CHAPTER

15 Hybrid couplers

Hybrid couplers, also known as hybrid junctions or simply “hybrids,” are lossless passive four-port devices used to make interconnections between circuit elements. Hybrids are used as power dividers (“signal splitters”) and combiners. They are also used in mixers and TR (transmit/receive) switches. A useful schematic representation for a hybrid, Figure 15.1, shows the four connection points (ports).

RF hybrids usually have unbalanced ports designed for coaxial transmission lines, so all four ports share a common ground, indicated in Figure 15.1 by a dotted ground symbol (usually not shown). Each port has a characteristic impedance. Most packaged RF hybrids with coaxial ports are made so that the characteristic impedance of all four ports is 50 or 75 ohms. The symbol in Figure 15.1 shows signal flow paths; power incident on Port 1 splits and exits through Ports 2 and 3. If both of these ports are properly terminated there will be no reflections and the impedance seen looking into Port 1 will be equal to the characteristic impedance of that port. In this case no power will reach Port 4 as opposite ports are isolated. But if Port 2 and/or Port 3 are not terminated in their own characteristic impedances, the power exiting these ports will be partially or completely reflected back into the hybrid. The reflection, which depends on the mismatch, is calculated exactly as if the power had exited from a transmission line whose impedance is equal to that of the respective port. Any power reflected back into the hybrid splits and follows the signal paths, just as if it had come from an external source. You can see that, with arbitrary terminations and arbitrary signals, the situation could become complicated. But usually we deal with continuous wave (cw) sinusoidal signals so, rather than analyze multiple reflections in the time domain, we only have to solve for the forward and reverse wave amplitudes on each of the four internal paths. In most applications, things are even simpler; hybrids usually have proper terminations and the signal flows are simple and can be determined by inspection of the signal flow diagram.

15.1 Directional coupling

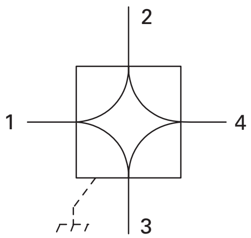


Figure 15.1. Schematic symbol for a hybrid coupler.

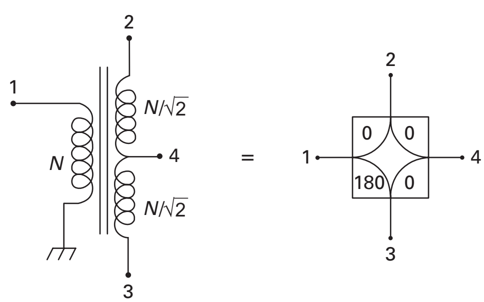
From inspection of the signal paths in Figure 15.1, we see that with the ports matched and with power flowing from Port 1 to Port 2 there will also be power flowing out of Port 3 but none out of Port 4. If the power is now reversed, to flow from Port 2 to Port 1, there will be power flowing out of Port 4 but none from Port 3. Port 3 is therefore coupled to power flowing from 1 to 2. Likewise, Port 4 is coupled to power flowing from 2 to 1. Therefore, a hybrid is a *directional coupler*, and can be used to determine how much power is flowing in each direction on a transmission line. Here we will use the term *hybrid* only for 3-dB directional couplers, i.e., directional couplers that split the incident power in half.

15.2 Transformer hybrid

The name *hybrid transformer* was first applied around 1920[1] to the simple center-tapped transformer shown in Figure 15.2. The primary winding has N turns while each half of the secondary winding has $N/\sqrt{2}$ turns. For this transformer hybrid (hybrid composed of a transformer), the characteristic impedances of Ports 1, 2, and 3 are equal and are twice the impedance of Port 4. (Here the port impedances are R, R, R, and $R/2$, where the value of R is arbitrary, as long as the transformer behaves as an ideal transformer, i.e., its magnetizing inductance has a reactance substantially larger than R.)

Let us confirm that this circuit has the power splitting and isolation characteristics of a hybrid. First consider a signal connected to Port 1. If Ports 2 and 3 have identical terminations, they will have equal and opposite voltages. The voltage at Port 4, since it is midway between the voltages at Ports 2 and 3, must be zero and Port 4 is indeed isolated from Port 1. Next note that a signal applied to Port 4 will appear unchanged at Ports 2 and 3 but will not appear at Port 1. (The currents to Ports 2 and 3 are in opposite directions so there is no net flux in the transformer to provide a voltage at Port 1 or produce an IX_L drop.) You can verify that Ports 2 and 3 are also isolated from each other (see Problem 15.1). Figure 15.2 also shows the symbol appropriate

Figure 15.2. Transformer hybrid.



[1] The origin of the term seems to be lost – a hybrid of what? – but it came from the telephone industry, where the terms *hybrid transformer* and *hybrid coil* were both common.

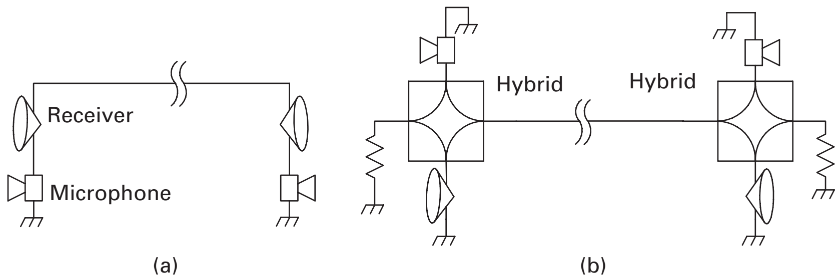


Figure 15.3. Hybrids allow full-duplex communication over a single line.

for this hybrid. The labels 0, 0, 0, and 180 indicate the phase shifts through the respective branches. A signal incident on Port 1, for example, appears at Port 2 with the phase unchanged (shifted 0°) and at Port 3 with its polarity inverted (shifted 180°). Any hybrid with these four phase shifts is called a *180° hybrid*.

15.2.1 Applications of the transformer hybrid

In telephony or other wired communication, hybrids allow a transmission line to carry independent signals in each direction. Figure 15.3 shows two telephone circuits. In the simple series circuit of Figure 15.3(a), each user hears his own voice as well as the voice from the other end. In the circuit of Figure 15.3(b), hybrids isolate each receiver from its own microphone.

If we are using the transformer hybrid of Figure 15.2, we would terminate Port 4 with a resistor of value $Z_0/2$, where Z_0 is the characteristic impedance of the phone line. The microphones and receivers must have impedances equal to Z_0. This arrangement provides two-way signaling , “full duplex,” over a single cable.[2]

The circuit of Figure 15.4 uses two hybrids and two amplifiers to make a bidirectional repeater for a long (lossy) line. Here the hybrids let the signals in each direction be independently amplified without feedback and consequent oscillation.

It is convenient to make the characteristic impedances be the same for all four ports of a general-purpose hybrid. The transformer hybrid, fixed up to have equal impedances, is shown in Figure 15.5. This is the kind of circuit found inside an off-the-shelf wideband 3-dB hybrid. Transformer hybrids made with toroidal cores (ferrite beads) can work over large bandwidths, e.g., 10 KHz to 20 MHz and 1 MHz to 500 MHz.

[2] In telephony, the circuit is deliberately unbalanced – just enough so that the users, hearing their own voices or ambient noise, will sense that the call is connected, but not enough that the users hold the receiver (and hence the microphone) away from their heads. Of course cancellation should be as great as possible when this kind of full-duplex circuit carries two-way digital data.

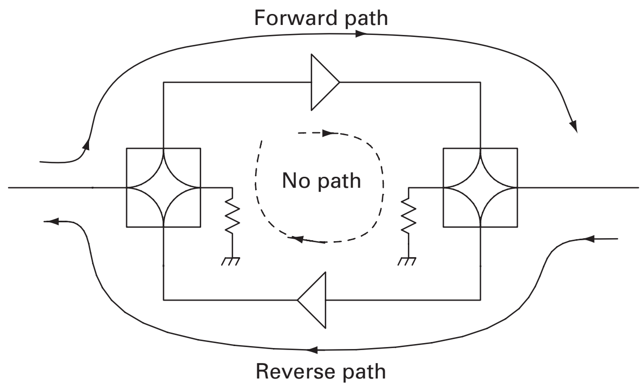


Figure 15.4. Two-way telephone repeater for long lines.

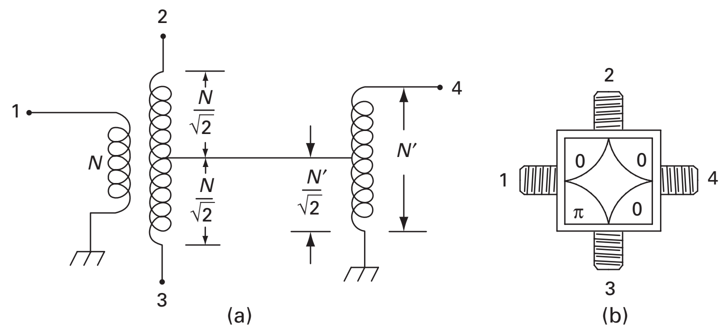
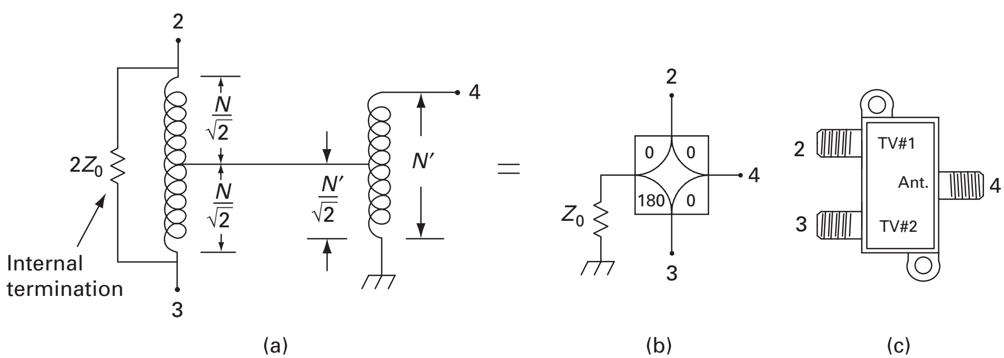


Figure 15.5. Two-transformers make a hybrid with the same impedance at all four ports.

Hybrids are often used in circuits like those illustrated above, as well as signal splitting and combining, where one port is terminated in its characteristic impedance. An easy way to terminate Port 1 of the hybrid of Figure 15.5 is to put a resistor of value $2Z_0$ between Ports 2 and 3. When this is done, the Port 1 winding on the hybrid transformer can be eliminated. The resulting circuit, shown in Figure 15.6a, is

Figure 15.6. Internally terminated hybrid is a two-way splitter/combiner.



commonly found in the signal splitters used to connect two receivers to a single antenna (c) and in other packaged 2:1 splitter/combiners. When it is acceptable for the impedance of Port 4 to be $Z_0/2$, the right-hand transformer can be omitted, and the hybrid consists only of the center-tapped *hybrid coil*.

15.3 Quadrature hybrids

The transformer hybrid is naturally a 180° hybrid. Other circuits are natural 90° hybrids, the symbol for which is shown in Figure 15.7.

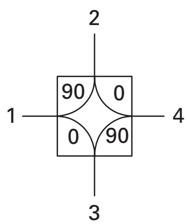


Figure 15.7. Symbol for a 90° hybrid.

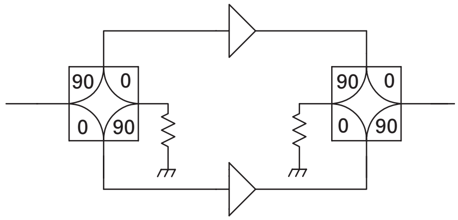
Let us look at an interesting application of the 90° hybrid (often called a *quadrature hybrid*). Here the internal phase paths are zero and 90°. A 90° path means a phase shift equal to that produced by a quarter-wave length of cable. For circuit analysis, one deals with hybrids in terms of voltages. We will consider the hybrid to be connected to transmission lines (of the same impedance as the hybrid) in order to describe the signals in terms of incident and reflected waves. A signal incident at Port 1 will be split equally into signals exiting Ports 2 and 3. Since the power division is equal, the magnitudes of the voltages of the signals exiting Ports 2 and 3 will be $1/\sqrt{2}$ times the magnitude of the incident voltage. The phases of the exiting signals will be delayed as indicated on the symbol for the hybrid. For the hybrid of Figure 15.7, the signal exiting Port 3 has no additional phase shift but the signal exiting Port 2 is multiplied by $e^{-j\pi/2}$. Suppose a signal is also incident at Port 4. It will also split into signals exiting from Ports 2 and 3. The total voltage of the waves exiting Ports 2 and 3 is just the superposition of the waves originating from Ports 1 and 4.

15.3.1 Balanced amplifier

A common application for 90° hybrids is the balanced amplifier circuit shown in Figure 15.8. As long as the two amplifiers are identical they can have arbitrary input and output impedances but the overall circuit will have input and output impedances of Z_0.

To see how this happens, suppose that the hybrids are 50-ohm devices but that the input impedance of the amplifiers is not 50 ohms. Imagine that the interconnections are made using 50-ohm transmission line. The input lines have equal lengths and the output lines have equal lengths. The amplifiers are identical so the two signals have equal phase changes upon reflection. An input signal is split by the input hybrid; half the power will be incident on the top amplifier and half on the bottom amplifier. But reflections from the amplifiers will be out of phase by 180° when they arrive back at the input of the hybrid because the signal on the upper path will have made a round trip through the 90° arm of the hybrid. The two reflections therefore cancel and there is no net

Figure 15.8. A balanced amplifier has constant input and output impedances.

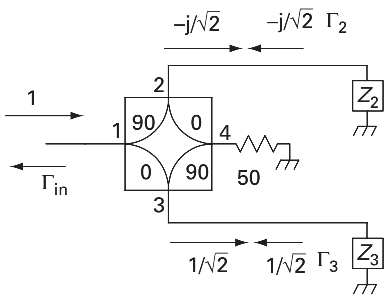


reflection. The input impedance of the overall amplifier will be the characteristic impedance of the hybrid (the value of the resistors, if transformer hybrids are used). The output side works the same way, and this combination of two arbitrary but identical amplifiers produces an amplifier with ideal constant input and output impedances.

15.4 How to analyze circuits containing hybrids

Let us work out an example problem to illustrate the way voltages are added in the manner of transmission line analysis. In Figure 15.9, a 50-ohm hybrid has arbitrary impedances, Z_2 and Z_3 terminating Ports 2 and 3. Port 4 is terminated in 50 ohms, so no power exiting Port 4 will be reflected back into the hybrid. We want to find the impedance seen looking into Port 1. Rather than work directly with impedances, we work with the equivalent reflection coefficients, $\Gamma = (Z - Z_0)/(Z+Z_0)$. Once we have found Γ_{in}, it can be converted to Z_{in}. The input signal is denoted by the forward arrow at Port 1. We will give it unity amplitude.

Figure 15.9. An example circuit problem: finding the impedance looking into Port 1.



The signal incident on Z_2 is therefore $(1/\sqrt{2})e^{-j\pi/2} = -j/\sqrt{2}$ where $1/\sqrt{2}$ is the reduction in amplitude due to the equal power split and $-\pi/2$ is the phase shift of the 90° path between Ports 1 and 2. The signal reflected back into Port 2 is $[-j\sqrt{2}]\Gamma_2$, i.e., the signal incident on Z_2 multiplied by the reflection coefficient of Z_2. This reflected signal will split as it enters Port 2 and its contribution to the wave leaving Port 1 will be its amplitude multiplied by $(1/\sqrt{2})e^{-j\pi/2}$ as it is split and phase shifted by the 90° path or $[\,-j(\sqrt{2})]\Gamma_2 \times -j(\sqrt{2}) = -\Gamma_2/2$.

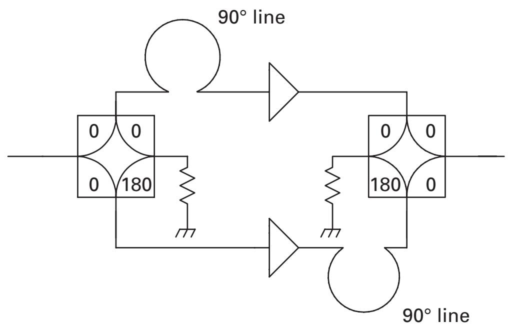


Figure 15.10. A balanced amplifier built with 180° hybrids.

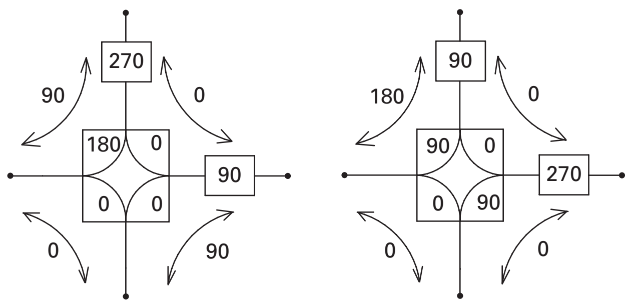


Figure 15.11. Conversions between 90° and 180° hybrids

Similarly, the contribution to Γ from the reflection at Z_3 is $\Gamma_3/2$, and the overall reflection is given by $\Gamma = -\Gamma_2/2 + \Gamma_3/2$, which solves the problem. The reader can work out the general case where Port 4 is also terminated by an arbitrary impedance, Z_4. Hint: let V_2, V_3, and V_4 be the wave amplitudes flowing out of Ports 2, 3, and 4. The waves flowing into these ports will therefore have amplitudes $\Gamma_2 V_2$, $\Gamma_3 V_3$, and $\Gamma_4 V_4$. By examination of the circuit, write three equations for V_2, V_3, and V_4, the three unknowns.

A balanced amplifier can also be built with 180° hybrids if two 90° lines are added as shown in Figure 15.10.

This use of cables to allow substitution of 180° hybrids for 90° hybrids in this circuit can be taken much farther; any hybrid can be converted to any other hybrid by adding lengths of transmission line to the ports. Figure 15.11 shows how to convert a 180° hybrid into a 90° hybrid and vice versa.

15.5 Power combining and splitting

An obvious way to power N loads from a single 50-ohm source is to transform the impedance of each load to $50N + j0$ ohms and then connect the transformed

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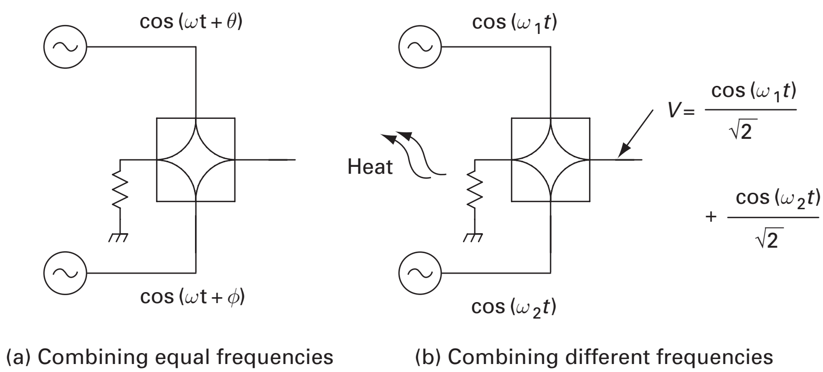

Figure 15.12. Hybrids used as power combiners.

loads in parallel across the generator. However, if any load changes, the power delivered to the other loads will change. A similar argument applies to combining the power from several sources: if any source changes amplitude or phase, the combined output will change. Hybrids provide a way to combine or split power without using impedance transformation and in a way that isolates multiple sources or multiple loads from each other.

To use a hybrid as a two-input power combiner, the unused port is terminated, either externally, as shown in Figure 15.12 or internally, as shown in Figure 15.6. The two signals to be combined should have equal amplitudes, as well as the correct phase relationship, to steer the total available power into the desired port. If the phase difference is changed by 180°, all the power will flow into the terminated port. Note that if one source fails, the other source will not know it; it still sees a matched load. This provides a fail-safe circuit, although the power output will drop by 75%.

When a hybrid is used to combine two signals of different frequencies, as in Figure 15.12(b), half the power of each signal will always be lost in the terminated port. Circuits to combine signals of different frequencies without loss are known as diplexers. (How would you make one?)

Power splitting is, of course, just time-reversed power combining. When hybrids are used as splitters and the source impedance is equal to the port impedance, the signal at any output port will remain constant when the loads on the other output ports are removed, shorted, or changed in any way.

15.5.1 Hybrid trees

Trees of hybrids are often used to make multi-input combiners or multiple-output splitters, as shown in Figure 15.13. High-power transmitters often use such tree structures to combine the power of many low-power solid-state amplifiers.

These trees have the same advantages as single hybrids, when used for power combining or splitting. A 2^N-to-1 combiner/splitter has $2^N - 1$ internal hybrids.

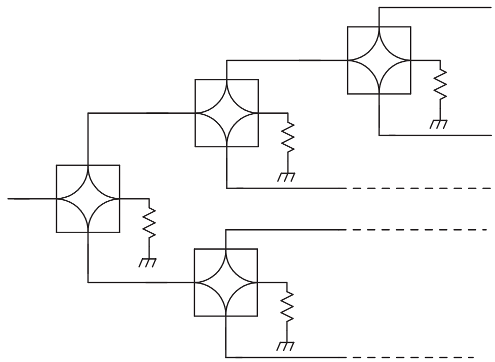

Figure 15.13. A tree of hybrids.

If you buy a three-way TV antenna splitter, you may find that each output provides only one quarter, rather than one third of the input power. What would have been a fourth output port is internally terminated. Or, if the splitter contains two hybrids, one output port can supply half the input power while the other two output ports can each supply one quarter of the input power.

15.6 Other hybrids

There are many ways to make hybrids without transformers. Most of them are circuits whose elements are transmission lines or capacitors and inductors. Unlike the ideal transformer hybrid, these hybrids are all frequency dependent – perfect hybrids only at their center frequency. However, most have a useful bandwidth of about an octave, which is usually sufficient for RF and microwave applications. You can use straightforward circuit analysis to analyze these hybrids.

15.6.1 Wilkinson power divider (or combiner)

This hybrid, shown in Figure 15.14 (in a 50-ohm version), uses two quarter-wave pieces of 70.7 $(50\sqrt{2})$ ohm transmission line. It has only three external ports; the fourth port is internally terminated, that is, connected to a load equal to its characteristic impedance.

It is easy to see that power applied to Port 1 will divide between Ports 2 and 3. By symmetry, the voltages at Ports 2 and 3 must be identical so no power is dissipated in the internal termination. Fifty-ohm loads at Ports 2 and 3 are transformed by the 90° cables to 100 ohms. The parallel connection of 100 ohms and 100 ohms at Port 1 produces the desired 50-ohm input impedance. The Wilkinson divider is usually classified as a 180° hybrid since its outputs have the same phase, even though this phase is 90° rather than 0°.

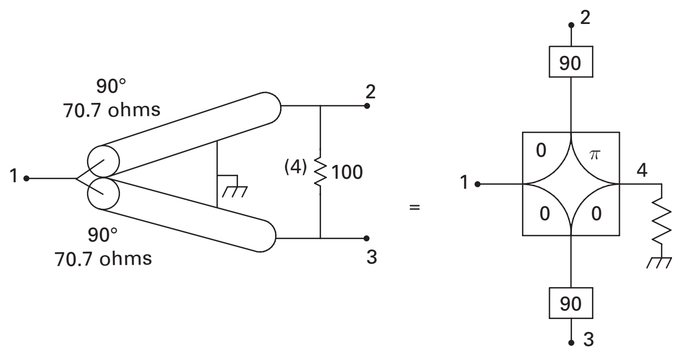

Figure 15.14. Wilkinson power divider.

15.6.2 Ring hybrid

The ring hybrid, shown in Figure 15.15, uses four pieces of transmission line. To have 50-ohm ports, the hybrid must be built from 70.7-ohm transmission line.

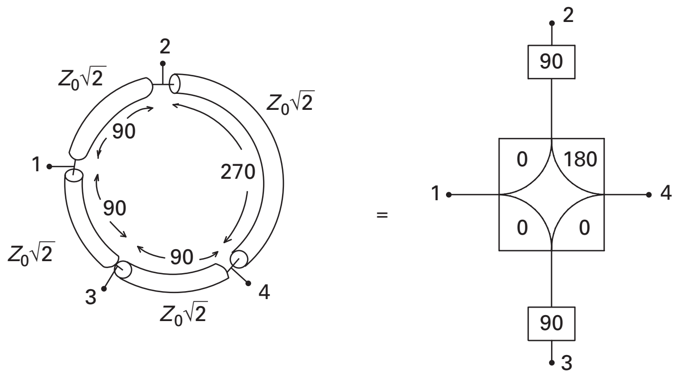

Figure 15.15. Ring hybrid.

Branch-line hybrids

These are ladder networks made of quarter-wave lengths of transmission line. The simplest is shown in Figure 15.16. More complicated versions have more branches and provide more bandwidth.

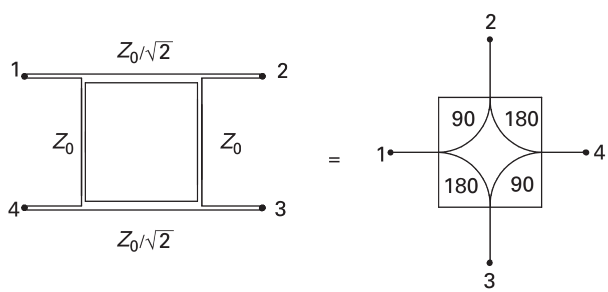

Figure 15.16. Branch line hybrid.

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15.6.3 Lumped element hybrids

Two examples of 50-ohm lumped element hybrids are shown in Figure 15.17. These two circuits are obtained by replacing the Z_0 and $Z_0/\sqrt{2}$ arms of the simple branch line hybrid with the pi (or T) lumped circuit impedance inverters discussed in Chapter 13.

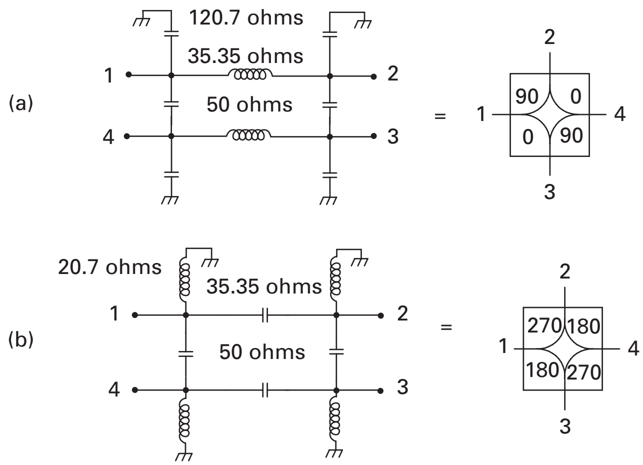

Figure 15.17. Two lumped-element hybrids.

15.6.4 Backward coupler

The *backward coupler* is shown in Figure 15.18 in shielded pair and stripline versions. The coupled transmission lines have both electric and magnetic coupling. When power flows from left to right, between Ports 1 and 2, coupled power flows left (backwards) out of Port 3. This type of coupler can be designed so that the voltage coupling coefficient, c, between Ports 1 and 3 is anywhere between 0 and 1. If designed for $c = 1/\sqrt{2}$, the coupler is a hybrid (a 3-dB directional coupler). This coupler can be analyzed in terms of its common mode and differential characteristic impedances, Z_{CM} and Z_{DIFF}, which are defined as follows. If the two inner conductors are driven together as a single (though split) center conductor, the characteristic impedance of the line is Z_{CM}. When the two inner conductors are regarded as a balanced shielded transmission line, its characteristic impedance is Z_{DIFF}. Two equivalent parameters are defined: $Z_{EVEN} = 2\,Z_{CM}$ and $Z_{ODD} = Z_{DIFF}/2$. Analysis using simultaneous forward and reflected common mode and differential mode signals (see Problem 15.8) produces the formulas:

$$Z_{ODD} = Z_0([1-c]/[1+c])^{1/2} \text{ and } Z_{EVEN} = Z_0([1+c]/[1-c])^{1/2} \quad (15.1)$$

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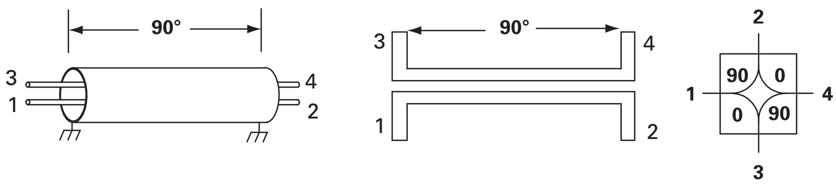


Figure 15.18. Backward couplers (coupled transmission line hybrids).

where Z_0 is the desired port impedance. However, these are only indirect design formulas, since electromagnetic analysis is needed to find physical dimensions to produce the required values for Z_{ODD} and Z_{EVEN}.

The *magic-T* of Figure 15.19 is a four-port waveguide junction that combines an *E*-plane tee junction with an *H*-plane tee junction. This waveguide hybrid, whose operation is surprisingly simple, is discussed in Chapter 16.

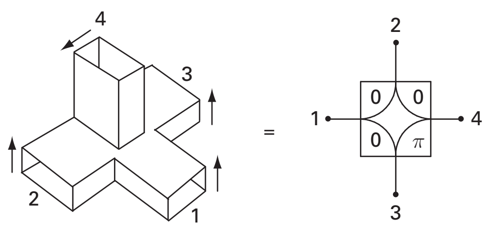


Figure 15.19. Waveguide magic-T hybrid.

Problems

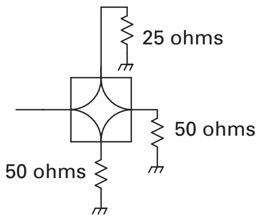
Problem 15.1. Verify for the transformer hybrid of Figure 15.2 that Port 2 is isolated from Port 3, i.e., show that with a generator connected to Port 3, the voltage at Port 2 will be zero provided the impedance terminating Port 1 is twice the impedance terminating Port 4.

Problem 15.2. Explain why the duplex telephone circuit and the two-way telephone repeater could be built with either 90° or 180° or any other hybrids.

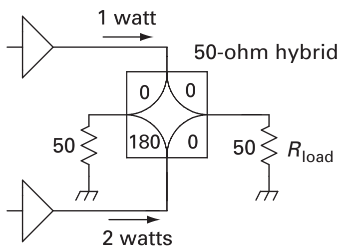
Problem 15.3. (a) Calculate the overall gain of the balanced amplifier of Figure 15.8 if the (identical) individual amplifiers each have gain G_0. (Answer: G_0)

(b) Calculate the overall gain if one of the interior amplifiers is dead, i.e., has zero gain. (Answer: $G_0/4$.)

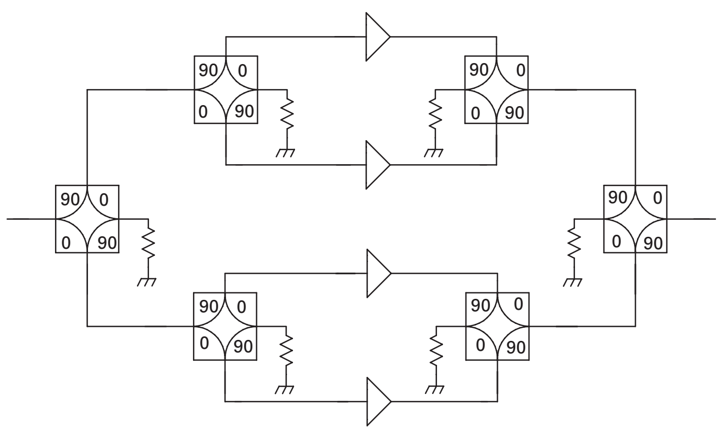
Problem 15.4. The 50-ohm hybrid shown below is properly terminated by 50-ohm resistors on two of its ports. The third port is terminated by a 25-ohm resistor. If a 50-ohm generator is connected to the remaining port, what fraction of the incident power will be reflected back into the generator? (Answer: 1/36.)



Problem 15.5. In the figure below, a 50-ohm hybrid is fed power from two amplifiers. The signals from these amplifiers have the same frequency and the same phase but the upper amplifier supplies 1 watt while the lower amplifier supplies 2 watts. In this seemingly unbalanced configuration, how much power reaches the load resistor? (Answer: 2.91 watts.)



Problem 15.6. Four identical 1-watt amplifiers and six hybrids are used to make a 4-watt amplifier. If one of the interior amplifiers fails, how much power will be delivered to the load? (Answer: 2.25 watts.)



Problem 15.7. Design an op-amp circuit to replace the telephone hybrids in Figure 15.3(b). Put a low-value resistor in series with the line so that a differential

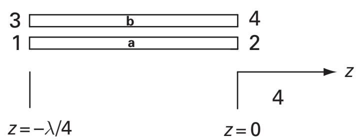
amplifier will produce a voltage proportional to the current × Z_0. Use op-amps to produce signals that are the sum and difference of this voltage and the line voltage.

Problem 15.8. Analyze the coupled-line hybrid to derive Equations (15.1) and (15.2). Let $V_a(z)$ and $V_b(z)$ denote respectively the voltages on the bottom conductor and top conductors. You can write these voltages as the superposition of a common-mode ("even") wave and its reflection plus a differential-mode ("odd") wave and its reflection. Putting $z=0$ at the right-hand end, confirm that these voltages take the form

$$V_a(x) = \mathrm{V_e}\mathrm{e}^{-\mathrm{j}kx} + \mathrm{V_e}\Gamma_e\mathrm{e}^{\mathrm{j}kx} + \mathrm{V_0}\mathrm{e}^{-\mathrm{j}kx} + \mathrm{V_0}\Gamma_0\mathrm{e}^{\mathrm{j}kx}$$

$$V_b(x) = V_e\mathrm{e}^{-\mathrm{j}kx} + V_e\Gamma_e\mathrm{e}^{\mathrm{j}kx} - V_0\mathrm{e}^{-\mathrm{j}kx} - V_0\Gamma_0\mathrm{e}^{\mathrm{j}kx}$$

where, as usual, $k=2\pi/\lambda$. The reflection coefficients, Γ_e and Γ_0, are calculated in terms of the even and odd characteristic impedances, $Z_e = 2\ Z_{CM}$ and $Z_0 = Z_{DIFF}/2$, in the usual way, i.e., $\Gamma_{e,0} = (Z_0 - Z_{e,0})/(Z_0 + Z_{e,0})$ (see Chapter 10). Note that Z_0 is the *load* impedance for the even and odd waves.



Assume a unity amplitude wave incident at Port 1, i.e., $V_a(-\lambda/4)=1$. The voltage at the isolated port must be zero, i.e., $V_b(0)=0$. Use these two equations to find V_e and V_0. Then use V_e and V_0 to show that the coupled voltage, $V_3 = V_b(-\lambda/4)$ is given by $c=(Z_e-Z_o)/(Z_e+Z_o)$. Finally, impose the (match) condition that $I_1 = 1/Z_0$, i.e., $I_a(-\lambda/4)) = 1/Z_0$, to show that $Z_eZ_o = Z_0^2$.

Reference

Montgomery, C. G., Dicke, R. H. and Purcell, E. M., *Principles of Microwave Circuits*, Volume 8 of the MIT Radiation Laboratory Series, New York: McGraw Hill, 1948. Reprinted London: Peter Peregrinus, 1987.