CHAPTER

11 Oscillators

Oscillators are autonomous dc-to-ac converters. They are used as the frequency-determining elements of transmitters and receivers and as master clocks in computers, frequency synthesizers, wristwatches, etc. Their function is to divide time into regular intervals. The invention of mechanical oscillators (clocks) made it possible to divide time into intervals much smaller than the Earth's rotation period and much more regular than a human pulse rate. Electronic oscillators are analogs of mechanical clocks.

11.1 Negative feedback (relaxation) oscillators

The earliest clocks used a "verge and foliot" mechanism which resembled a torsional pendulum but was not a pendulum at all. These clocks operated as follows: torque derived from a weight or a wound spring was applied to a pivoted mass. The mass accelerated according to Torque $= I\,\mathrm{d}^2\theta/\mathrm{d}t^2$ (the angular version of $F = ma$). When θ reached a threshold, θ_0, the mechanism reversed the torque, causing the mass to accelerate in the opposite direction. When it reached $-\theta_0$ the torque reversed again, and so on. The period was a function of the moment of inertia of the mass, the magnitude of the torque, and the threshold setting. These clocks employed negative feedback; when the controlled variable had gone too far in either direction, the action was reversed. Most home heating systems are negative feedback oscillators; the temperature cycles between the turn on and turn off points of the thermostat. Negative feedback electronic oscillators are called "relaxation oscillators." Most of these circuits operate by charging a capacitor until its voltage reaches an upper threshold and then discharging it until the voltage reaches a lower threshold voltage. In Figure 11.1(a), when the voltage on the capacitor builds up to about 85 V, the neon bulb fires. The capacitor then discharges quickly through the ionized gas (relaxes) until the voltage decays to about 40 V. The bulb then extinguishes and the cycle begins anew.

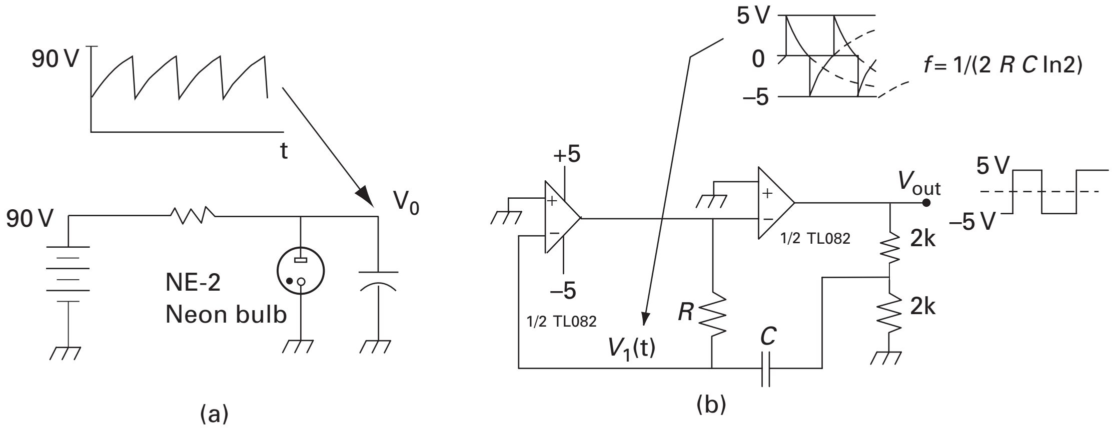


Figure 11.1. Relaxation (negative feedback) oscillators.

The circuit of Figure 11.1(b) alternately charges the capacitor, C, until its voltage reaches 2.5 V, and then discharges it until the voltage has fallen to −2.5 V. ($V_1(t)$ decays alternately toward +5 or −5 volts. When it reaches zero volts, the left-hand op-amp abruptly saturates in the opposite direction, kicking $V_1(t)$ to the voltage it had been approaching. The voltage then begins to decay in the opposite direction, and so forth.) Voltage-to-frequency converters are usually relaxation oscillators in which the control voltage determines the slope, and hence the oscillation period, of a fixed-amplitude sawtooth wave. Relaxation oscillators typically contain waveforms that are ramps or exponential decays. In the verge and foliot clock, the angle $\theta(t)$ consists of a sequence of parabolic arcs. Note that relaxation oscillators are nonlinear circuits which switch alternately between a charge mode and a discharge mode. Positive feedback oscillators, the main subject of this chapter, are nominally linear circuits. They generate sine waves.

11.2 Positive feedback oscillators

Clock makers improved frequency stability dramatically by using a true pendulum, a moving mass with a restoring force supplied by a hair spring or gravity.[1] As first observed by Galileo, a pendulum has its own natural frequency, independent of amplitude. It moves sinusoidally in simple harmonic motion. A pendulum clock uses *positive* feedback to push the pendulum in the direction of its motion, just as one pushes a swing to restore energy lost to friction.

[1] The Salisbury Cathedral clock, when installed around 1386, used a verge and foliot mechanism. Some 300 years later, after Christian Huygens invented the pendulum clock based on Galileo's observations of pendulum behavior, the Salisbury clock was converted into a positive feedback pendulum clock, its present form.

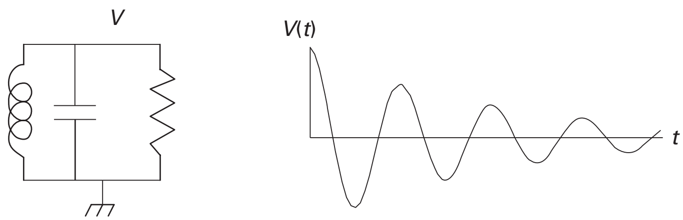


Figure 11.2. Damped oscillation in a parallel *LCR* circuit.

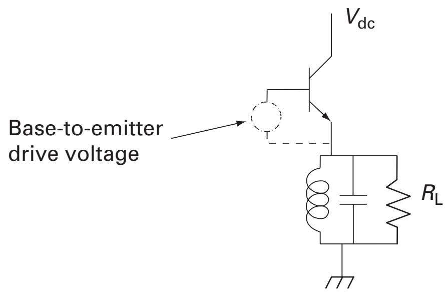


Figure 11.3. Transistor and dc supply replace energy lost to damping.

Electronic versions of the pendulum clock are usually based on resonators such as parallel or series *LC* circuits or electromechanical resonators such as quartz crystals. They use positive feedback to maintain the oscillation. A resonator with some initial energy (inductor current, capacitor charge, or mechanical kinetic energy) will oscillate sinusoidally with an exponentially decaying amplitude as shown in Figure 11.2. The decay is due to energy loss in the load and in the internal loss of the finite-Q resonator. In Figure 11.2 the resonator is a parallel *LCR* circuit.

To counteract the exponential decay, a circuit pumps current into the resonator when its voltage is positive and/or pulls current out when its voltage is negative. Figure 11.3 shows how a transistor and a dc supply can provide this energy. In this example circuit, the transistor is shown in the emitter-follower configuration simply because it is so easy to analyze; the emitter voltage tracks the base voltage and the base draws negligible current. The single transistor cannot supply negative current but we can set it up with a dc bias as a class-A amplifier so that current values less than the bias current are equivalent to negative current.

All that remains to complete this oscillator circuit is to provide the transistor drive, i.e., the base-to-emitter voltage. We want to increase the transistor's conduction when the output voltage (emitter voltage) increases, and we see that the emitter voltage has the correct polarity to be the drive signal. Since an emitter follower's voltage gain is slightly less than unity, the base needs a drive

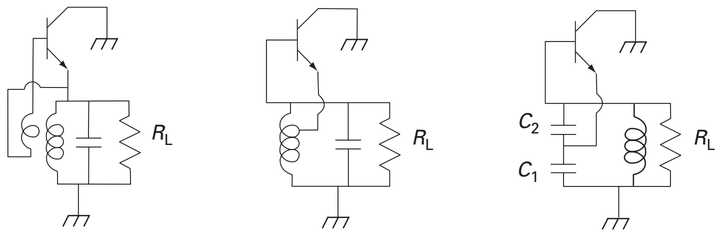


Figure 11.4. Feedback loop details define (a) Armstrong; (b) Hartley; (c) Colpitts oscillators.

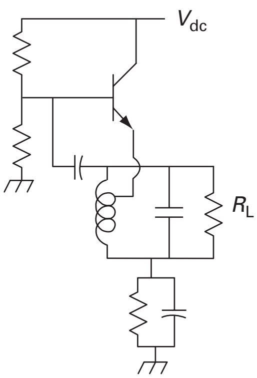


Figure 11.5. Hartley oscillator circuit including bias circuitry.

signal with slightly more amplitude than the sine wave on the emitter. Figure 11.4 shows three methods to provide this drive signal. The Armstrong oscillator adds a small secondary winding to the inductor. The voltage induced in the secondary adds to the emitter voltage. The Hartley oscillator accomplishes the same thing by connecting the emitter to a tap slightly below the top of the inductor. This is just an autotransformer version of the Armstrong oscillator if the magnetic flux links all the turns of the inductor. But the top and bottom portions of the inductor do not really have to be magnetically coupled at all; most of the current in the inductor(s) is from energy stored in the high-Q resonant circuit. This current is common to the two inductors so they essentially form a voltage divider. (Note, though, that the ratio of voltages on the top and bottom portions of the inductor ranges from the turns ratio, when they are fully coupled, to the square of the turns ratio, when they have no coupling.) If we consider the totally decoupled Hartley oscillator – no mutual inductance – and then replace the inductors by capacitors of equal (but opposite) reactance and replace the capacitor by an inductor, we get the Colpitts oscillator. Note that each oscillator in Figure 11.4 is an amplifier with a positive feedback loop. No power supply or biasing circuitry is shown in these figures; they simply indicate the ac signal paths.

Using the Hartley circuit as an example, Figure 11.5 shows a practical circuit. It includes the standard biasing arrangement to set the transistor's operating point. (A resistor voltage divider determines the base voltage and an emitter resistor then determines the emitter current, since V_{be} will be very close to 0.7 V.) A blocking capacitor allows the base to be dc biased with respect to the emitter. A bypass capacitor puts the bottom of the resonant circuit at RF ground.

In practice, one usually finds oscillators in grounded emitter circuits, as shown in Figure 11.6. The amplitude of the base drive signal must be much smaller than the sine wave on the resonant circuit. Moreover, the polarity of the base drive signal must be inverted with respect to the sine wave on the collector. You can inspect these circuits to see that they do satisfy these conditions. But, on closer inspection, you can note the circuits are *identical* to the circuits of Figure 11.4, except that the ground point has been moved from the collector to

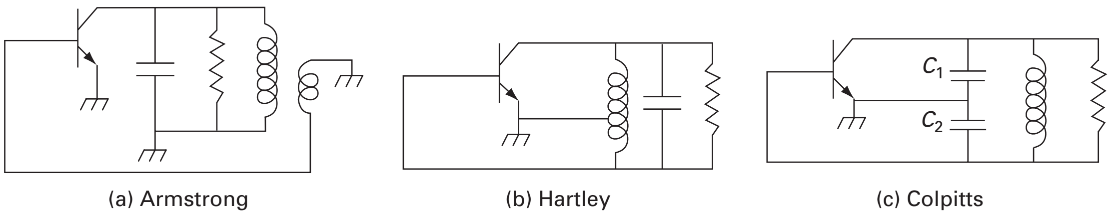


Figure 11.6. Grounded-emitter oscillator circuits.

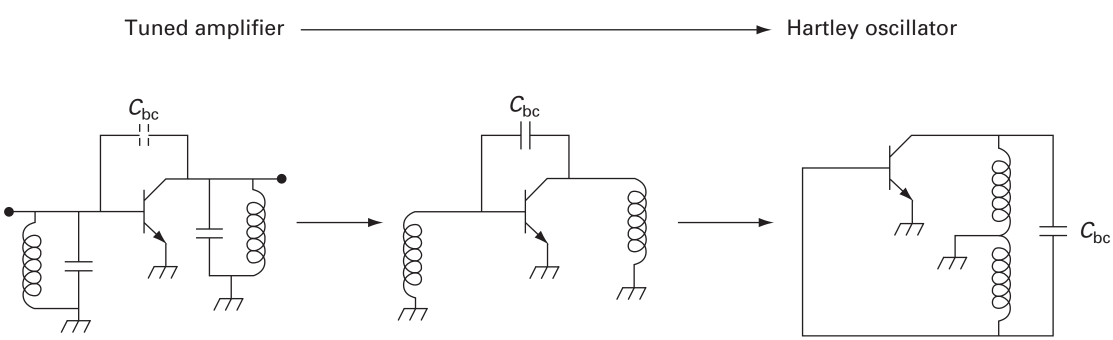
the emitter. In these oscillators, an amplifier is enclosed in a positive feedback loop. But, because there is no input signal to have a terminal in common with the output signal, oscillators, unlike amplifiers, do not have common-emitter, common-collector, and common-base versions.

The Colpitts oscillator, needing no tap or secondary winding on the inductor, is the most commonly used circuit. Sometimes the transistor's parasitic collector-to-emitter capacitance is, by itself, the top capacitor, C_1, so this capacitor may appear to be missing in a circuit diagram. A practical design example for the Colpitts circuit of Figure 11.6(c) is presented later in this chapter.

11.2.1 Unintentional oscillators

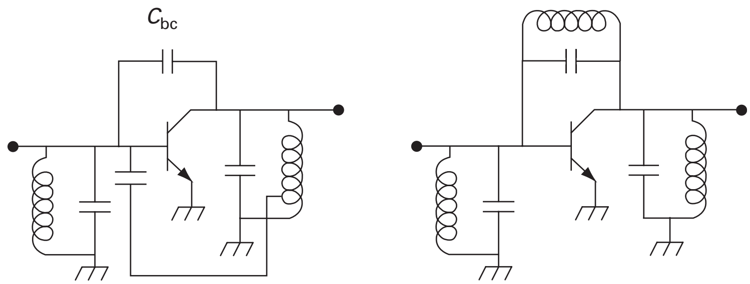
In RF work it is common for a casually designed amplifier to break into oscillation. One way this happens is shown in Figure 11.7. The circuit is a basic common-emitter amplifier with parallel resonant circuits on the input and output (as bandpass filters and/or to cancel the input and output capacitances of the transistor). When the transistor's parasitic base-to-collector capacitance is included, the circuit has the topology of the decoupled Hartley oscillator. If the feedback is sufficient, it will oscillate. The frequency will be somewhat lower than that of the input and output circuits so that they look inductive as shown in the center figure. This circuit known as a TPTG oscillator, form Tuned-Plate Tuned-Grid, in the days of the vacuum tube.

Figure 11.7. Tuned amplifier as an oscillator



With luck, the loop gain of any amplifier will be less than unity at any frequency for which the total loop phase shift is 360° and an amplifier will be stable. If not, it can be *neutralized* to avoid oscillation. Two methods of neutralization are shown in Figure 11.8.

Figure 11.8. Amplifier neutralization.

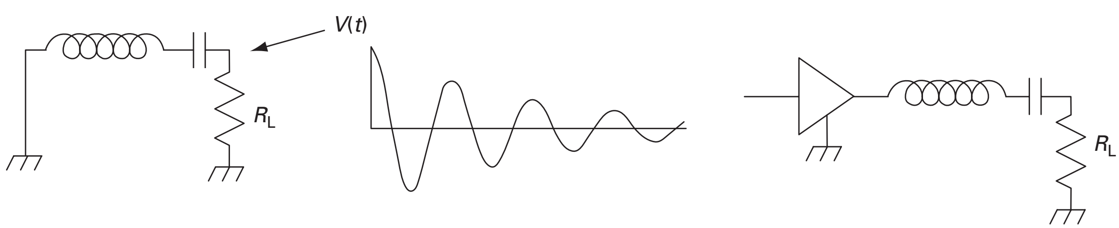


In Figure 11.8(a), a secondary winding is added to provide an out-of-phase voltage which is capacitor-coupled to the base to cancel the in-phase voltage coupled through C_{bc}. In Figure 11.8(b), an inductor from collector to base resonates C_{bc} to effectively remove it (a dc blocking capacitor would be placed in series with this inductor). In grounded-base transistor amplifiers and grounded-grid vacuum tube amplifiers the input circuit is shielded from the output circuit. These are stable without neutralization (but provide less power gain than their common-emitter and common-cathode-counterparts).

11.2.2 Series resonant oscillators

The oscillators discussed above were all derived from the parallel resonant circuit shown in Figure 11.2. We could just as well have started with a series *LCR* circuit. Like the open parallel circuit, a shorted series *LCR* circuit executes an exponentially damped oscillation unless we can replenish the dissipated energy. In this case we need to put a voltage source in the loop which will be positive when the current is positive and negative when the current is negative, as shown in Figure 11.9.

Figure 11.9. Series-mode oscillator operation.



While a bare transistor with base-to-emitter voltage drive makes a good current source for a parallel-mode oscillator, a low-impedance voltage source is needed for a series-mode oscillator. In the series-mode oscillator shown in Figure 11.10, an op-amp with feedback is such a voltage source.

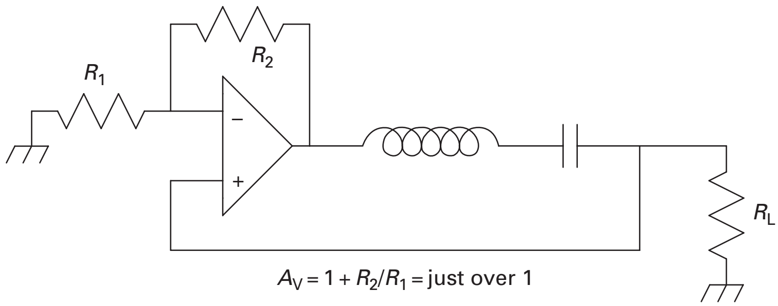
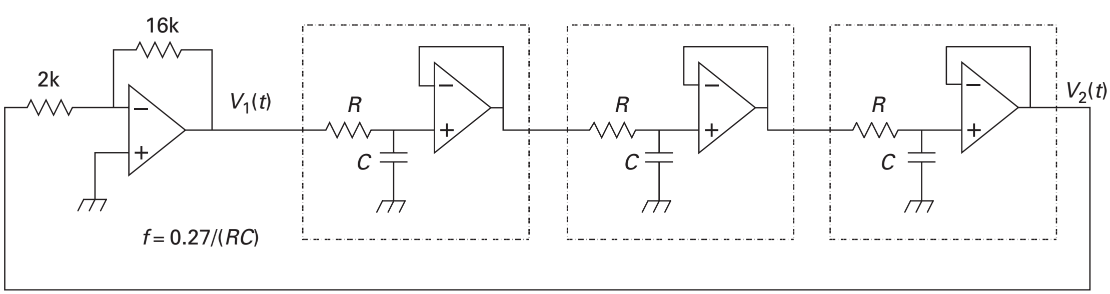


Figure 11.10. An op-amp series-mode oscillator.

Since no phase inversion is provided by the tank circuit, the amplifier is connected to be noninverting. An emitter-follower has a low output impedance and can be used in a series-mode oscillator (see Problem 11.4). When the series *LC* circuit is replaced by a multisection RC network, the resulting oscillator is commonly known as a *phase-shift oscillator* (even though every feedback oscillator oscillates at the frequency at which the overall loop phase shift is 360°). An *RC* phase-shift oscillator circuit is shown in Figure 11.11. Op-amp voltages followers make the circuit easy to analyze.

For the three cascaded *RC* units, the transfer function is given by $V_2(t)/V_1(t) = 1/(\omega RC+1)^3$. The inverting amplifier at the left provides a voltage gain of $-16/2 = -8$, so $V_1(t)/V_2(t) = -8$. Combining these two equations yields a cubic equation with three roots: $\omega RC = 3j$, $\sqrt{3}$, and $-\sqrt{3}$. The first root corresponds to an exponential decay of any initial charges on the capacitors while the two imaginary roots indicate that the circuit will produce a steady sine-wave oscillation whose frequency is given by $\omega RC = \sqrt{3}$. In practice, the 16k resistor would be increased to perhaps twice that value to ensure oscillation. Note that

Figure 11.11. An *RC* phase-shift oscillator.



this circuit is a positive-feedback sine-wave oscillator even though it does not contain a resonator. When the 16k resistor value is increased, the loop gain for the original frequency becomes greater than unity, but for the new gain, there will be a nearby complex frequency, $\omega - j\alpha$, for which the loop gain is unity. The time dependence therefore becomes $e^{j(\omega - j\alpha)t} = e^{j\omega t}e^{\alpha t}$, showing that the oscillation amplitude grows as $e^{\alpha t}$. This circuit illustrates how any linear circuit with feedback will produce sine-wave oscillations if there is a (complex) frequency for which the overall loop gain is unity and the overall phase shift is 360°. (Of course α must be positive, or the oscillation dies out exponentially.)

11.2.3 Negative-resistance oscillators

In the circuits described above, a transistor provides current to an *RLC* circuit when the voltage on this circuit is positive, i.e., the transistor behaves as a negative resistance. But the transistor is a three-terminal device and the third terminal is provided with a drive signal derived from the *LCR* tank. Figure 11.12 shows how two transistors can be used to make a two-terminal negative resistance that is simply paralleled with the *LCR* tank to make a linear sine wave oscillator that has no feedback loop.

The two transistors form an emitter-coupled differential amplifier in which the resistor to $-V_{ee}$ acts as a constant current source, supplying a bias current, I_0. The input to the amplifier is the base voltage of the right-hand transistor. The output is the collector current of the left-hand transistor. The ratio of input to output is $-4V_T/I_0$, where V_T is the thermal voltage, 26 mV. This ratio is just the negative resistance, since the input and output are tied together. This negative-resistance oscillator uses a parallel-resonant circuit, but a series-resonant version is certainly possible as well.

Any circuit element or device that has a negative slope on at least some portion of its I–V curve can, in principle, be used as a negative resistance. Tunnel diodes can be used to build oscillators up into the microwave frequency range. At microwave frequencies, single-transistor negative-resistance oscillators are common. A plasma discharge exhibits negative resistance and provided a pre-vacuum tube method to generate coherent sine waves. High-efficiency

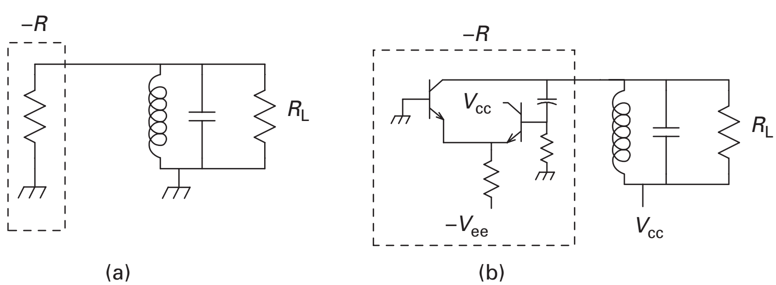


Figure 11.12. A negative-resistance oscillator.

Poulsen arc transmitters, circa World War I, provided low-frequency RF power exceeding 100 kW.

11.3 Oscillator dynamics

These resonant oscillators are basically linear amplifiers with positive feedback. At turn-on they can get started by virtue of their own noise if they run class A. The tiny amount of noise power at the oscillation frequency will grow exponentially into the full-power sine wave. Once running, the signal level is ultimately limited by some nonlinearity. This could be a small-signal nonlinearity in the transistor characteristics. Otherwise, the finite voltage of the dc power provides a severe large-signal nonlinearity, and the operation will shift toward class-C conditions. The fact that amplitude cannot increase indefinitely shows that some nonlinearity is operative in every real oscillator. Any nonlinearity causes the transistor's low- frequency $1/f$ noise to mix with the RF signal, producing more noise close to the carrier than would exist for linear operation. An obvious way to mitigate large-signal nonlinearity is to detect the oscillator's output power and use the detector voltage in a negative feedback arrangement to control the gain. This can maintain an amplitude considerably lower than the power supply voltage. Alternatively, if the oscillator uses a device (transistor or op-amp circuit) with a soft saturation characteristic, the amplitude will reach a limit while the operation is still nearly linear. For example, the amplifier in the oscillator of Figure 11.10 might have a small cubic term, i.e., $V_{\mathrm{OUT}} = AV_{\mathrm{IN}} - BV_{\mathrm{IN}}^3$, where B/A is very small (see Problem 11.5).

11.4 Frequency stability

Long-term (seconds to years) frequency fluctuations are due to component aging and changes in ambient temperature and are called *drift*. Short-term fluctuations, known as oscillator noise, are caused by the noise produced in the active device, the finite loaded Q of the resonant circuit, and nonlinearity in the operating cycle. The higher the Q, the faster the loop phase-shift changes with frequency. Any disturbances (transistor fluctuations, power supply variations changing the transistor's parasitic capacitances, etc.) that tend to change the phase shift will cause the frequency to move slightly to reestablish the overall 360° shift. The higher the resonator Q, the smaller the frequency shift. Note that this is the *loaded* Q, so the most stable oscillators, besides having the highest Q resonators, are loaded as lightly as possible. In LC oscillators, losses in the inductor almost always determine the resonator Q. A shorted piece of transmission line is sometimes used as a high-Q inductor. Chapter 24 treats oscillator noise in detail.

11.5 Colpitts oscillator theory

Let us look in some detail at the operation of the Colpitts oscillator. Figure 11.13 shows the Colpitts oscillator of Figure 11.6(c) redrawn as a small-signal equivalent circuit (compare the figures). The still-to-be-biased transistor is represented as a voltage-controlled current source. The resistor r_{be} represents the small-signal base-to-emitter resistance of the transistor.

The parallel combination of L and the load resistor, R, is denoted as Z, i.e., $Z = \mathrm{j}\omega LR/(\mathrm{j}\omega L+R) = j\omega L_S + R_S$, where L_S and R_S are the component values for the equivalent series network. Likewise, it is convenient to denote r_{be}^{-1} as g. The voltage V_{be}, a phasor, is produced by the current I (a phasor) from the current source. This is a linear circuit, so V_{be} can be written as $V_{be} = I\,Z_T$, where Z_T is a function of ω. We will calculate this "transfer impedance" using standard circuit analysis. Since the current I is proportional to V_{be}, we can write an equation expressing that, in going around the loop, the voltage V_{be} exactly reproduces itself :

$$-\,g_m V_{be} Z_T = V_{be} \quad \text{or} \quad \frac{1}{Z_T} = -g_m. \tag{11.1}$$

This equation will let us find the component values needed for the circuit to oscillate at the desired frequency, i.e., the values that will make the loop gain equal to unity and the phase shift equal 360°.

We can arbitrarily select L, choosing an inductor whose Q is high at the desired frequency. Equation (11.1), really two equations (real and imaginary parts), will then provide values for C_1 and C_2. To derive an expression for Z_T, we will assume that $V_{be} = 1$ and work backward to find the corresponding value of I. With this assumption, inspection of Figure 11.13 shows that the current I_1 is given by

$$I_1 = \mathrm{j}\omega C_2 + g. \tag{11.2}$$

Now the voltage V_c is just the 1 volt assumed for V_{be} plus $I_1 Z$, the voltage developed across Z:

$$V_c = 1 + (\mathrm{j}\omega C_2 + g)Z. \tag{11.3}$$

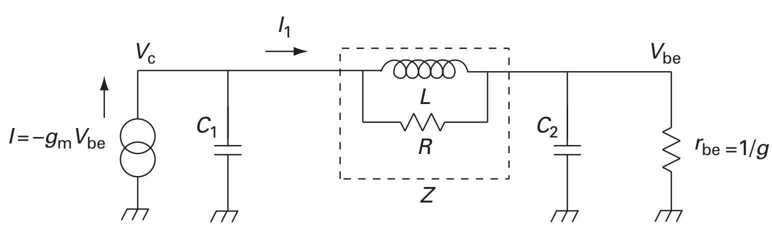


Figure 11.13. Colpitts oscillator small-signal equivalent circuit.

Finally, the current I is just the sum of I_1 plus $V_c\,j\omega C_1$, the current going into C_1:

$$I = (j\omega C_2 + g) + [1 + (j\omega C_2 + g)Z]\,j\omega C_1. \tag{11.4}$$

Since we had assumed that $V_{be} = 1$, we have $Z_T = 1/I$ or

$$\frac{1}{Z_T} = j\omega C_2 + g + [1 + (j\omega C_2 + g)Z]\,j\omega C_1. \tag{11.5}$$

Using this, the condition for oscillation, Equation (11.1) becomes

$$g_m + j\omega C_2 + g + [1 + (j\omega C_2 + g)Z]\,j\omega C_1 = 0. \tag{11.6}$$

The job now is to solve Equation (11.6) for C_1 and C_2. If we assume that ω is real i.e., that the oscillation neither grows nor decays, we find from the imaginary part of this equation, that

$$\frac{C_2 + C_1 + gR_S C_1}{L_S C_1 C_2} = \omega^2 \tag{11.7}$$

and, from the real part, that

$$\omega^2 C_1 C_2 R_S + g(\omega^2 L_S C_1 - 1) = g_m. \tag{11.8}$$

Solving Equations (11.7) and (11.8) simultaneously for C_2 and C_1 produces

$$C_2 = \frac{g_m L_S}{2R_S}\left(1 + \sqrt{\frac{4R_S(1 + gR_S)(g_m + g)}{g_m{}^2 L_S \omega^2}}\right) \tag{11.9}$$

and

$$C_1 = \frac{C_2}{\omega^2 L_S C_2 - 1 - gR_S}. \tag{11.10}$$

Normally C_2 will have a much larger value than C_1 and $\omega \approx 1/\sqrt{LC_1}$. Moreover, the second term in the square root of Equation (11.9) is usually much less than unity so $C_2 \approx g_m L_S/R_S$.

11.5.1 Colpitts oscillator design example

Let us design a practical grounded-emitter Colpitts oscillator. Suppose this oscillator is to supply 1 mW at 5 MHz and that it will be powered by a 6 V dc supply. Assuming full swing, the peak output sine wave voltage will be 6 V. The output power is given by $0.001\ \mathrm{W} = (6\ \mathrm{V})^2/(2R_L)$ so the value of the load resistor, R_L, will be 18 k ohms. Assuming class-A operation, the bias current in the transistor is made equal to the peak current in the load: $I = I_{pk} = 6\ \mathrm{V}/18\ \mathrm{k} = 0.33\ \mathrm{mA}$. If we let the emitter biasing resistor be 1.5 k, the emitter bias voltage will be $1500 \times 0.33\ \mathrm{mA} = 0.5\ \mathrm{V}$. Assuming the typical 0.7 V offset between the base and emitter, the base voltage needs to be 1.2 V. A voltage divider using a 40 k resistor and a 10 k resistor will produce 1.2 V from the 6 V

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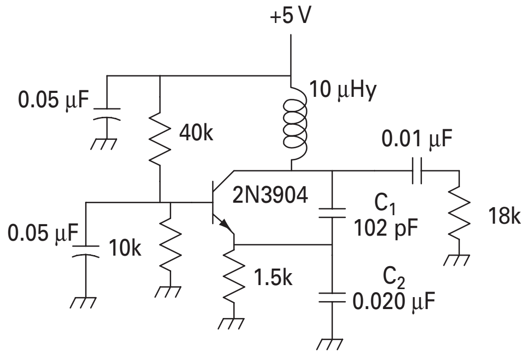

Figure 11.14. Colpitts oscillator: 5 MHz, 1 mW.

supply. These bias components are shown in the schematic diagram of Figure 11.14.

A 0.05 μF bypass capacitor pins the base to ac ground and another bypass capacitor ensures that the dc input is held at a firm RF ground. Note that the 1.5 k emitter bias resistor provides an unwanted signal path to ground. This path could be eliminated by putting an inductor in series with the bias resistor as an RF choke, but this is not really necessary; the1.5k resistor is in parallel with C_2, which will have such a low reactance that the resistor will divert almost no current from it.

With the biasing out of the way, we now deal with the signal components. The transconductance of the transistor is found by dividing the bias current, I_0, by 26 mV, the so-called thermal voltage,[2] i.e., $g_m = 0.33$ mA/ 26 mV = 0.013 mhos. The small-signal base-to-emitter resistance, r, is given by $r = \beta V_{\text{thermal}}/I_0$. For a typical small-signal transistor, such as a 2N3904, β is about 100, so r_{in} = 100·0.026 V/0.33 mA = 8000 ohms.

Using Equation (11.9) and (11.10), the values of C_1 and C_2 are 102 pF and 0.023 μF, respectively. These are the values for which the oscillator theoretically will maintain a constant amplitude. In practice, we increase the feedback by decreasing the value of C_2 to ensure oscillation. This produces a waveform that grows exponentially until it reaches a limit imposed by circuit nonlinearity. The frequency becomes complex, i.e., ω becomes $\omega - j\alpha$ and the time dependence therefore becomes $e^{j(\omega - j\alpha)t} = e^{j\omega t}e^{\alpha t}$. Suppose we want α to be, say 10^5, which will cause oscillation to grow by a factor e every 10 μsec. (Fast growth would be important if, for example, the oscillator is to be rapidly pulsed on and off.) How do we find the value of C_2 to produce the desired α? To avoid doing more analysis, it is convenient to use a standard computer program such as Mathcad to find the root(s) of Equation (11.6) for trial values of C_2. In this example, if we decrease C_2 to 0.020 μF, we obtain the desired α.

[2] The thermal voltage is given by $V_{\text{thermal}} = 0.026\text{V} = kT/e$, where k is Boltzman's constant, T is the absolute ambient temperature, and e is the charge of an electron.

Problems

Problem 11.1. Draw a schematic diagram (without component values) for a bipolar transistor Colpitts oscillator with the collector at ground for both dc RF. Include the biasing circuit. The oscillator is to run from a positive dc supply.

Problem 11.2. Design (without specifying component values) a single-transistor series-mode oscillator based on the emitter follower circuit.

Problem 11.3. A simple computer simulation can illustrate how an oscillator builds up to an amplitude determined by the nonlinearity of its active element. The program shown below models the negative-resistance oscillator of Figure 11.12(a). The *LC* resonant frequency is 1 Hz. This network is in parallel with a negative-resistance element whose voltage vs. current relation is given by $I = -(1/R_n)*(V - \varepsilon V^3)$, to model the circuit of Figure 11.12. The small-signal (negative) resistance is just $-R_n$. The term $-\varepsilon V^3$ makes the resistance become less negative for large signals. The program integrates the second-order differential equation for $V(t)$ and plots the voltage versus time from an arbitrary initial condition, $V = 1$ volt.

Run this or an equivalent program. Change the value of the load resistor R. Find the minimum value of R for sustained oscillation. Experiment with the values of R and R_n. You will find that when the loaded Q of the *RLC* circuit is high, the oscillation will be sinusoidal even when the value of the negative resistor is only a fraction of R. When Q is low (as it is for $R = 1$), a low value of R_n such as $R_n = 0.2$ will produce a distinctly distorted waveform.

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'QBasic simulation of negative-resistance oscillator of Figure 11.12a.
SCREEN 2
R = 1: L = 1 / 6.2832: C = L 'the parallel RLC circuit: 1 'ohms, 1/2pi henries, 1/2pi farads
RN = .9 'run program also with RN=.2 to see non-'sinusoidal waveform
E = .01 'negative resistance: I= (1/RN)*(V-EV^3)
V = 1: U = 0 'initial conditions, V is voltage, U is 'dV/dt
DT = .005 'step size in seconds
FOR I = 1 TO 3000
T = T + DT 'increment the time
VNEW = V + U * DT
U = U + (DT / C) * ((1 / RN) * (U - 3 * E * V * V * U) - V / L - U / R)
V = VNEW
PSET (40 * T, 100 + 5 * V) 'plot the point
NEXT I
```

Problem 11.4. In the oscillator shown below, the voltage gain of the amplifier decreases with amplitude. The voltage transfer function is $V_{out} = 2\ V_{in} - 0.5V_{in}^3$. This characteristic will limit the amplitude of the oscillation.

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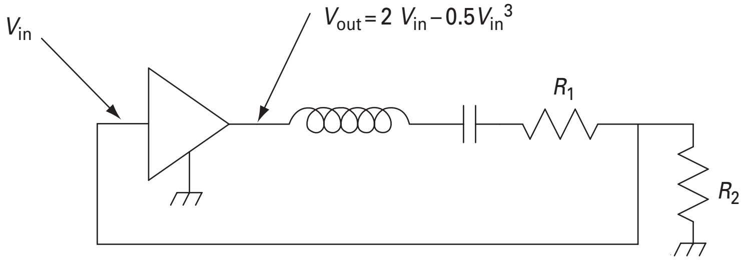

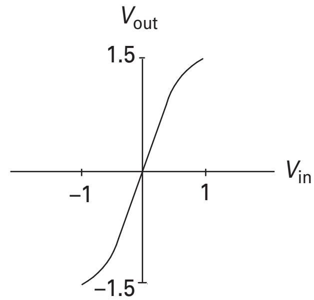

Find the ratio R_2/R_1 in order that the peak value of the sine wave V_{in} will be one volt. Hint: assume $V_{in} = \sin(\omega t)$. The amplifier output is then $2\ \sin(\omega t)\ -0.5\sin^3(\omega t)$. The second term resembles the sine wave but is more peaked. The *LC* filter will pass the fundamental Fourier component of this second term. Find this term and add it to $2\sin(\omega t)$. Then calculate the ratio R_2/R_1 so that the voltage divider output is $\sin(\omega t)$.