CHAPTER

14 Transformers and baluns

Transformers are harder to understand than resistors, capacitors, and single inductors. First, transformers have two terminal pairs rather than one, so we must deal with two voltages and two currents. Second, we can be misled by the deceptive simplicity of the simplest mathematical model, the "ideal transformer." In this chapter we discuss the conventional transformers used in power supplies, switching power supplies, amplifiers, and RF matching networks. We will then examine transmission line transformers, which work to higher frequencies, and *baluns*, which are devices used to connect *bal*anced circuits to *un*balanced circuits.

Figure 14.1(a) shows two inductors (here, wire coils) with arbitrary placement. The region does not have to be otherwise empty; it can contain any distribution of clumped and/or continuous magnetic materials.

The inductance values ("self-inductances") of these coils are L_1 and L_2, each measured with the other coil open circuited, so that it carries no current. We will refer to these coils as L_1 and L_2. Two representative magnetic flux lines are shown, corresponding to a current in L_1. Note that one of these flux lines is encircled by three turns of L_2. Therefore, when L_1 carries an ac current, by Faraday's law there will be an ac voltage induced in L_2, proportional to the time derivative of the encircled magnetic flux. We can write Faraday's law for this situation as

$$V_2 = \mathrm{j}\omega t M I_1 + \mathrm{j}\omega t L_2 I_2, \tag{14.1}$$

where the constant M is known as the "mutual inductance." The jω factor represents the time derivative since we are using standard ac circuit analysis, where the time dependence is contained in an implicit factor $e^{\mathrm{j}\omega t}$. The second term, containing the self-inductance, L_2, is a voltage induced by the current, if any, flowing in L_2 itself. Note that we have assumed that the wires have negligible resistance – no IR voltage drop.[1] The directions of the currents are

[1] To take account of the winding resistances, we would add a term I_2R_2 to the right-hand side of Equation (14.1) and a term I_1R_1 to the right-hand side of Equation (14.2). The effect of the resistance distributed in the windings is the same as if the resistance were consolidated into two external resistors, R_1 and R_2, in series with the windings.

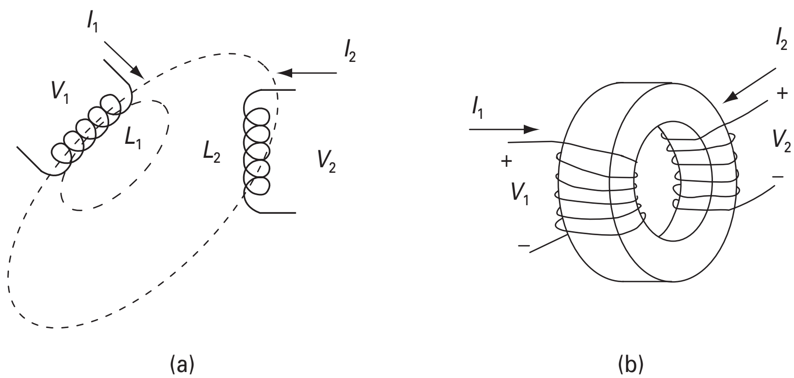

Figure 14.1. (a) Coupled inductors; (b) prototype transformer.

defined by arrows in Figure 14.1 as entering the positive end of each coil. This symmetric assignment produces a symmetric equation for V_1:

$$V_1 = \mathrm{j}\omega t L_1 I_1 + \mathrm{j}\omega t M I_2. \tag{14.2}$$

Note that both equations contain the same constant M. There is no need to write M_{12} and M_{21}, since the mutual inductances are always equal. This can be seen quite easily for the arrangement of coupled coils shown in Figure 14.1(b). This a conventional transformer, in which a toroid of iron or other magnetic material effectively contains all the field lines, forcing them to thread through every turn of both L_1 and L_2. An ac current in L_1 produces a flux proportional to N_1, the number of turns in L_1. The ac voltage induced in L_2 is proportional to N_2 times the flux. Hence M_{21} is proportional to the product N_1N_2. Likewise the ac current in L_2 produces an ac voltage in L_1 with the same proportionality to N_1N_2, so M_{12} and M_{21} are equal.[2]

If all the flux lines thread both windings, the transformer is said to be *perfectly coupled*. A *coupling coefficient*, k, is defined by $M = k(L_1 L_2)^{1/2}$. The value of k ranges from zero, for inductors with no coupling, to unity, for perfect coupling. In the transformer of Figure 14.1(b), perfect coupling is approached by using a core material of extreme magnetic permeability. For such a transformer, it makes no difference whether the windings are side-by-side, as shown, or wound one on top of the other. Nor is it necessary that they be wound tightly around the core; loose windings can be used to allow circulation in an oil-cooled power transformer.

14.1 The "ideal transformer"

If a transformer is perfectly coupled and the windings have negligible resistance, then the ratio of primary-to-secondary[3] voltages is equal to the turns ratio

[2] The equality of M_{12} and M_{21} is a general reciprocity relation that holds true for any passive two-port network, e.g., any network made from resistors, capacitors, inductors, and transformers.

[3] The names "primary" and "secondary" are arbitrary and refer only to the way the transformer is used; power usually flows into the primary and out of the secondary.

and these voltages have the same phase. This strict proportionality of voltages follows directly from Faraday's law, since the time derivative of the magnetic flux is equal in both windings. The situation with the currents is not as simple. The primary and secondary currents are not strictly proportional. This follows from the transformer's ability to store magnetic energy. If the transformer could *not* store energy, the instantaneous net power into the transformer would have to be zero. The proportionality of primary and secondary voltages would demand that the currents be proportional, i.e., setting $V_p I_p = V_s I_s$, we would have $I_s/I_p = V_p/V_s = \text{constant} = n_P/n_S$, the effective turns ratio. This hypothetical transformer, which stores no energy, is known as the *ideal transformer* and is a useful abstraction. We will discuss below a circuit with two inductors and an ideal transformer which together, are equivalent to a real transformer. But first let us emphasize how the ideal transformer, by itself, is an unrealistic model.

Consider an ideal transformer with an effective turns ratio n_1/n_2. If an impedance Z_{load} is connected to the secondary, the ratio of primary voltage to primary current will be $(n_1/n_2)\ V_2/[(n_2/n_1)I_s] = (n_1/n_2)^2\ V_s/I_s = (n_1/n_2)^2 Z_{load}$ times the secondary current. The primary will therefore present an impedance of $(n_1/n_2)^2 Z_{load}$, a simple impedance multiplication. If Z_{load} is infinite (an open circuit), the impedance looking into the primary of the ideal transformer is also infinite. But inspection of Equation (14.2) shows that, in this case, the impedance looking into a real transformer is $j\omega L_1$. Another "unreal" feature of an ideal transformer is that it contains no magnetic field! The field (or flux) from the primary winding is exactly cancelled by the field from the secondary winding. And with no magnetic field there would be no $d\Phi/dt$ and therefore no voltage across either winding. A real transformer approaches the ideal transformer model only when the number of turns approaches infinity and the magnetic coupling approaches 100%. Transformers used in practice are usually far from ideal. This is in contrast to resistors and capacitors which, at least at low frequencies, are almost ideal components obeying the relations $Z_R = R$, $Z_C = 1/j\omega C$, and $Z_L = j\omega L$.

14.2 Transformer equivalent circuit

An equivalent circuit for a real transformer is shown in Figure 14.2(b).

This model circuit consists of two inductors plus an ideal transformer having an equivalent turns ratio n_1/n_2. The shunt inductor at the left is known as the *magnetizing inductance* and its value is L_1, the inductance of the primary. Note that, even if the secondary is left open, a voltage V applied to the primary will produce a primary current $I = V/(j\omega L_1)$. The series inductor at the right is known as the *leakage inductance*. Its value is $L_2\ (1-\ k^2)$, so a perfectly coupled transformer ($k = 1$) has no leakage inductance. In practice, maximum coupling is limited to maybe 98% at low frequencies and less at RF frequencies. The useful frequency range of a transformer is determined by these two inductances. Suppose we put a transformer between a resistive load and a signal generator.

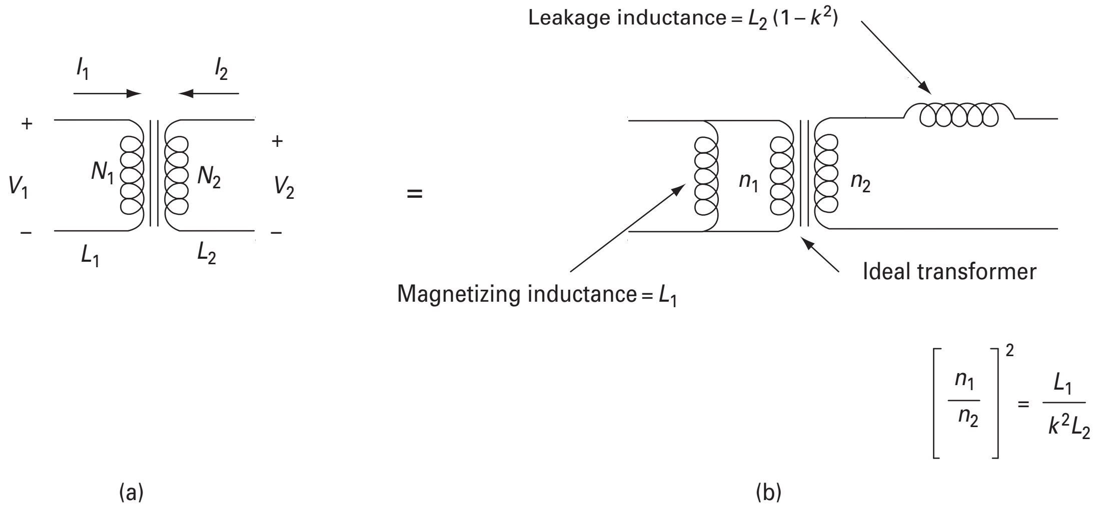

Figure 14.2. (a.) Transformer symbol with voltage and current assignments; (b.) an equivalent circuit.

Below the useful frequency range, the magnetizing inductance becomes a short circuit across the generator. Above the useful range, the leakage inductance becomes a high impedance in series with the load. In both extremes, the power delivered to the load becomes negligible.

Let us demonstrate that the model circuit of Figure 14.2(b) does, indeed, agree with the fundamental equations (14.1) and (14.2), i.e., that it really is an equivalent circuit. We have already seen that the value of the magnetizing inductance must be L_1, the inductance value of the primary winding. Leaving the secondary open, so that $I_2 = 0$, the fundamental equations (14.1) and (14.2) produce the relation $V_2/V_1 = M/L_1 = k\sqrt{L_1L_2}/L_1 = k\sqrt{L_2/L_1}$. In this situation, the model circuit gives $V_2/V_1 = n_2/n_1$. For the model to agree with the fundamental equations, $n_2/n_1 = k\sqrt{L_2/L_1}$. Finally, consider the situation in which the primary is shorted, so that $V_1 = 0$. The secondary current given by Equations (14.1) and (14.2) must be the same as the current predicted by the model. Equation (14.1) produces $I_1 = -I_2M/L_1$. Putting this into Equation (14.2) gives $V_2/I_2 = \mathrm{j}\omega\ (L_2/M - M^2\ /L_1) = \mathrm{j}\omega L_2(1-k^2)$. Looking at the model, the impedance at the secondary, with the primary shorted, is just jω time the leakage inductance, so the leakage inductance must be assigned the value $L_2(1-k^2)$. With these assignments for the values of the magnetization and leakage inductances, the model correctly reproduces Equations (14.1) and (14.2).

This is not the only possible equivalent circuit. We could just as well have constructed this equivalent circuit with the magnetizing inductance on the right side and the leakage inductance on the left side. Or we could "push" either the magnetizing inductance or the leakage inductance through the transformer, correcting the inductance by a factor $(n_1/n_2)^2$ or $(n_2/n_1)^2$, so that they are both on the same side. Another equivalent circuit, is shown in Figure 14.3. Using arguments like those presented above, you can show that this circuit also

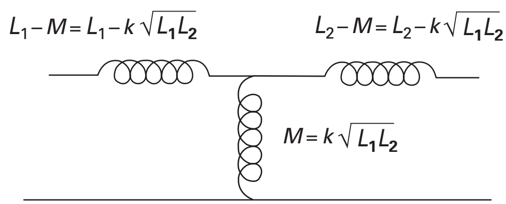

Figure 14.3. An all-inductor equivalent circuit.

satisfies Equations (14.1) and (14.2). This circuit contains no unphysical ideal transformer (and therefore has no dc isolation between primary and secondary, making it not quite as equivalent). But you can see from the labels in the figure that, in general, one of the inductors must have an unphysical negative inductance. However, note that if $k < L_1/L_2$ and $k < L_2/L_1$ all the inductors are positive. As the turns ratio becomes close to unity, the inductors all remain positive as the coupling is increased. In the case of a 1:1 transformer with perfect coupling, the values of the series inductors go to zero and the equivalent circuit is just a single shunt inductor, the magnetizing inductance. (You could put a 1:1 ideal transformer on each side of this inductor to produce a symmetric equivalent circuit that preserves dc isolation.) And, of course, the circuit of Figure 14.3 could be converted from the T configuration to an equivalent pi configuration.

To approach the ideal, a transformer must have a very high magnetizing inductance and a very small leakage inductance. You can increase the magnetizing inductance by increasing the number of turns (keeping the turns ratio constant) but, in practice, this only increases the leakage inductance and the ohmic resistance of the windings. You can decrease the leakage inductance by using fewer turns, but this lowers the magnetizing inductance. A compromise is generally needed. However, we will see that there are applications in which the leakage and/or magnetizing inductances become useful circuit components.

14.3 Power transformer operation

Power transformers are usually iron-core transformers with high coupling. The best power transformer would be an ideal transformer; its stored energy, excitation current and leakage inductance would all be negligibly small. The primary is connected to the ac line, which can be regarded as a perfect voltage source with negligible source impedance. If the coupling is high enough to make the leakage inductance negligible and the winding resistances are low, the secondary voltage will be constant, $V_2 = (n_2/n_1)V_1$, independent of the load. The magnetizing inductance will draw a constant "magnetizing" current from the line, $I_M = V/(j\omega L_1)$. Since this current is 90° out of phase with respect to the primary voltage, it consumes no average power, but does cause "excitation" energy to slosh in and out of the transformer. When a resistive load is connected to the secondary, additional "working" currents flow in both the primary and the secondary. These currents have the ratio n_2/n_1. They are in phase with the primary voltage and transfer power from the source to the load. Sometimes a capacitor is placed across the primary to resonate with the magnetizing inductance. The excitation energy will then slosh back and forth between the capacitor and the magnetizing inductance. This corrects the power factor; the power line now only has to supply the component of current that is in phase with the voltage. In practice, the magnetizing current may be comparable to the maximum working current. In a power transformer, the magnetic core is used close to saturation. When the magnetizing current is at its maximum, the inductance of the core is reduced. This nonlinear behavior of the core distorts the otherwise sinusoidal waveform of the magnetizing current. Nevertheless, the voltages on the primary and secondary remain proportional and sinusoidal, because of the low source impedance of the power line, low IR drops in the windings, and negligible leakage inductance.

14.4 Mechanical analogue of a perfectly coupled transformer

A transformer transfers ac power, usually with a step-up or step-down in voltage. Figure 14.4 shows how a lever could be used to step down the velocity of a sinusoidally reciprocating arm. The resistive load on the right-hand side is a dashpot (damper), which produces a reaction force proportional to velocity. A voltage step-down transformer increases current. This lever steps down velocity (and amplitude) and provides increased force. For an ideal transformer (infinite magnetizing inductance) or an ideal lever (zero mass) the input power (primary voltage times primary current or primary velocity times primary force) is equal to the output power at every instant. But, for a real transformer with finite magnetizing inductance, there is also the “excitation” current, lagging the voltage by 90°, pumping energy in and out of the core. Likewise, for a real lever, with nonzero mass, there is an additional component of input force, leading the velocity by 90°, that pumps mechanical kinetic energy in and out of the lever. For both the transformer and the lever the average reactive power is zero but the excitation current or force can be considerable.

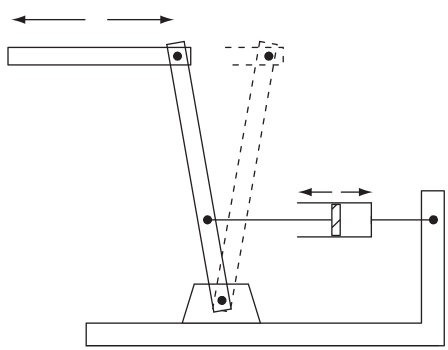

Figure 14.4. Mechanical analogue of a transformer.

14.5 Magnetizing inductance used in a transformer-coupled amplifier

In Chapter 3 we saw a circuit whose operation cannot be explained if its transformer is modeled as an ideal transformer. That circuit, a transformer-coupled class-A amplifier, is shown in Figure 14.5. Since the transformer windings have almost no dc resistance, the average voltage at the collector must be V_{cc}. Under maximum signal conditions the collector voltage swings between 0 and $2V_{dc}$, applying a peak-to-peak voltage of $2V_{dc}$ to the transformer primary.

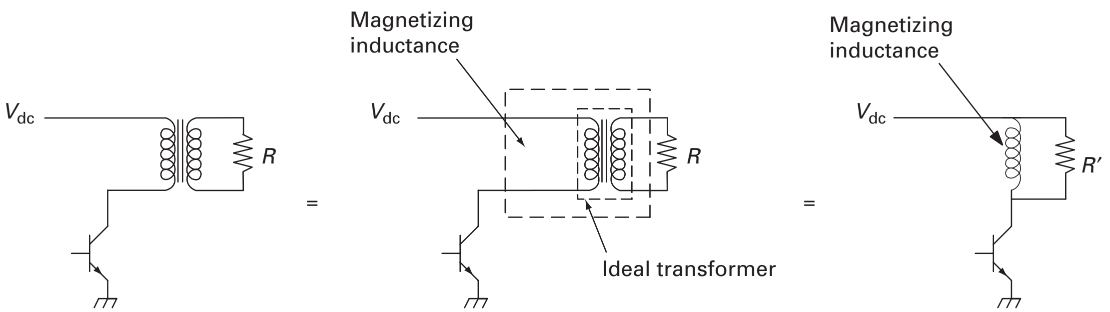

Figure 14.5. Transformer-coupled amplifier.

If we do not include the magnetizing inductance, the transformed load is a pure resistance. We would mistakenly conclude that the quiescent collector voltage must be $V_{cc}/2$ rather than V_{cc} and that the largest peak-to-peak collector signal would be V_{cc} rather than $2V_{cc}$. We would also conclude incorrectly that the frequency response would be unlimited, rather than being limited at low frequencies by the magnetizing inductance and limited at high frequencies by leakage inductance.

14.6 Double-tuned transformer: making use of magnetization and leakage inductances

Leakage inductance and magnetizing inductance limit the performance of transformers used in audio and other baseband applications. But in RF work these parasitic inductances can be tuned out with capacitors. Sometimes the leakage and magnetizing inductance can be intentionally used as in the band-pass filter of Figure 14.6(a).

To see how this circuit works, consider Figure 14.6(b), where the transformer has been replaced by its equivalent circuit. Only the leakage and magnetizing inductances are shown; the ideal transformer in the equivalent circuit of the transformer is either one-to-one or the resistor and capacitor on the right-hand side have been multiplied by its ratio. This equivalent circuit, with its vertical

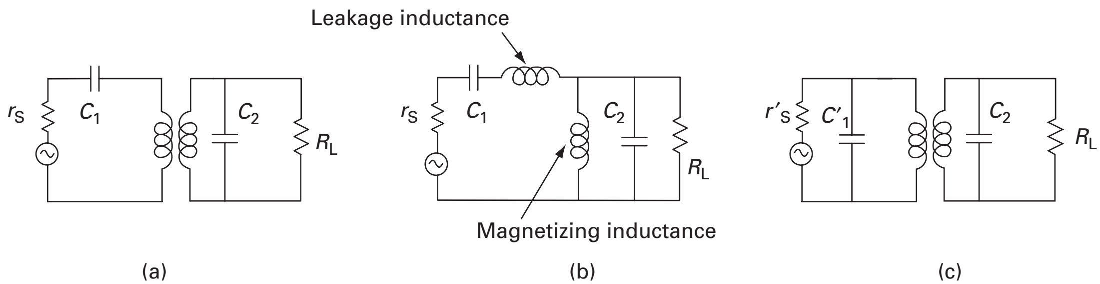

Figure 14.6. (a) Bandpass filter made with a loosely-coupled transformer; (b) equivalent circuit; (c) alternate circuit with two shunt capacitors.

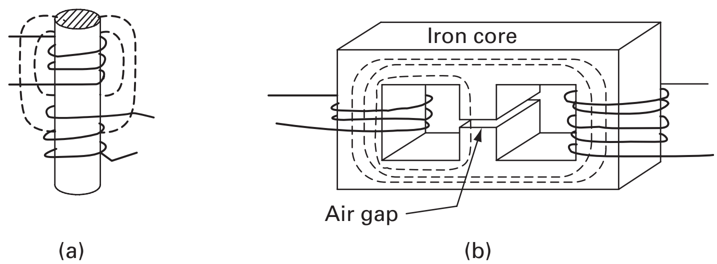

Figure 14.7. Loosely coupled transformers: (a) with powdered iron core; (b) with iron core for low frequencies.

parallel resonator and its horizontal series resonator, is a canonical two-section bandpass filter, as discussed in Chapter 4. The transformer, to have enough intentional leakage inductance, may be air-wound or may be wound on a permeable rod as shown in Figure 14.7(a). The alternate circuit of Figure 14.6(c) uses a parallel capacitor on each side of the transformer. The values of r'_S and C'_1 can be determined from the values of r_S and C_1 by noting that the equivalent Thévenin generator containing r'_S and C'_1 must have an impedance equal to the impedance of $r_S + jC_1$. (Note: whenever you see capacitors across both windings of a transformer, you can guess that the coupling is less than unity – otherwise the two capacitors would be effectively in parallel, and a single capacitor could be used.)

Power transformers are sometimes designed to have intentional leakage inductance to provide short-circuit protection (the leakage inductance limits the current). One way to build an iron core transformer with leakage inductance is shown in Figure 14.7(b). The magnetic path containing the air gap effectively shunts some of the flux generated by one winding from reaching the other winding. This design also provides magnetic shielding, in that the fringing fields are contained within the body of the transformer.

14.7 Loss in transformers

Large power distribution transformers are designed to have efficiencies around 99%. Inexpensive transformers are designed with enough efficiency to avoid premature burn-out. (Plug-in "wall-wart" transformers can be hot to the touch even without a load.) High-frequency transformers have efficiency limits set by core materials and the "skin effect" that excludes high-frequency currents from the interior of conductors, thus increasing the effective resistance of the windings.

Resistance of the windings ("copper loss") is an obvious loss mechanism. We have seen that this can be included easily in a transformer equivalent circuit by simply putting a resistor in series with the primary and another in series with the secondary.

A magnetic core made of a conductive material such as iron will dissipate energy as ordinary I^2R loss since closed paths in the iron core will act as shorted turns around magnetic flux lines. To minimize these *eddy current* losses, any such closed paths are kept short by making the core a stack of thin sheet iron laminations separated by insulating varnish or oxide. The core of a toroidal transformer can be a bundle of insulated iron wire rings, a stack of varnished sheet metal toroids, or a toroid wound of varnished thin metal tape. High-frequency transformers use cores made of magnetic particles, held together in an inert binder material. Eddy current losses can be included in the transformer equivalent circuit as a resistor in parallel with the ideal transformer.

The final loss mechanism comes from magnetic hysteresis in the core. Ideally, the magnetic flux density is proportional to the magnetic force, $B = \mu H$, where μ is the permeability. But B often lags H, as magnetic domains exhibit a kind of static friction before they break loose and reverse their direction. As a result, B vs. H through the ac cycle forms a closed curve whose included area is the energy loss per unit volume per cycle. Hysteresis loss is associated with the magnetizing current, since it is the magnetizing current that produces H, which induces B. We can therefore include hysteresis loss in the equivalent circuit as a resistor in parallel with the magnetizing inductance. This is satisfactory for a power transformer, where the magnetizing current is constant and the frequency is constant. But note that the hysteresis loss is proportional to frequency, since the B-H loop is traversed once per cycle. It also has a nonlinear amplitude dependence. Thus a simple resistor is not adequate to model hysteresis loss in a wideband transformer or a transformer operating over a range of input voltages.

14.8 Design of iron-core transformers

A transformer designer usually strives to find the smallest, lightest, and least expensive (usually synonymous) transformer that conforms to a set of electrical specifications. To see the issues involved, let us consider the design of a 60-Hz

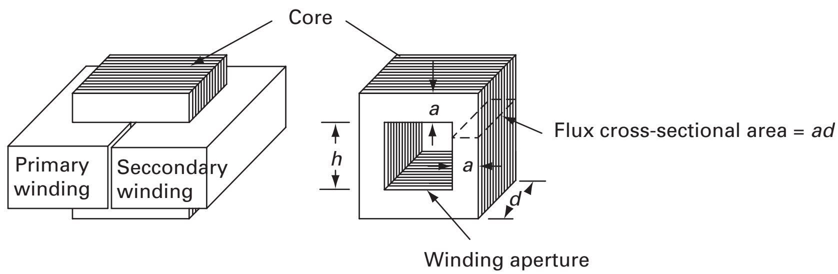

Figure 14.8. Iron-core transformer geometry.

power transformer. Suppose the primary voltage is 220 volts rms and the power delivered to the load is 500 watts. The efficiency is to be 96% and the magnetizing current must be no greater than the "working" (in-phase) current.

We will pick a silicon-steel core material for which the maximum flux density before saturation, B_{max}, is 1.5 webers/m^2. The core, a square toroid, is shown in Figure 14.8. For minimum copper loss (neglecting the excitation current) the primary and secondary windings will have equal loss and will each occupy half of the winding aperture.

The transformer will be specified by four parameters: the number of turns on the primary, N, and the three linear dimensions of the core, a, h, and d. To determine these four parameters we must write equations for the maximum B field, the loss, and the inductance of the primary winding. Faraday's law of induction gives us the maximum B field:

$$V_{\max} = N\frac{\mathrm{d}\phi}{\mathrm{d}t} = N\omega a d B_{\max}. \tag{14.3}$$

Since the rms primary voltage is 220, $V_{\max} = 220\sqrt{2}$ and we have

$$B_{\max} = \frac{220\sqrt{2}}{\omega N a d} < 1.5\ \text{webers/m}^2. \tag{14.4}$$

The copper loss in the primary winding will be I^2R_p where I is the rms current in the primary and R_p is the resistance of the winding. The number of turns on the primary, N, is given by

$$N = \frac{(h^2/2)}{\sigma} \tag{14.5}$$

where $h^2/2$ is the winding area for the primary and σ is the cross-sectional area of the wire. The mean length per turn is given by $2(a+d+h)$ so the primary resistance is found to be

$$R_p = \rho\frac{\text{length}}{\sigma} = \frac{4\rho N^2(a+d+h)}{h^2}. \tag{14.6}$$

As for core losses, the 60-Hz loss for the selected core material (when B_{max}= 1.5) is 0.6 watts/lb = 11 000 watts/m^3. The overall loss is the sum of the winding losses and the core loss. Since this loss is to be 500(1−0.96) = 20 watts, we can write

$$\mathrm{Loss_{watts}} = 11\,000(4ad(a+h)) + \left(\frac{500}{220}\right)^2 \frac{4\rho N^2(a+d+h)}{h^2} = 20. \quad (14.7)$$

Finally, the specification on the magnetizing current is equivalent to specifying that the reactance of the primary, ωL, is greater than the equivalent input load resistance, or

$$\omega L \geq \frac{220^2}{500}. \quad (14.8)$$

The inductance of the primary, L, can be written as

$$L = \frac{\mu N^2(\text{flux area})}{\text{mean flux path length}}. \quad (14.9)$$

The mean flux path length, from Figure 14.8, is $4(h+a)$, so

$$L = \frac{\mu N^2 ad}{4(h+a)}. \quad (14.10)$$

We must use Equations 14.4, 14.7, and 14.8 to find transformer parameters, a, d, h, and N that will satisfy the given specifications and minimize the size of the transformer. This is not quite as simple as solving four equations in four unknowns. The equations are really inequalities and, in general, there will not be a solution that simultaneously produces the maximum allowable flux density, the maximum allowable loss, and the minimum allowable inductance. Instead, this problem in linear programming is most often solved by cut-and-try iterative methods, conveniently done using a spreadsheet program. In this particular example, such a procedure led to the set of parameters: $d = 5$ cm, $a = 2$ cm, $h = 5$ cm and $N = 580$ turns. These dimensions give a core weight of 5.1 lbs, a loss of 19.3 watts and B_{max} of 1.42. The reactance of the primary, assuming a relative permeability of 1000, is 5.9 times the input load resistance – five times more than the minimum required reactance. Note: in the equations presented above, no consideration was made for the space occupied by wire insulation and lamination stacking but these can be accounted for by simply increasing the value of the winding wire resistivity, ρ, and decreasing the permeability.

14.8.1 Maximum temperature and transformer size

The heat generated by a transformer makes its way to the outside surface to be radiated or conducted away. The interior temperature buildup must not damage the insulation or reach the Curie temperature where the ferromagnetism quits (a consideration with high-frequency ferrite cores). "Class-A" insulation materials

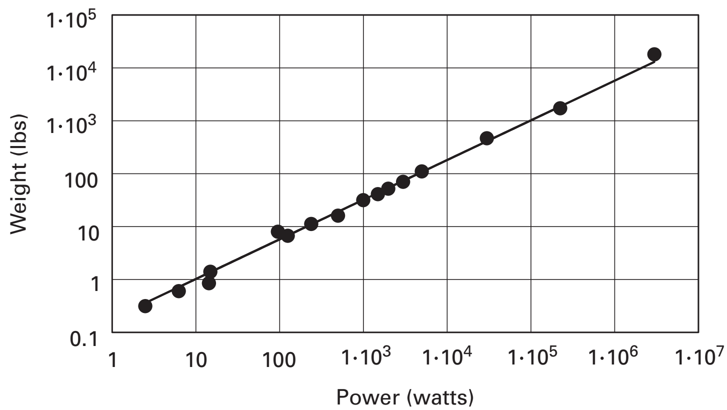

Figure 14.9. Weight vs. power rating for 16 commercial 60-Hz power transformers. The data points fit the solid line, for which (weight in lbs) = 0.18 (power in watts)$^{3/4}$.

(cotton, silk, paper, phenolics, varnishes) are limited to a maximum temperature of 105 °C. If reliable theoretical or empirically determined equations are available to predict internal temperatures, they can be included in the iterative design procedure described above. Rules of thumb can be used, at least as a starting point to determine core sizes for conventional transformers. One such rule for transformers up to, say, 1 kW is that the flux cross-sectional area of the core ($a \times d$ in Figure 14.8) in square inches should be about $0.25\sqrt{\text{Power in watts}}$. Transformer weight vs. power rating (from catalog specifications) is plotted in Figure 14.9 for sixteen 60-Hz power transformers, ranging from 2.5 watts to 3 megawatts.

The solid line, which fits the data, shows that the weight is proportional to the power raised to the exponent 3/4. Transformer manufacturers seem to use the rule-of-thumb that makes core area proportional to power$^{1/2}$ since this results in the volume and weight being proportional to power$^{3/4}$.

14.9 Transmission line transformers

Leakage inductance and distributed capacitance eventually determine the high-frequency limit of conventional transformers. For wideband applications we cannot simply resonate away these parasitics. Wideband *transmission line transformers* [3] are built like ordinary core-type transformers except that the windings are made with transmission line – either a coaxial cable, as shown in Figure 14.10, or a bifilar winding. The core must have high permeability but modest loss is acceptable (cores in chokes and transformers store little energy so high Q is not necessary). The effect of the core is to choke off any common-mode current in the transmission line, leaving only differential currents. When the transformer is wound with a piece of coaxial cable, as shown in Figure 14.10, the core suppresses current flowing on the outside of the shield,

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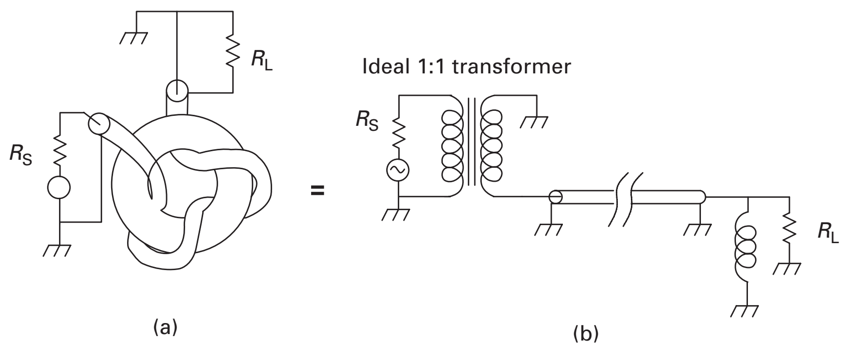


Figure 14.10. Transmission line reversing transformer.

leaving only the equal and opposite currents on the inside of the shield and the inner conductor.

This circuit is a reversing transformer, i.e., $V_{out} = -V_{in}$. The polarity flip is achieved by reversing the transmission line connections at the load end where the center conductor is grounded. Normally this would simply short the generator, but the inductance provided by the magnetic core chokes off the otherwise short-circuit current. An equivalent circuit model is shown in (b). Here the reversal is done with an ideal transformer. The length of the coax is the same, to duplicate the additional phase shift between the generator and the load. The inductor in parallel with the load represents the inductance of the cable winding around the core. At the lowest frequencies, this inductor diverts current from the load, just as the magnetization inductance limits the low-frequency response of a conventional transformer. At high frequencies, however, the circuit becomes just a piece of transmission line and its response does not fall off. There is effectively no leakage inductance nor stray capacitances. The time lag through the transmission line, however, will shift the phase from the nominal 180° as the frequency increases. Nevertheless, if an application calls for a pair of signals, identical except for polarity, the "reference" signal can be provided by using an identical piece of transmission line to provide an identical delay.

Transmission line transformers extend the range of ordinary transformers by two octaves or more. In addition to this reversing transformer, many other transformers can be made with the transmission line technique [3, 4]. Commercial hybrids good from 0.1 MHz to 1000 MHz use transmission line transformers. Miniature transmission line transformers are commercially available as standard components.

14.10 Baluns

A balun is any device that converts a *bal*anced (double-ended symmetric) signal into an *un*balanced (single-ended) signal. Baluns are commonly used to feed

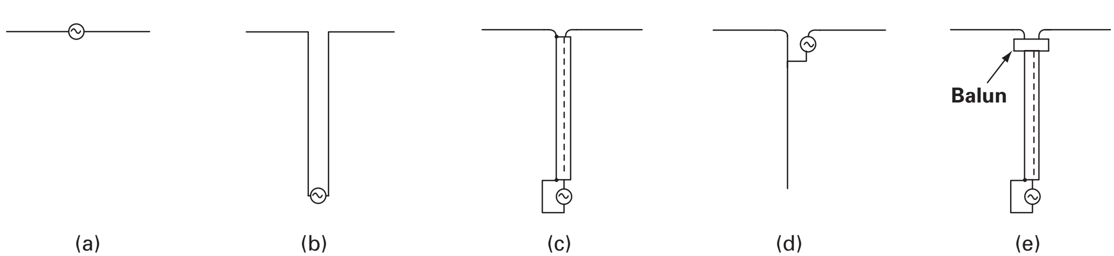


Figure 14.11. A symmetric dipole antenna fed (a) at the antenna feed point, (b) with a balanced feedline, (c) with an unbalanced coaxial feedline, (d) equivalent circuit for (c) showing how the antenna is modified by current on the feedline, (e) Balun provides symmetric feed.

symmetric antennas (e.g., dipoles) from unbalanced coaxial feed lines. Figure 14.11 shows what happens if we feed a dipole directly with a coaxial transmission line. In (a), the generator is at the feed point of the dipole so there is no question of balance or imbalance. In (b), a balanced feedline is used. Everything is still symmetric. At any point along the feedline, the current in one side is equal and opposite to the current in the other side. The spacing between the conductors is very small compared to the wavelength, so “cancellation” assures there is negligible radiation from the line. In (c) a coaxial line feeds the dipole improperly; the shield of the coax tied to the left-hand element of the dipole. An equivalent circuit (d) shows how the outer conductor of the coax becomes part of the left-hand dipole element. The antenna now has one straight element and one L-shaped element. The radiation pattern will not be the intended dipole pattern and there will be RF current flowing on the outside of the feedline. In (e) a balun at the end of the coaxial feedline provides equal and opposite voltages to each side of the dipole and eliminates any current from the outside of the feedline.

Figure 14.12 illustrates the requirement for a balun; with equal Z_1 and Z_2, i.e., a load structure symmetric with respect to ground, we want V_1 and V_2 to be equal and opposite with respect to ground.

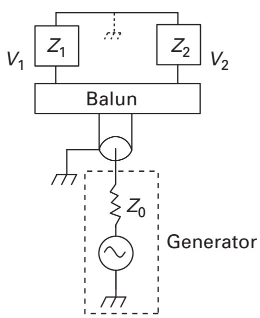


Figure 14.12. Balun operation: unbalanced-to-balanced.

The dotted ground symbol indicates that this point of symmetry will have zero voltage (when $Z_1 = Z_2$) and can be grounded if necessary or desirable. Figure 14.13 shows the equivalent situation with the load at the unbalanced side. When V_1 and V_2 are in phase (common mode) there must be no excitation of Z. But when V_1 and V_2 are 180° apart (differential mode) the load, Z, is fully excited. Baluns are normally reciprocal devices so the name “Unbal” is not needed.

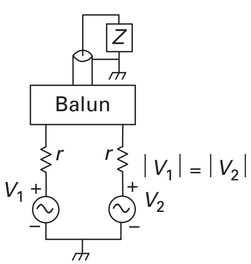


Figure 14.13. Balun operation: balenced-to-unbalanced.

From a transmitting standpoint, the balun eliminates common mode current on the feedline which otherwise would radiate and affect the pattern of the antenna. Figure 14.14(a) shows a reversing transformer used as a balun for this application. Note that this balun is also a 4-to-1 impedance transformer since the voltage across the dipole is $2V$. In Figure 14.14(b), the reversing transformer is replaced by a half-wave length of transmission line. The phase shift through this piece of line transforms V into $-V$ just as the transformer did, but the reversal is only correct over a narrow frequency band. (Note that the half-wave line can have any value for Z_0.)

The simple reversing transformer in Figure 14.14(a) can be replaced by the wideband transmission line transformer of Figure 14.10. The phase shift this transformer picks up at increasing frequencies is compensated by using a

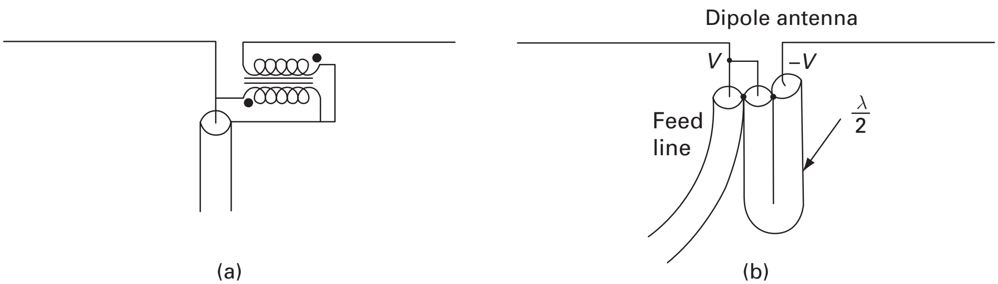


Figure 14.14. 4:1 baluns.

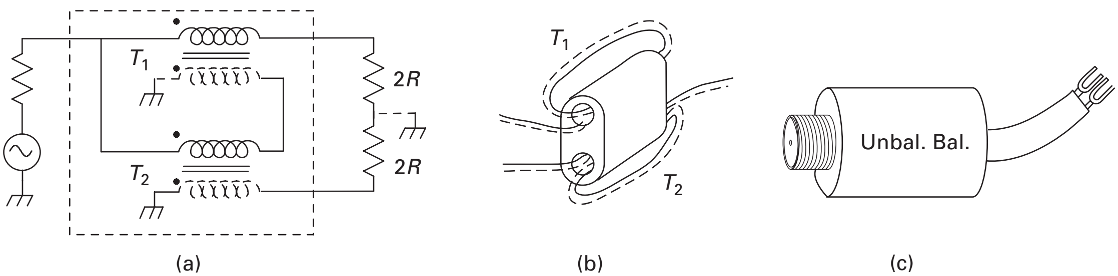


Figure 14.15. Wideband 4:1 balun made from two 1:1 transmission line transformers.

second transformer, identical except with no reversal, to provide an equal frequency-dependent phase shift. The combination of these transformers, connected in parallel at one end and in series at the other end, as shown in Figure 14.15(a), makes a very wideband 4-to-1 balun.

This is the circuit most often found in the television balun of Figure 14.15(c). The transformers are often wound on a "binocular core" (Figure 14.15b). This core operates as two separate cores, i.e., there is nominally no magnetic coupling between the two transformers, T_1 and T_2. For clarity, the figure shows the transformers wound with only two turns; in practice several turns are used.

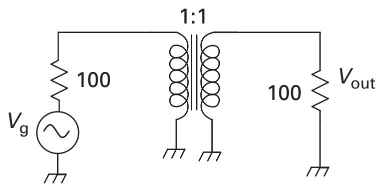
Problems

Problem 14.1. Use Equations (14.1) and (14.2) to show that when the primary and secondary windings of a transformer are connected in series the total inductance is given by $L = L_1 + L_2 \pm 2M$ where M, the mutual inductance, is given by $M = k\sqrt{L_1 L_2}$ and the ± changes when one of the windings is reversed. (This is a standard method for measuring mutual inductance.)

Problem 14.2. Consider the following transformer: $L_1 = 0.81$ H (inductance of the primary winding), $L_2 = 1$ H (inductance of the secondary winding), $k = 0.9$ (coupling coefficient).

(a) If the secondary is open circuited and 1 volt (ac, of course) is applied to primary, show that the secondary voltage is 1 V. (b) If the primary is open circuited and 1 volt is applied to the secondary, show that the primary voltage is 0.81 V.

Problem 14.3. Calculate the low-frequency cutoff (half-power frequency) for a resistive load coupled by a particular transformer to a generator. The transformer has perfect coupling and a 1:1 turns ratio. The source and load impedances are both 100 ohms and the reactance of the transformer primary is 50 ohms at 20 Hz.



Problem 14.4. Upgrade your ladder network analysis program (Problem 1.3) to handle conventional transformers. Let the transformer be specified by its primary inductance, secondary inductance, and coupling coefficient.

Example answer: For the MATLAB example solution given in Problem 1.3, add the element, "XFRMR" by inserting the following sequence of statements in the "elseif chain":

```
elseif strcmp(component,'XFRMR')==1
ckt_index=ckt_index+1; Lpri=ckt{ckt_index}; %primary inductance
ckt_index=ckt_index+1; Lsec=ckt{ckt_index};% secondary inductance
ckt_index=ckt_index+1; k =ckt{ckt_index}; %coupling coefficient
V=V+I*(1j*w*Lsec*(1-k^2));
ratio= sqrt(Lpri/(k^2*Lsec));
V=V*ratio; I=I/ratio;
I=I+V/(1j*w*Lpri);
```

Problem 14.5. A lossless transformer is placed between a 50-ohm signal generator and a 4.5-ohm load. (a) Use your ladder network analysis program (or an equivalent program) to plot the relative power at the load vs. frequency. Use the following parameters: primary inductance=100 μH, secondary inductance=10 μH, coupling coefficient k=0.9. (b) Find the values of a capacitor to be shunted across the primary (i.e., in parallel with the magnetizing inductance) and another capacitor to be placed in series with the secondary (i.e., in series with the leakage inductance) so that the magnetizing and leakage inductances will be cancelled (resonated out) at 0.5 MHz. Plot the resulting frequency response to verify that the transmission is now perfect at 0.5 MHz.

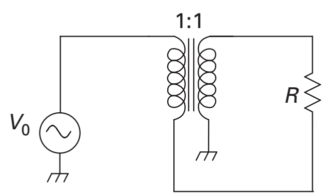
Problem 14.6. When a power transformer is first turned on, i.e., connected to the line, there is sometimes an initial inrush of current strong enough to dim lights on the same circuit and produce an audible "grunt" from the transformer itself. Decide whether this effect is strongest when the circuit is closed at a zero crossing of the line voltage or at a maximum of the line voltage. (This involves the magnetizing inductance of the transformer so simply analyze the transient when an inductor is connected to an ac line.)

Problem 14.7. (a) Suppose you have a power transformer designed to be fed from 220 V, 60 Hz but you want to use it in a country where the power line supplies 220 V, 50 Hz. Why is the transformer likely to overheat when fed with 50 Hz power? Consider the magnetizing current, copper losses, and core losses. (b) Consider the reverse situation. Would there be any mechanism to cause extra power dissipation if a 50 Hz transformer is used on a 60 Hz line?

Problem 14.8. Two identical perfectly coupled 1:1 transformers are connected in series, i.e., the secondary of the first is connected to the secondary. The primary and secondary inductances of each transformer are L. Show that this combination is equivalent to a single transformer and find its magnetizing inductance L'. If you enjoy algebra, assume the transformers are not perfectly coupled and find L' and k'.

Problem 14.9. Consider a lossless transformer with primary and secondary inductances L_1 and L_2. Suppose the coupling coefficient has a value that results in a 1:1 ideal transformer in the transformer's equivalent circuit. Find the values of the magnetizing inductance and the leakage inductance.

Problem 14.10. The transformer in the figure has a turns ratio of 1:1. The primary and secondary inductances are both L. The amplitude of the sine wave from the generator is V_0. Assume the transformer has no leakage inductance. Find an expression for the current in the resistor. Hint: use the equivalent circuit for the transformer: an inductor together with an ideal transformer.



References

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