier, phase noise increases faster than ω'^{-2}. This additional noise is usually attributed to nonlinearity in the active device. We have made the assumption that the device is linear but some nonlinearity *must* be present in any actual oscillator or its amplitude would either decay or increase without limit. The nonlinearity can translate baseband $1/f$ noise up to the radio frequencies.

Problems

Problem 24.1. A transistor with noise parameters, F_{min}, G_{opt}, B_{opt}, and R_{n}, is provided with a noise matching network to produce an amplifier with $F'_{\mathrm{min}} = F_{\mathrm{min}}$, $B'_{\mathrm{opt}} = 0$, and $G'_{\mathrm{opt}} = 1/50$. What is the value of R'_n?

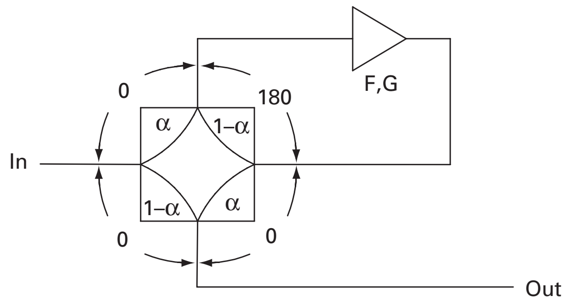
Problem 24.2. Can a transistor be fitted with an input network such that the correlation admittance, Y_{c}, is zero for the overall amplifier?

Problem 24.3. The portion of the noise current, i_{c}, that is correlated with the noise voltage cannot exceed the total noise current, i.e., $\langle |\, i_{\mathrm{c}}|^2 \rangle \leq \langle |\, i|^2 \rangle$. Use this to prove the following inequality:

$$F_{\mathrm{min}} - 1 \leq 4R_n G_{\mathrm{opt}}.$$

Problem 24.4. Show that when two identical transistors are paralleled, the minimum noise figure of the combination is the same as the minimum noise figure of the individual transistors.

Problem 24.5. A standard amplifier for use in 50-ohm systems has a gain G and a noise figure F. This amplifier is combined with a 50-ohm directional coupler to form the feedback amplifier shown below. The 180° coupler has a power gain of α in its "main channel" and therefore a power gains of $1-\alpha$ in the adjacent arms. The 180° path of the coupler provides the negative feedback. (Assume that the amplifier has zero phase shift.)



(a) Find the gain, G', of the feedback amplifier.
(b) Find the noise figure, F', of the feedback amplifier.
(c) Combine the results of (a) and (b) to show that the noise measure of the feedback amplifier is the same as the noise measure of the imbedded amplifier, i.e., show that

$$\frac{F'-1}{1-1/G'} = \frac{F-1}{1-1/G}.$$

This problem provides a simple demonstration of noise measure remaining invariant when an amplifier is imbedded in a lossless network. The negative feedback reduces the noise figure but it also reduces the gain and the noise measure stays the same. (What would positive feedback do − short of causing oscillation?)

Problem 24.6. It is certainly possible (and common) for an amplifier to have a noise temperature less than its physical temperature. Is it possible to build an *electronic cold load*, i.e., an active one-port circuit whose impedance is resistive but which generates less noise power than a resistor at ambient temperature?

Problem 24.7. A differential amplifier, e.g., an op-amp, is a three-port device (output and two inputs). Find an equivalent circuit which represents an op-amp as a noiseless device with three external noise sources. Arrange for these noise sources to be on the input side of the amplifier.

Problem 24.8. Which of the following statements best describes the operation of an oscillator?
(a) Noise is necessary to get an oscillator started. Once running, the oscillation is self-sustaining and noise plays no role.
(b) The output of an oscillator consists of amplified noise; the very narrow line shape derives from a resonator whose high loaded Q is effectively increased by the circuit's positive feedback.

Problem 24.9. Discuss the consequences (sound or picture quality) of a noisy oscillator used as a local oscillator in (a) an AM receiver; (b) an FM receiver; (c) an NTSC television receiver; (d) an ATSC television receiver.

Problem 24.10. Sketch a design for an oscillator that somehow regulates its own gain in order to stay as linear as possible, i.e., runs with just enough gain to maintain oscillation.

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