CHAPTER

9 Class-C, D, and E power RF amplifiers

Class-C, D, and E RF power amplifiers are all about high efficiency. They are used in large transmitters and industrial induction heaters, where high efficiency reduces the power bill and saves on cooling equipment, and also in the smallest transmitters, such as cell phones, where high efficiency increases battery life. These amplifiers are so nonlinear (the output signal amplitude is not proportional to the input signal amplitude), they might better be called synchronized sine wave generators. They consist of a power supply, at least one switching element (a transistor or vacuum tube), and an *LCR* circuit. The "*R*" is the load, R_L, often the radiation resistance of an antenna, equivalent to a resistor. The *LC* network is resonant at the operating frequency. The output sine-wave amplitude, while not a linear function of the input signal amplitude, *is* proportional to the power supply voltage. Thus, these amplifiers can be amplitude modulated by varying the supply voltage. Of course they can also be frequency modulated by varying the drive frequency (within a restricted bandwidth, determined by the *Q* of the *LC* circuit). Finally, they can be used as frequency multipliers by driving them at a subharmonic of the operating frequency.

9.1 The class-C amplifier

Figure 9.1 shows a class-C amplifier (a), together with an equivalent circuit (b). The circuit looks no different from the class-B amplifier of Figure 3.15 or a small-signal class-A amplifier. But here the active device (transistor or tube) is used not as a continuously variable resistor, but as a switch. To simplify the analysis, we consider the switch to have a constant on-resistance, *r*, and infinite off-resistance. This model is a fairly good representation of a power FET, when used as a switch.

The switch is closed for less than half the RF cycle, during which time the power supply essentially "tops off" the capacitor, restoring energy that the load has sapped from the resonant circuit during the cycle. The switch has internal resistance (i.e., loss), so in recharging the capacitor, some energy is lost in the

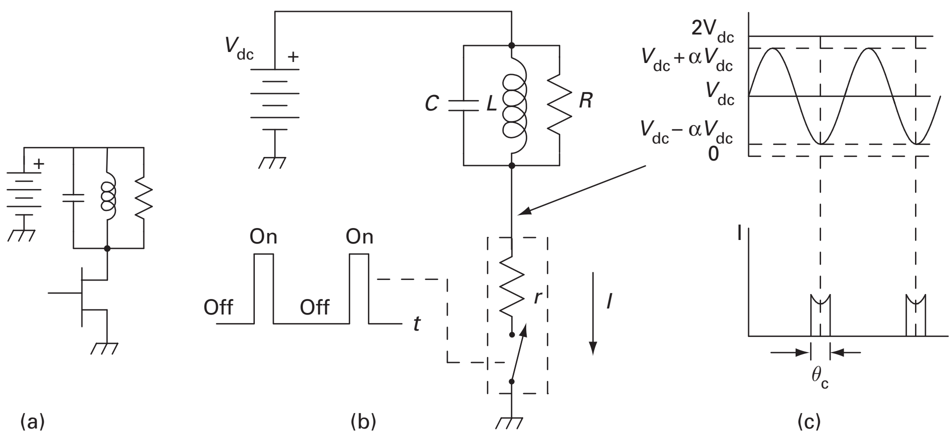

Figure 9.1. Class-C amplifier operation.

switch. Normally the LC circuit has a high Q (at least 5) so its flywheel action minimizes distortion of the sine wave caused by the abrupt pull-down of the switch and by the damping caused by the load. The drive is shown as a rectangular pulse but is often a sine wave, biased so that conduction takes place just around the positive tips. Class-C amplifiers are normally run in saturation, meaning that the switch, when on, always has its lowest possible resistance, ideally much less than the load resistance.

9.1.1 Simplified analysis of class-C operation

The simplified analysis, which we will also use later to analyze the class-E amplifier, is based on the assumption that the LC circuit provides enough flywheel effect to maintain a perfect sine wave throughout the cycle, including the interval when the resistive switch is closed. With this assumption, let us analyze the circuit of Figure 9.1 to find the output voltage and, thereby, the power and the efficiency. Referring to the figure, θ_c is the conduction angle, V_{dc} is the supply voltage, r is the on-resistance of the switch, and αV_{dc} is the peak voltage of the sine wave. We can find α as follows. The input power (the power supplied by the battery) must be equal to the sum of the output power (the power dissipated in R) plus the power dissipated in the switch resistance r. These terms are given respectively by the average of the battery voltage times the current, $(\alpha V_{dc})^2/(2R)$, and the average of I^2/r. The power equation becomes

$$\frac{1}{2\pi}\int_{\theta_c/2}^{\theta_c/2} V_{dc}\left(\frac{V_{dc}-\alpha V_{dc}\cos\theta}{r}\right)\mathrm{d}\theta = \frac{(\alpha V_{dc})^2}{2R} + \frac{1}{2\pi}\int_{-\theta_c/2}^{\theta_c/2}\left(\frac{V_{dc}-\alpha V_{dc}\cos\theta}{r}\right)^2 r\mathrm{d}\theta. \tag{9.1}$$

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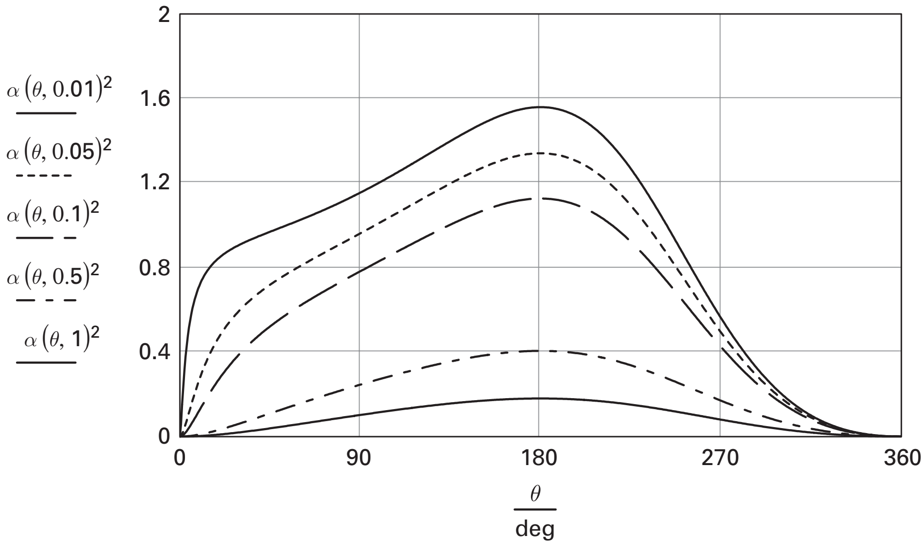


Figure 9. 2. Class-C output power ($\alpha 2$) vs. conduction angle for five values of r/R. The efficiency is given by the output power divided by the power supplied by the battery.

Rearranging this equation, we get

$$\frac{1}{\pi r/R}\int_{0}^{\theta_c/2}(1-\alpha\cos\theta)(\alpha\cos\theta)\mathrm{d}\theta-\frac{\alpha^2}{2}=0. \tag{9.2}$$

Carrying out the integral in Equation (9.2) and solving for α, we find

$$\alpha(\theta_c, r/R)=\frac{2\sin(\theta_c/2)}{\theta_c/2+\sin(\theta_c)/2+\pi r/R}. \tag{9.3}$$

The quantity α^2, proportional to the output power, is plotted in Figure 9.2 for five values of r/R, the ratio of the switch resistance to the load resistance.

The middle value, $r/R = 0.1$, is typical in actual practice. Maximum power is produced for a conduction angle of 180°, i.e., if the switch is closed during the entire negative voltage loop. If the conduction angle exceeds 180°, the incursions into the positive loop extract energy from the tuned circuit and the power is reduced. Note that α^2, and therefore α, can be greater than unity, especially when the conduction angle is 180°. We will see below, however, that much higher efficiency is obtained for conduction angles substantially less than 180°. The efficiency is given by the output power divided by the power supplied by the battery:

$$\eta(\theta_c, r/R)=\frac{(\alpha V_{\mathrm{dc}})^2/2R}{\frac{1}{2\pi}\int_{-\theta_c/2}^{\theta_c/2}V_{\mathrm{dc}}(V_{\mathrm{dc}}-\alpha V_{\mathrm{dc}}\cos\theta'/r)\mathrm{d}\theta'}. \tag{9.4}$$

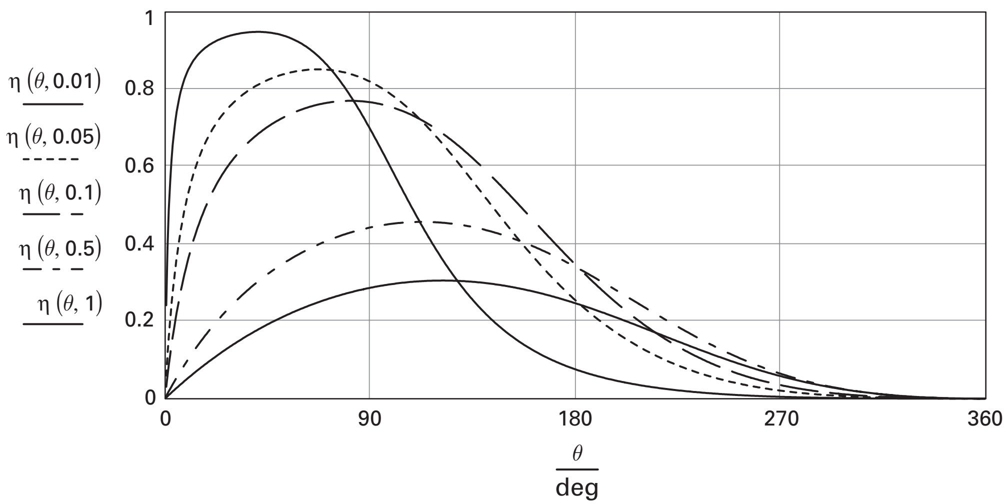


Figure 9.3. Class-C efficiency vs. conduction angle for five values of *r*/*R*.

Evaluating the integral in Equation (9.4) we find

$$\eta(\theta_c, r/R) = \frac{(\pi r/R)\alpha^2}{\theta_c - 2\alpha \sin(\theta_c/2)}, \tag{9.5}$$

where α is given by Equation (9.3). This expression for efficiency is plotted in Figure 9.3 for the same five values of r/R. For $r/R = 0.1$, the efficiency is a maximum at about 90°.

Note that this amplifier model, assuming a constant resistance in the switch, can be solved exactly by finding the general solution for the transient waveform during the switch-off period, as well as the transient waveform during the switch-on period. Because the differential equations are of second order, these general solutions will each have two adjustable parameters. The parameters are found by imposing the boundary conditions that, across the switch openings and closings, the voltage on the capacitor is continuous and the current through the inductor is continuous.

9.1.2 General analysis of a class-C operation with a nonideal tube or transistor

The above analysis will not give accurate results for class-C amplifiers made with tubes or bipolar transistors, because these devices do not have the simple constant on-resistance characteristic of a FET. Nevertheless, their nonlinear characteristics are specified graphically on data sheets, and accurate class-C analyses can be done numerically. The method is basically the same as the simplified analysis; one assumes the resonant LC circuits at the input and output have enough Q to force the input and output waveforms to be sinusoidal. For this analysis, the device characteristics are plotted in a "constant current" format: in the case of a tube, curves of constant plate current are plotted on a graph whose axes are plate voltage and grid voltage. Sinusoidal plate and grid voltages are assumed and a numerical integration of plate current × plate voltage, averaged through one complete cycle, gives the power dissipated in the device. The power supply voltage times the *average* current gives the total input power. The difference is the power delivered to the load. The designer selects a device and a power supply voltage and assumes trial waveforms with different bias points and sine-wave amplitudes. Usually several trial designs are needed to maximize output power with the given device or to maximize efficiency for a specified output power. A class-C amplifier can approach 100% efficiency, but only in the limit that the output power goes to zero. We will see below that class-D and E amplifiers can approach 100% efficiency and still produce considerable power.

9.1.3 Drive considerations

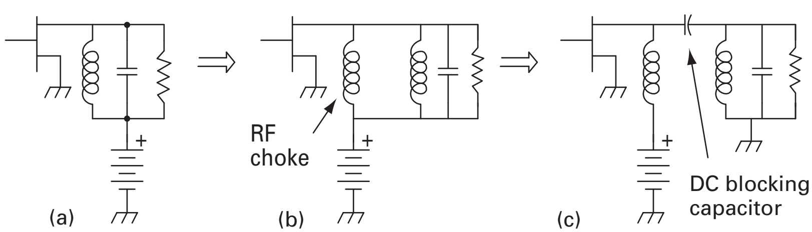
Class-C amplifiers using vacuum tubes[1] nearly always drive the control grid positive when the tube is conducting. The grid draws current and dissipates power. Data sheets include grid current curves so that the designer can use the procedure outlined above to verify that the chosen operating cycle stays within both the maximum plate dissipation rating and the maximum grid dissipation rating. When tetrodes are used, a third analysis must be done to calculate the screen grid dissipation.

9.1.4 Shunt-fed class-C amplifier

In Figure 9.1, since the switch and the *LCR* tank circuit are in series, they can be interchanged, allowing the bottom of the tank to be at ground potential, which is often a convenience. But the usual way to put the tank at dc ground is to use the shunt-fed configuration shown in Figure 9.4(c).

In the shunt-fed circuit, an RF choke connects the dc supply to the transistor and a blocking capacitor keeps dc voltage off the tank circuit. The RF choke has

Figure 9.4. Equivalence of series-fed and shunt-fed circuits.



[1] A triode vacuum tube is analogous to an NPN transistor. The tube's plate, control grid, and cathode correspond respectively to the transistor's collector, base, and emitter. Tetrode tubes have an additional grid, the screen grid, between the control grid and the plate. The screen grid is usually run at a fixed bias voltage and forms an electrostatic shield between the control grid and the plate [5].

a large inductance, so the current through it is essentially constant. The switch pulls pulses of charge from the blocking capacitor. This charge is replenished by the current through the choke. Figure 9.4 shows, from right to left, the equivalence of the shunt-fed and series-fed circuits. Note that the series-fed and shunt-fed equivalence applies as well to amplifiers of class-A, B, or C and for large-signal or small-signal operation.

9.1.5 The class-C amplifier as a voltage multiplier

An important property of the saturated class-C amplifier is that the peak voltage of the output RF sine wave is directly proportional to the supply voltage ($V_{pk} = \alpha V_{dc}$). In the standard saturated operation, the proportionality constant, α, is about 0.9. The class-C amplifier is therefore equivalent to a voltage multiplier which forms the product of a nearly unit-amplitude sine wave times the power supply voltage. Modulating (varying) the power supply voltage of a class-C amplifier is the classic method used in AM transmitters. (Note that this useful property of a class-C (or D or E) amplifier would be considered a defect for an op-amp circuit – poor power supply rejection.)

9.1.6 The class-C amplifier as a frequency multiplier

If the class-C amplifier drive circuit furnishes a turn-on pulse only every other cycle, you can see that the decaying oscillation of the tank circuit voltage will execute two cycles between refreshes. Of course there will be a greater voltage droop, producing less output power, but note that the circuit becomes a frequency doubler. If the circuit is pulsed only on every third cycle, it becomes a tripler, etc. It is common to use a cascade of frequency multipliers to produce a high RF frequency that is a multiple of the frequency of a stable low-frequency oscillator. Class-D and class-E amplifiers can also be used as frequency multipliers.

9.2 The class-D RF amplifier

Class-D amplifiers can, in principle, achieve 100% efficiency. At least two switches are required, but neither is forced to support simultaneous voltage and current. The *class-D series resonant amplifier* is shown in Figure 9.5(a).

A single-pole double-throw switch produces a square-wave voltage. The series LC filter lets the fundamental sine-wave component reach the load, R_L. The bottom of the switch could be connected to a negative supply but ground will work since the resonating capacitor provides ac coupling to the load resistor. A real circuit is shown in Figure 9.5(b); two transistors form the switch. This is a push–pull circuit, i.e., the transistors are driven out-of-phase so that when one is on the other is off. Let us find the voltage on the load. Since the capacitor also acts as a dc block, we can consider this square wave to be

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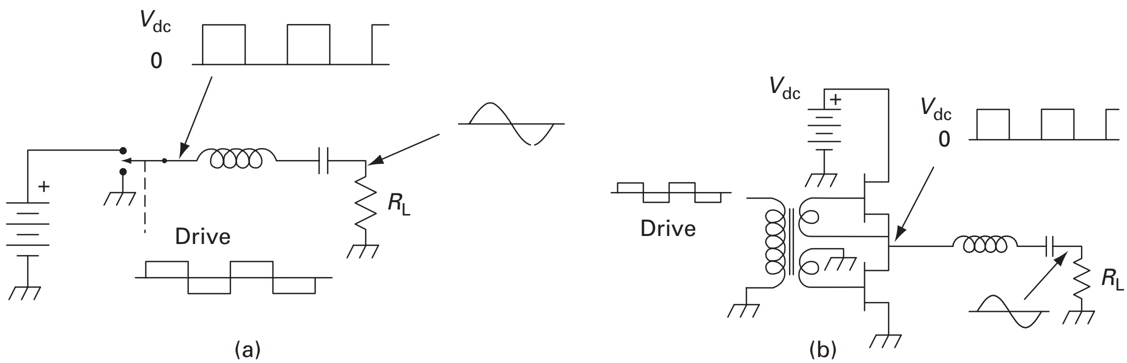

Figure 9.5. Class-D series resonant amplifier.

symmetric about zero, swinging from $-V_{dc}/2$ to $+V_{dc}/2$. The square wave is equivalent to a Fourier series, i.e., a sum of sine waves whose frequencies are the fundamental, and odd multiples of the fundamental. The series resonant LC filter passes the fundamental sinusoidal component of the square wave. We can find the amplitude of the output sine wave by equating the dc input power to the sine-wave output power. They must be equal, since the circuit has no lossy element other than R_L, the load. Since both the voltage and current change sign during the negative half of the cycle, the power is the same in the negative half-cycle as in the positive half-cycle. The power delivered by the dc supply is therefore the product of the square-wave voltage, $V_{dc}/2$, times the average of the positive current in the series LCR. The current can be written as $I(t) = I_{pk}\sin(\omega t)$ and the average positive current is $\langle|I|\rangle = (2/\pi)\ I_{pk} = (2/\pi)\ V_{pk}/R_L$, where V_{pk} is the peak value of the sine-wave voltage on the load. Equating the power from the supply to the sine-wave power on the load, we have $(V_{dc}/2)\cdot(2/\pi)(V_{pk}/R_L) = 2V_{dc}^2/(R_L)$. Solving for V_{pk}, we find $V_{pk} = 2V_{dc}/\pi$.

To estimate the loss, we will assume the FETs have constant on-resistance, r. The current through the load passes through one or the other of the switches, so the ratio of output power to switch power is $(I^2R_L)/(I^2\,r)$ and the efficiency is $\eta = R_L/(R_L+r)$. You can see that, inas much as $r \ll R_L$, the efficiency can approach 100%. However, there is another, more important, source of loss in this class-D amplifier. Each switching transistor has parasitic capacitance which is abruptly charged and discharged through the transistor's resistance once per cycle. The energy lost is ½CV^2 per transition so, for the circuit of Figure 9.5, the switching losses would be $4 \times$ ½$CV_{dc}^2 \times f$. Suppose the frequency, f, is 10 MHz and the FET switches each have, say, 200 pF of parallel capacitance. With a supply voltage of 200 volts, this would produce a loss of 160 watts! This loss can be avoided by using an alternate topology, the parallel resonant class-D amplifier.

9.2.1 Parallel-resonant class-D RF amplifiers

The class-D amplifier of Figure 9.6 consists of a square-wave *current source* driving a parallel LCR circuit. In this circuit, a large inductor (RF choke)

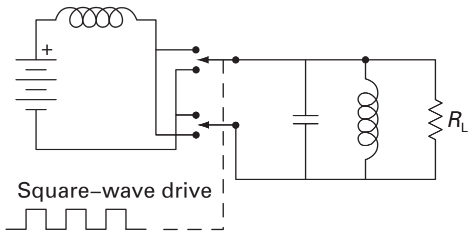

Figure 9.6. Class-D parallel-resonant RF amplifier operation.

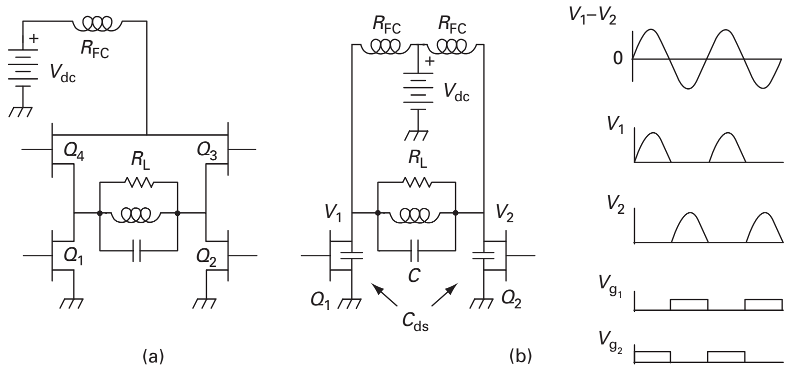

Figure 9.7. Practical class-D parallel-resonant RF amplifiers.

provides the constant current. A DPDT switch commutates the load, effectively forming a square-wave current source.

Two practical versions of this circuit are shown in Figure 9.7. In (a) the peak sine-wave voltage on the load, R_L, will be $\pi V_{dc}/2$ and the dc supply current will be $\pi^2 V_{dc}/(8R_L)$. For the appealing circuit in (b), these quantities are πV_{dc} and $\pi^2 V_{dc}/(2R_L)$. (See problem 9.4.) But the beauty of the parallel class-D amplifier is that the voltage on the parasitic capacitances of the transistors (drain-to-source capacitance, shown in dotted lines in Figure 9.7b) is zero at the instants the switches open or close. Thus, there is no lossy abrupt charging or discharging of these parasitic capacitors. (Refer to the waveforms in Figure 9.7b. Note that the value of C is effectively increased by C_{ds}, the parasitic capacitance of one transistor – first one, then the other.)

9.3 The class-E amplifier

We have seen that a class-C amplifier can be built with a single transistor and that its efficiency is high, relative to class-B and class-A amplifiers. The class-D switching amplifier, on the other hand can, in principle, achieve 100% efficiency, but requires two transistors. The *class-E amplifier* [4], has both virtues: single transistor operation and up to 100% efficiency. Figure 9.8 shows the circuit.

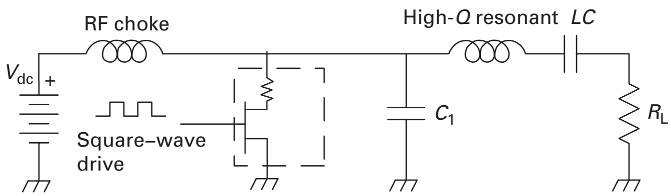

Figure 9.8. Class-E amplifiers.

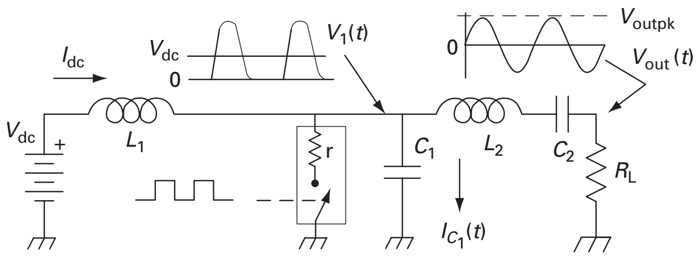

Figure 9.9. Class-E amplifier equivalent circuit.

The transistor is used as a switch: fully on or fully off. An equivalent circuit is shown in Figure 9.9, where the transistor is represented as a switch. The switch operates with a 50% duty factor, so the voltage at the switch, $V_1(t)$, is forced to be zero for half the cycle (or almost zero if we consider the switch to have a small series resistance, r).

During the other half-cycle, $V_1(t)$ consists of a rounded positive pulse. The power supply is connected through L_1, an RF choke (a high-value inductor). The choke and the switch form a *flyback* circuit in which the power supply pumps energy into the inductor while the switch is closed and the inductor pumps energy into the rest of the circuit while the switch is open. Since there can be no dc drop across the choke, you can see that the pulses in $V_1(t)$ at the switch must have an average amplitude equal to twice the power supply voltage. The key to the efficiency of this circuit is that it can be designed so that the voltage $V_1(t)$ has fallen to zero at precisely the instant the switch closes, i.e., the capacitor C_1 is shorted out without a sudden lossy discharge. Note that C_1 includes the transistor's parasitic capacitance. The RF choke, L_1, is large enough to ensure that the dc supply current, I_{dc}, has essentially no ac component, i.e., the inductor current's increase and decrease during the flyback cycle are much smaller than the average current.

A simplified description of the circuit operation is as follows. The pulses at C_1 are not square, but must have an average voltage of $2V_{dc}$. The waveform is a rough approximation to a sine wave with a peak voltage of V_{dc} plus an equal dc offset. The bandpass filter formed by L_2 and C_2 passes a good sine wave to the load, R_L. Let us now look in detail at the circuit operation and design.

9.3.1 Class-E amplifier design procedure

We have seen that we can expect this amplifier to furnish the load with a sine wave whose amplitude is something like V_{dc}, so the RF power output will be approximately ${V_{dc}}^2/(2R_L)$. If a different power is desired, the resistance of the load, R_L, can be transformed to a different value using any of the techniques of Chapter 2. Next, we can simply pick a value for L_2 that will give a reasonably high Q, maybe 10, so that the waveform at the load will be a good sine wave, i.e., have minimal harmonic content. We are left with finding values for C_1 and C_2. So far, the only constraint on the circuit is the condition that $V_1(t)$ have fallen to zero at $\tau/2$, the instant of switch closure. It turns out that this can be satisfied over a range of combinations of C_1 and C_2 and that this provides a way to set the output power. Combinations with lower values of C_1 reduce the output power. However, it is possible and beneficial to impose a second constraint which will require a unique combination of C_1 and C_2. This constraint is that $\mathrm{d}V_1/\mathrm{d}t$, as well as $V_1(t)$, be zero at the instant the switch is closed. If this condition is met, the frequency of the amplifier can be shifted somewhat without seriously violating the condition that $V_1(\tau/2)$ be zero. Moreover, at high frequencies, where the transistor's switching time is relevant, $V_1(t)$ will at least remain close to zero during the switching process. Before outlining the analysis, we summarize the resulting design procedures as follows:

1. Pick a value for Q, say $Q = 10$. Then $\omega L_2 = QR_L$.

$$\text{Pick } \omega \mathrm{L}_1 \text{ to be, say, } 100R_L. \tag{9.6}$$

2. Calculate C_1 as follows:

$$\frac{1}{\omega C_1} = \frac{\pi}{2}\left(1 + \frac{\pi^2}{4}\right)R_L = 5.45R_L. \tag{9.7}$$

3. Calculate C_2 as follows:

$$\frac{1}{\omega C_2} = \omega L_2 - \frac{\left(\frac{\pi^2}{8} - \frac{1}{2}\right)}{\left(1 + \frac{\pi^2}{4}\right)}\frac{1}{\omega C_1} = \omega L_2 - 0.212\frac{1}{\omega C_1}. \tag{9.8}$$

4. The amplitude of the output sine wave (the voltage on R_L) is

$$V_{\mathrm{out_{pk}}} = V_{dc}\frac{2}{\sqrt{1 + \frac{\pi^2}{4}}} = 1.07V_{dc}. \tag{9.9}$$

5. The output power is given by

$$P_{\mathrm{out}} = \frac{(V_{\mathrm{out_{pk}}})^2}{2R_L} = 1.154\frac{V_{dc}^2}{2R_L}. \tag{9.10}$$

6. The average current drawn from the power supply is

$$I_{\rm dc} = \frac{V_{\rm out_{pk}}}{2R_{\rm L}} = 0.577\frac{V_{\rm dc}}{R_{\rm L}}. \tag{9.11}$$

9.3.2 Class-E circuit analysis

This section is included for the reader who wants to understand the derivation of the above design formulas. We will analyze the circuit assuming $r=0$ and afterward estimate the loss when r is small compared with $R_{\rm L}$. The switch will be open whenever $\sin(\omega t)$ is positive, i.e., it opens at $t=0$, closes at $\omega t=\pi$, etc. Because of the high Q of the series combination, L_2C_2, we will again use the simplifying assumption that the load voltage is a perfect sine wave, $V_{\rm RL}=V_{\rm out_{pk}}(t)\sin(\omega t+\phi)$ where ϕ is a phase shift to be determined [3]. As with the class-C amplifier, an exact analysis requires finding the properly connected switch-open and switch-closed transient solutions. Assuming the sine wave, the current into the load is given by $I_{\rm RL}(t) = (V_{\rm out_{pk}}/R_{\rm L})\sin(\omega t+\phi)$.

By inspection of Figure 9.9, we can immediately write an equation for the current through the capacitor C_1 while the switch is open:

$$I_{C_1}(t) = C_1\frac{{\rm d}V_1}{{\rm d}t} = I_{\rm dc} - \frac{V_{\rm out_{pk}}}{R_{\rm L}}\sin(\omega t+\phi). \tag{9.12}$$

Imposing the condition that I_{C_1} be zero at $\omega t=\pi$, we find that

$$\sin(\phi) = \frac{-I_{\rm dc}R_{\rm L}}{V_{\rm out_{pk}}}. \tag{9.13}$$

We can integrate Equation (9.12) to get $V_1(t)$, the voltage on the capacitor while the switch is open:

$$V_1(t) = \frac{1}{C_1}\left(I_{\rm dc}t + \frac{V_{\rm out_{pk}}}{\omega R_{\rm L}}\cos(\omega t+\phi) - \frac{V_{\rm out_{pk}}}{\omega R_{\rm L}}\cos(\phi)\right), \tag{9.14}$$

where the last term is a constant of integration, added to satisfy the condition $V_1(0)=0$, imposed by the switch having been in its closed state before $t=0$. Next, imposing the condition that V_1 be zero also at $\omega t=\pi$, Equation (9.14) gives us

$$\cos(\phi) = \frac{\pi}{2}\frac{I_{\rm dc}R_{\rm L}}{V_{\rm out_{pk}}}. \tag{9.15}$$

We can combine Equations (9.13) and (9.15), using $\cos^2\theta+\sin^2\theta=1$, to find

$$\frac{V_{\rm out_{pk}}}{I_{\rm dc}R_{\rm L}} = a \quad \text{and} \quad \sin(\phi) = \frac{-1}{a}, \quad \text{where} \quad a = \sqrt{\frac{\pi^2}{4}+1}. \tag{9.16}$$

From this point, it remains to express $V_1(t)$ and the load current as complex constants times $e^{j\omega t}$ (the actual voltage and current are, of course, the real parts) and calculate the ratio, which must be equal to the complex impedance looking into the $L_2C_2R_L$ series circuit, $Z = R_L + j(\omega L_2 - 1/(\omega C_2))$.

The load current, $(V_{pk}/R_L)\sin(\omega t+\phi) = a\ I_{dc}\ \sin(\omega t+\phi)$, corresponds to the complex current $I(t) = -j\ a\ I_{dc}\ e^{j(\omega t+\phi)}$, since $\mathrm{Re}(-j\ a\ I_{dc}\ e^{j(\omega t+\phi)}) = a\ I_{dc}\ \sin(\omega t+\phi)$. Since the voltage $V_1(t)$ is not a sine wave, we have to find the complex representation of its fundamental Fourier component by evaluating the integral

$$\text{complex } V(t) = e^{j\omega t}\frac{2}{\tau}\int_0^{\pi/\omega} V(t)e^{-j\omega t}dt. \tag{9.17}$$

Note that the upper limit of the integral would normally be $2\pi/4$ but by integrating over only the first half-cycle, we account for the fact that the closed switch forces $V_1(t)$ to be zero during the second half-cycle. We can rewrite Equation (9.14) in terms of a and I_{dc}:

$$V_1(t) = \frac{I_{dc}}{C_1}\left(t + \frac{a}{\omega}\cos(\omega t + \phi) - \frac{a}{\omega}\cos(\phi)\right). \tag{9.18}$$

Evaluating the integral in Equation (9.17), and then setting the ratio *complex* $V_1(t)$ / *complex* $I(t)$ equal to the impedance, $R_L + j(\omega L - 1/(\omega C_2))$, results in the formulas for C_1 and C_2 (Equations 9.7 and 9.8).

9.3.3 Efficiency of the class-E amplifier

We will now assume that r is not zero, but still small enough that we can use the currents derived above, where r was taken as zero. Power is dissipated in r when the switch is closed. The current through r is just

$$I_r(t) = I_{dc} - \frac{V_{out_{pk}}}{R_L}\sin(\omega t + \phi), \tag{9.19}$$

which is the same as Equation (9.12), except that now we assume any current flowing in the capacitor is negligible compared with the current flowing through the closed switch.

The instantaneous power dissipated in r is

$$(I_r(t))^2 r = \left(I_{dc} - \frac{V_{out_{pk}}}{R_L}\sin(\omega t + \phi)\right)^2 r \tag{9.20}$$

and the average power dissipation in r is, therefore, given by

$$\begin{aligned}\langle (I_r(t))^2 r\rangle &= \left\langle \left(I_{dc} - \frac{V_{out_{pk}}}{R_L}\sin(\omega t + \phi)\right)^2\right\rangle r \\ &= \frac{1}{\tau}\int_{\tau/2}^{\tau}\left(I_{dc} - \frac{V_{out_{pk}}}{R_L}\sin(\omega t + \phi)\right)^2 r\,dt,\end{aligned} \tag{9.21}$$

where the period, τ, is just $2\pi/\omega$. Evaluating this integral and using Equations (9.15) and (9.16), we find

$$\langle (I_r(t))^2 r \rangle = \frac{r}{R_L^2}\left(\frac{3{V_{\text{out}_{\text{pk}}}}^4}{8{V_{\text{dc}}}^2} + \frac{{V_{\text{out}_{\text{pk}}}}^2}{4}\right). \tag{9.22}$$

To get the fractional power loss, just divide this by the power out, $V_{\text{out}_{\text{pk}}}/(2R_L)$.

9.3.4 Class-E design example

The formulas above were used to design a class-E amplifier designed to produce a 1-MHz, 12-V peak sine wave on a 50-ohm load, using a 12-V power supply. The component values were as follows: $L_1 = 1$ mH, $C_1 = 584$ pF, $L_2 = 79.6$ µH, $C_2 = 360$ pF, and $R_L = 50$ ohms. If the transistor has an on-resistance of 0.5 ohms, the fractional loss will be 1.9% (98% efficiency). A SPICE analysis of this circuit shows the second harmonic power at the load to be 26 dB below the fundamental power. The 3-dB bandwidth is about 15%.

9.4 Which circuit to use: class-C, class-D, or class-E?

Remember that the high-efficiency amplifiers discussed in this chapter are narrowband and nonlinear. They are ideal for producing conventional signals with AM, pulse, or phase modulation, such as radio broadcast signals and signals from cell phones. Lower efficiency linear class-A, AB, or B amplifiers are used with more complicated signals such as the superposition of many individual signals from a cellular base station or from a direct broadcast television satellite.[2] AM and FM broadcast transmitters have historically used class-C vacuum tube amplifiers. This type of transmitter is still widely used. Tubes are available with maximum plate dissipations of up to more than 1 MW. Typical class-C amplifiers have efficiencies of 75–85%, so a single large tube can produce in excess of 6 MW of RF power. The highest power used in radio broadcasting is about 2 MW, but amplifiers (or class-C oscillators) for much higher power are found in industrial heating applications such as curing plywood and welding. Many new designs use class-D and class-E solid-state amplifier modules, with the power from multiple modules being combined to achieve the desired total power. The complexity of using many modules is sometimes offset by the ability to "hot swap" defective modules without interrupting operations. The switching amplifiers, class-D and class-E, benefit from advances in transistor technology and have reached GHz frequencies.

[2] In principle, class-C, D, and E amplifiers can reproduce any type of band-limited signal by using simultaneous amplitude and phase modulation, as in the efficient amplification of single-sideband signals discussed in Chapter 8.

Problems

Problem 9.1. For the shunt-fed class-C amplifier of Figure 9.4(c), sketch the waveforms of the voltage on each side of the blocking capacitor and of the current through the blocking capacitor.

Problem 9.2. Suppose the simple series class-D amplifier of Figure 9.5(b) is driving a 50-ohm resistive load at a frequency of 1 MHz. The values of the inductor and capacitor are 9.49 μH and 2.67 nF (equal and opposite reactances at 1 MHz). This resonant circuit passes the 1-MHz component of the square wave and greatly reduces the harmonics. By what factor is the 3-MHz (i.e., third harmonic) power delivered to the load lower than the fundamental (1-MHz) power? Hints: in a square-wave, the amplitude of the third harmonic is one-third of the amplitude of the fundamental. Remember that at 1 MHz, $X_L = X_C$ while at 3 MHz, X_L is increased by a factor of 3 and X_C is decreased by a factor of 3.

Problem 9.3. A single-tube class-C amplifier with 75% efficiency is providing 500 kW of continuous cw output power. The supply voltage is 60 kV and the conduction angle is 90°. What is the average current drawn from the power supply? Answer: 11.1 amperes. What is the average current when the tube is on? Answer: 44.4 amperes.

Problem 9.4. (a) Consider the class-D amplifier of Figure 9.7(a). Show that the peak sine-wave voltage on the load, R_L, will be $\pi V_{dc}/2$ and the dc supply current will be $\pi^2 V_{dc}/(8R_L)$. Hint: since the efficiency is 100% (there are no lossy components except the load), the power supplied by the dc source will be $I \cdot V_{dc}$ where I is the constant current flowing through the RF choke. But the power from the supply can also be written as V_{dc} times the average of the absolute value of the sine wave on the load. (The average of $|A\sin(\theta)|$ is $2A/\pi$.)

(b) For the amplifier of Figure 9.7(b), show that the peak sine-wave voltage on the load will be πV_{dc} and the dc supply current will be $\pi^2 V_{dc}/(2R_L)$.

References

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