CHAPTER

2 Impedance matching

Matching normally means the use of a lossless (nonresistive) network between a signal source and a load in order to maximize the power transferred to the load. This presupposes that the source is not capable of supplying infinite power, i.e., that the source is not just an ideal voltage generator or an ideal current generator. Rather, the source is assumed to be an ideal voltage generator in series with a source impedance, i.e., a Thévenin equivalent circuit, or an ideal current generator in parallel with a source admittance, a Norton equivalent circuit. Note that these equivalent circuits are themselves equivalent; each can be converted into the form of the other. An antenna that is feeding a receiver is an example of an ac signal source connected to a load. Figure 2.1 shows the simplest situation, a dc generator driving a resistive load. The generator is represented in Thévenin style (a) and in Norton style (b).

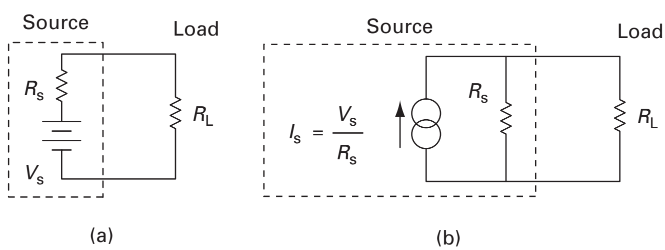


Figure 2.1. DC generator driving a resistive load, Equivalent circuits.

You can see the equivalence by inspection: the generators have the same open-circuit voltage and the same short-circuit current. Maximum power is transferred when the load resistance is made equal to the source resistance. You can show this by differentiating the expression for the power, $P_{load} = [V_S R_L/(R_L+R_S)]^2/R_L$. Figure 2.2 plots the relative transferred power (Pwr/MaxPwr) as a function of the normalized load resistance ($r = R_L/R_S$). In (a) the scales are linear and in (b) the scales are logarithmic so the relative power is expressed in dB. Note that R_L can differ by a factor of 10 from R_S and the power transferred is still 33% of the maximum value.

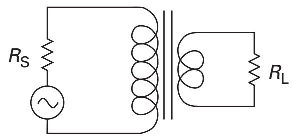


Figure 2.3. Transformer converts R_L to R_S for maximum power transfer.

2.1 Transformer matching

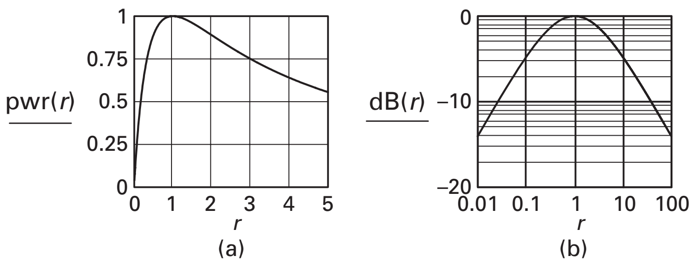


Figure 2.2. Relative power transfer as a function of R_L/R_S, (=r)

In the case of an ac source, a transformer can make the load resistance match (equal) the source resistance (and vice versa) as shown in Figure 2.3. The impedance is transformed by the square of the turns ratio.[1]

The ac situation often has a complication: the source and/or the load may be reactive, i.e., have an unavoidable built-in reactance. An example of a reactive load is an antenna; most antennas are purely resistive at only one frequency. Above this resonant frequency they usually look like a resistance in series with an inductor and below the resonant frequency they look like a resistance in series with a capacitor. An obvious way to deal with this is first to cancel the reactance to make the load and/or source impedance purely resistive and then use a transformer to match the resistances. In the circuit of Figure 2.4, an inductor cancels

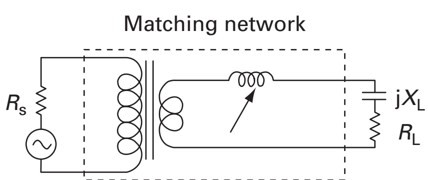


Figure 2.4. A series reactor makes the load a pure resistance.

[1] Let the secondary winding be the load side. Then $V_{sec} = V_{pri}\, N_{sec}/N_{pri}$. For energy to be conserved, $V_{pri}I_{pri} = V_{sec}I_{sec}$. Therefore $I_{sec} = I_{pri}\, N_{pri}/N_{sec}$ and $V_{pri}/I_{pri} = (V_{sec}/I_{sec})\,(N_{pri}/N_{sec})^2$ or $Z_{pri} = Z_{sec}(N_{pri}/N_{sec})^2$.

the reactance of a capacitive (but not purely capacitive) load. If we are working at 60 Hz, we would say the inductor corrects the load's power factor.

From the standpoint of the load, the matching network converts the source impedance, $R_S + j0$, into the complex conjugate of the load impedance. When a matching network is used between two devices, each device will look into an impedance that is the complex conjugate of its own impedance. As a result, the reactances cancel and the resistances are equal. Whenever the source and/or load have a reactive component, the match will be frequency dependent, i.e., away from the design frequency the match will not be perfect. In fact, with reactive sources and/or reactive loads, *any* lossless matching circuit will be frequency dependent – a filter of some kind – whether we like it or not.

2.2 L-networks

More often than not, matching circuits use no transformers (i.e., no coupled inductors). Figure 2.5 shows a two-element L-network (in this figure, a rotated letter L) that will match a source to a load resistor whose resistance is smaller than the source resistance. The trick is to put a reactor, X_P, in *parallel* with the *larger* resistance. Consider a specific example: $R_S = 1000$ and $R_L = 50$.

The impedance of the left-hand side is given by

$$Z_{\text{left}} = R_{\text{left}} + jX_{\text{left}} = \frac{1000\,jX_P}{1000 + jX_P} = \frac{(1000jX_P)(1000 - jX_P)}{(1000 + jX_P)(1000 - jX_P)}$$

$$= \frac{1000^2 jX_P + 1000X_P^2}{1000^2 + X_P^2}. \tag{2.1}$$

We can pick the value of X_P so that the real part of Z_{left} will be 50 ohms, i.e., equal to the load resistance. Using Equation (2.1), we find that $X^2_P = 52\,628$ so we can pick either $X_P = 229$ (an inductor) or $X_P = -229$ (a capacitor). The left-hand side now has the correct equivalent series resistance, 50 ohms, but it is accompanied by an equivalent series reactance, X_{left}, given by the imaginary part of Equation (2.1). We can cancel X_{left} by inserting a series reactor, X_S, equal to $-X_{\text{left}}$. Figure 2.6 shows the matching circuits that result when X_P is an inductor and when X_P is a capacitor.

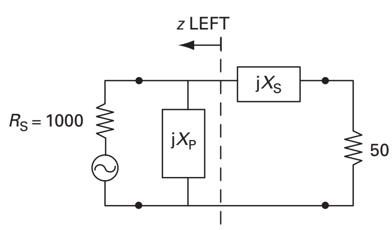


Figure 2.5. Two reactors in an L-network match R_L to R_S.

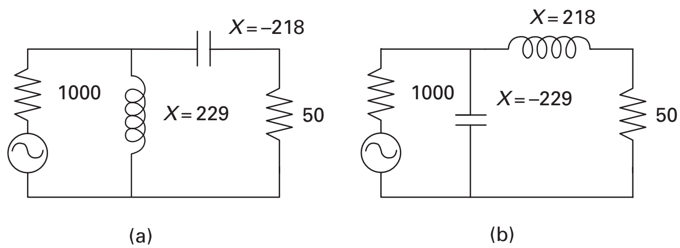


Figure 2.6. The two realizations for the L-network of Figure 2.5.

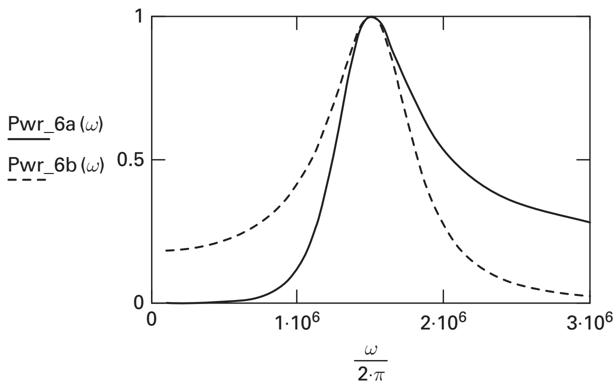


Figure 2.7. Frequency response (power vs. frequency) for the L-networks of Figure 2.6.

The final step is to find the values of L and C that produce the specified reactances at the given frequency. For the circuit of Figure 2.6(b), $\omega L = 218$. Suppose the design frequency is 1.5 MHz ($\omega = 2\pi \cdot 1.5 \cdot 10^6$), near the top of the AM broadcast band. Then $L = 23.1$ µH and $C = 462$ pF. Note that the values of the two reactors are completely determined by the source and load resistances. Except for the choice of which element is to be an inductor and which is to be a capacitor, there are no free parameters in this two-element matching circuit. The match is perfect at the design frequency but, away from that frequency, we must accept the resulting frequency response. The frequency responses (fractional power reaching the load vs. frequency) for the two circuits of Figure 2.6 are plotted in Figure 2.7. Note that around the design frequency, i.e., around the resonant peak, the curves are virtually identical. Otherwise, the complete cutoff at very low frequencies of Figure 2.6a and the complete cutoff at very high frequencies of Figure 2.6b can be predicted from inspection of the circuits.

Quick design procedure for L-networks

If you remember only that the parallel reactance goes across the larger resistance you will be able to repeat the steps used above and design L-networks. But if you are doing these things often it may be worth memorizing the following "Q factor" for L-network design:

$$Q_{\mathrm{EL}} = \sqrt{\frac{R_{\mathrm{high}}}{R_{\mathrm{low}}} - 1}. \tag{2.2}$$

You can verify (Problem 2.6) that the ratios $R_{\mathrm{high}}/X_{\mathrm{parallel}}$ and $X_{\mathrm{series}}/R_{\mathrm{low}}$ are both equal to this factor, Q_{EL}. Remember the definition of Q_{EL} and these ratios immediately give you the L-network reactance values. You can also verify that, when Q_{EL} is large, the two elements in an L-network have nearly equal and opposite reactances, i.e., together they resonate at the design frequency. In this case the magnitude of the reactances is given by the geometric mean of R_{high} and R_{low} (especially easy to remember).

When the ratio of the source resistance to the load resistance is much different from unity, an L-network produces a narrowband match, i.e., the match will be good only very close to the design frequency. Conversely, when the impedance ratio is close to unity, the match is wide. The width of any resonance phenomenon is described by a factor, *the effective Q* (or *circuit Q* or just *Q*), which is equal to the center frequency divided by the two-sided 3-dB bandwidth (the difference between the half-power points). Equivalently, Q_{eff} is the reciprocal of the fractional bandwidth. When an ideal voltage generator drives a simple *RLC* series circuit, Q_{eff} is given by X/R where X is either X_{L} or X_{C} at the center frequency (since they are equal). The L-network matching circuit is equivalent to a simple series *RLC* circuit, but Q_{EL} is twice Q_{eff} because the nonzero source resistance is also in series; the matching circuit makes the effective source resistance equal to the load resistance so the loop's total series resistance is twice the load resistance. As a result, the fractional bandwidth is given by $1/Q_{\mathrm{eff}} = 2/Q_{\mathrm{EL}}$. In many applications the bandwidth of the match is important and the match provided by the L-network (which is completely determined by the source and load resistances) may be too narrow or too wide. When matching an antenna to a receiver, for example, one wants a narrow bandwidth so that signals from strong nearby stations won't overload the receiver. In another situation the signal produced by a modulated transmitter might have more bandwidth than the L-network would pass. Networks described below solve these problems.

2.3 Higher *Q* – pi and T-networks

Higher *Q* can be obtained with back-to-back L-networks, each one transforming down to a center impedance that is lower than either the generator or the source resistance. The resulting pi-network is shown in Figure 2.8.

With the simple L-networks we had $Q_{\mathrm{EL}} = \sqrt{19} = 4.4$. In this pi-network both the 1000-ohm source and the 50-ohm load are matched down to a center impedance of 10 ohms (a free parameter). The bandwidth is equivalent to that of an L-network with $Q_{\mathrm{EL}} = 11.95$. When $R_{\mathrm{HIGH}} \gg R_{\mathrm{LOW}}$, the pi-network has a bandwidth equivalent to that of an L-network with $Q_{\mathrm{EL}} = \sqrt{R_{\mathrm{HIGH}}/R_{\mathrm{CENTER}}}$. Again, the fractional bandwidth is given by $1/Q_{\mathrm{eff}} = 2/Q_{\mathrm{EL}}$.

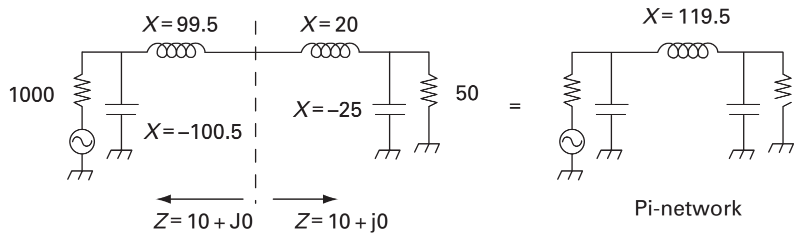


Figure 2.8. Pi-network (back-to-back L-networks) provides higher Q.

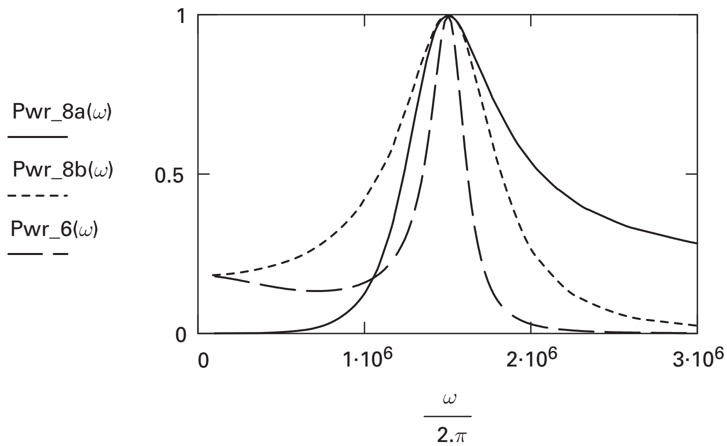


Figure 2.9. Response of the pi-network of Figure 2.8 compared with the L-networks of Figure 2.6.

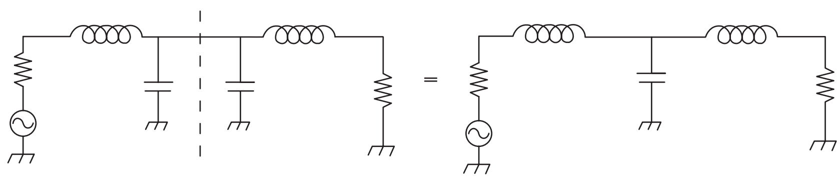


Figure 2.10. The T-network, like the pi-network, provides higher Q.

The response of this pi-network is shown in Figure 2.9 together with the responses of the L-networks of Figure 2.6.

You can guess that we could just as well have used "front-to-front" L-networks, each one transforming up to a center impedance that is higher than both the source and load impedances. This produces the T-network of Figure 2.10. Note that both the pi-network and the T-network have a free parameter (the center impedance) which gives us some control over the frequency response while still providing a perfect match at the center or design frequency.

2.4 Lower Q – the double L-network

In a double L-network (Figure 2.11) the first stage transforms to an impedance value between the source and load impedances. The second stage takes it the

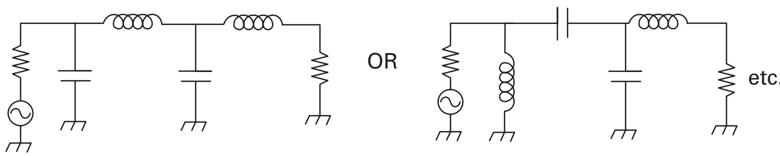


Figure 2.11. Double L-network for lower Q (wider bandwidth).

rest of the way. The process can, of course, be done in smaller steps with any number of cascaded networks. A long chain of L-networks forms an artificial transmission line that tapers in impedance to produce a frequency-independent match. Real transmission lines (i.e., lines with distributed L and C) are sometimes physically tapered to provide this kind of impedance transformation. A tapered transmission line is sometimes called a transformer, since, like the transformer in Figure 2.3, it provides frequency-independent matching.

2.5 Equivalent series and parallel circuits

To design the L-network we used the fact that a two-element parallel XR circuit, where $1/Z = 1/R_{\text{parallel}} + 1/\text{j}X_{\text{parallel}}$, has an equivalent series circuit, where $Z = R_{\text{series}} + \text{j}X_{\text{series}}$. Conversion between equivalent series and parallel representations is used so often it is worth a few more words. If you are given, for example, an antenna or a black box with two terminals and you make measurements at a *single frequency* you can only determine whether the box is "capacitive," i.e., equivalent to an RC combination, or is "inductive," i.e., equivalent to an RL combination. Suppose it is capacitive. Then you can represent it equally well as a series circuit where $Z = R_{\text{series}} + 1/\text{j}\omega C_{\text{series}}$ or as a parallel circuit where $1/Z = 1/R_{\text{parallel}} + \text{j}\omega C_{\text{parallel}}$. As long as you're working only at (or never very far from) the single frequency, either representation is equally valid, even if the box contains a complicated circuit with discrete resistors, capacitors, inductors, transmission lines, metallic and resistive structures, etc. If you measure the impedance at more than one frequency you might determine that the box does indeed contain a simple parallel RC or series RC circuit or that its impedance variation at least resembles that of a simple parallel circuit more than it resembles that of a simple series circuit.

2.6 Lossy components and efficiency of matching networks

So far we have considered networks made of ideal inductors and capacitors. Real components, however, are lossy due to the finite conductivity of metals, lossy dielectrics or magnetic materials, and even radiation. Power dissipated in nonideal components is power that does not reach the load so, with lossy components, we must consider a matching network's efficiency. As explained above, a lossy reactor can be modeled as an ideal L or C together with either a series or parallel resistor. Normally we can make the approximation that the values of L or C and the value of the associated resistor are constant throughout the band of interest. Let us consider the efficiency of the L-network that uses a series inductor and a parallel capacitor. We shall assume that the loss in the capacitor is negligible compared to the loss in the inductor. (This is very often the case with lumped components.) We shall model the lossy inductor as an ideal inductor in series with a resistor of value r_S. The ratio of the inductive reactance, X_L, to this resistance value is the *quality factor*, Q_U, where the subscript denotes "*unloaded Q*" or *component Q*. (Less series resistance certainly implies a higher quality component.) Note that this resistance, like the inductor, is in series with the load resistor so the same current, I, flows through both. The power delivered to the load is I^2R_L and the power dissipated in r_S is I^2r_S. Using the relations $X_S = Q_{EL}R_{load}$ and $Q_U = X_S/r_S$, we find the efficiency of the match is given by

$$\eta = \text{Efficiency} = \frac{\text{Power Out}}{\text{Power In}} = \frac{I^2R_L}{I^2R_L + I^2r_S} = \frac{1}{1 + Q_{EL}/Q_U}. \tag{2.3}$$

Efficiency is maximized by maximizing the ratio Q_U/Q_{EL}, i.e., the ratio of unloaded Q to loaded Q. If we model the lossy inductor as a parallel LR circuit and define the unloaded Q as r_P/X we would get the same expression for efficiency (Problem 2.7). Likewise, if the loss occurs in the capacitor we will also get this expression, as long as we define the unloaded Q of the capacitor again as parallel resistance over parallel reactance or as series reactance over series resistance. When the load resistance is very different from the source resistance, the effective Q of an L-network will be high so, for high efficiency, the unloaded Q of the components must be very high. The double L-network, with its lower loaded Q's, can be used to provide higher efficiency.

Q factor summary

Loaded Q, the Q factor associated with *circuits*, can be either high or low depending on the application. Narrowband filters have high loaded Q. Wideband matching circuits have low loaded Q. Loaded Q is therefore not a measure of quality. Unloaded Q, however, which specifies the losses in *components*, is indeed a measure of quality since lowering component losses always increases circuit efficiency.

Problems

Problem 2.1. A nominal 47-ohm, $\frac{1}{4}$-watt carbon resistor with 1.5 inch wire leads is measured at 100 MHz to have an impedance of 48+j39 ohms. Find the component values for (a) an equivalent series RL circuit, and (b) an equivalent parallel RL circuit.

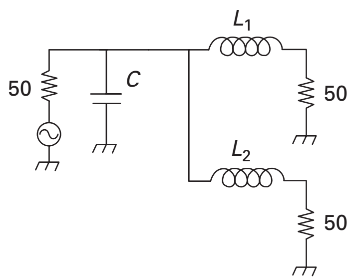
Problem 2.2. (a) Design an L-network to match a 50-ohm generator to a 100-ohm load at a frequency of 1.5 MHz. Let the parallel element be an inductor. Use your circuit analysis program (Problem 1.3) to find the frequency response of this circuit from 1 MHz to 2 MHz in steps of 50 kHz.

(b) Same as (a), but let the parallel element be a capacitor.

Problem 2.3. Design a double L matching network for the generator, source, and frequency of Problem 2.2(a). For maximum bandwidth, let the intermediate impedance be the geometric mean of the source impedance and the load impedance, i.e., $\sqrt{50 \cdot 100}$. Use your circuit analysis program (Problem 1.3) to find the response as in Problem 2.2.

Problem 2.4. Suppose the only inductors available for building the networks of Problems 2.2(a) and 2.3 have a Q_U (unloaded Q) of 100 at 1.5 MHz. Assume the capacitors have no loss. Calculate the efficiencies of the matching networks at 1.5 MHz. Check your results using your circuit analysis program.

Problem 2.5. The diagram below shows a network that allows a 50-ohm generator to feed two loads (which might be antennas). The network divides the power such that the top load receives twice as much power as the bottom load. The generator is matched, i.e., it sees 50 ohms. Find the values of X_{L1} , X_{L2} and X_C. Hint: transform each load first with an L-section network and then combine the two networks into the circuit shown.



Problem 2.6. Verify the prescription given for calculating the values of an L-network: $X_P = \pm\, R/Q$ and $X_S = \mp\, rQ$ where $R > r$ and $Q = \sqrt{R/r - 1}$.

Problem 2.7. At a single frequency, a lossy inductor can be modeled as a lossless inductor in series with a resistance or as a lossless inductor in parallel with a resistance. Convert the series combination r_S, L_S to its equivalent parallel combination r_P, L_P and show that Q_U defined as X_S/r_S is equal to Q_U defined as r_P/X_P.