7

Acoustics

Sounds are pressure waves. When an object moves suddenly in air, the air is compressed. This compression causes a change in pressure. The pressure change results in a push on the surrounding air, which causes it to move, disturbing air that is farther away. The result is a disturbance that propagates away from the object. These pressure changes are ordinarily quite small – the changes in a normal conversation are only about a millionth of atmospheric pressure.

7.1 Equations of Sound

We can derive a formula for the speed of sound in terms of the basic properties of air. The equations turn out to be similar to the transmission-line equations, with pressure playing the role of voltage and average velocity the role of current. We start by considering the effect of a pressure disturbance on a section of air of length l (Figure 7.1a). We will write the pressure as $P(z, t)$, where z is the distance and t is the time. If the length l is small, then we can write the difference in pressure between the left and right section approximately as $\frac{\partial P}{\partial z} l$. The pressure difference across the section of air will cause the section to accelerate, because there is more force on one side than the other. We can find the acceleration from Newton's second law,

$$F = ma, \tag{7.1}$$

where F is the force in newtons, m is the mass in kilograms, and a is the acceleration in m/s^2.

In considering acceleration due to pressure, it is convenient to think in terms of the mass per unit area, which we can write as ρl, where ρ is the volume density. We get

$$\frac{\partial P}{\partial z} l = -\rho l \frac{\partial U}{\partial t}, \tag{7.2}$$

where U is the average velocity. We cancel l and write

$$\frac{\partial P}{\partial z} = -\rho \frac{\partial U}{\partial t}. \tag{7.3}$$

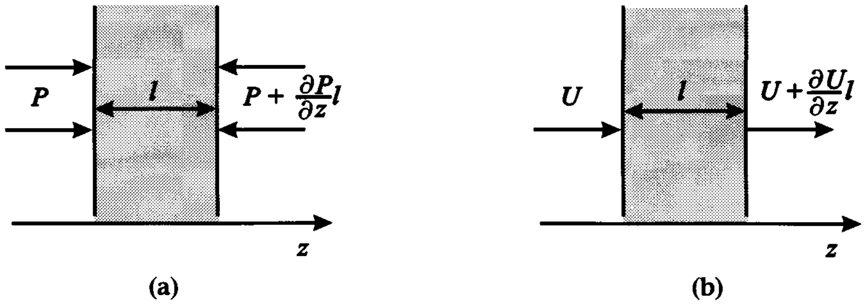

Figure 7.1. Effect of varying the pressure (a) and the velocity (b) across a section of air of length l.

This corresponds to the first transmission-line equation

$$\frac{\partial V}{\partial z} = -L\frac{\partial I}{\partial t}. \tag{7.4}$$

Thus density plays the same role as distributed inductance, and we can write the following correspondences:

$$V \Longleftrightarrow P, \tag{7.5}$$

$$I \Longleftrightarrow U, \tag{7.6}$$

$$L \Longleftrightarrow \rho. \tag{7.7}$$

Deriving the second equation is more difficult. We will consider the effect of a velocity variation across a section of air of length l (Figure 7.1b). If the velocity at each side is different, the length will change with time. For small l, we can write

$$\frac{dl}{dt} = l\frac{\partial U}{\partial z}. \tag{7.8}$$

We can relate the length l to P. For a gas, the pressure P and volume V are related by a power law,

$$PV^{\gamma} = \text{constant}, \tag{7.9}$$

where γ is an experimental constant, equal to 1.403 for air at room temperature and atmospheric pressure. This is an *adiabatic* relation, which means that the heat does not have time to flow from a compressed region to an expanded region. We can write the relation between l and P as

$$l = \alpha P^{-1/\gamma}, \tag{7.10}$$

where α is a constant. We are assuming here that the gas only compresses and expands in the z direction, and not in the x and y directions. We can write the derivative $\frac{dl}{dP}$ as

$$\frac{dl}{dP} = -\frac{\alpha P^{-1/\gamma-1}}{\gamma} = -\frac{l}{\gamma P}. \tag{7.11}$$

Now we can find $\frac{dl}{dt}$ by the chain rule as

$$\frac{dl}{dt} = \frac{dl}{dP} \cdot \frac{dP}{dt} = -\frac{l}{\gamma P} \cdot \frac{dP}{dt}. \tag{7.12}$$

If we substitute for $\frac{dl}{dt}$ in Equation 7.8, we get

$$l\frac{\partial U}{\partial z} = -\frac{l}{\gamma P} \cdot \frac{\partial P}{\partial t}. \tag{7.13}$$

If we divide by l, we get

$$\frac{\partial U}{\partial z} = -\frac{1}{\gamma P} \cdot \frac{\partial P}{\partial t}. \tag{7.14}$$

This formula is difficult to interpret. It is analogous to the second transmission-line equation

$$\frac{\partial I}{\partial z} = -C\frac{\partial V}{\partial t}. \tag{7.15}$$

One difficulty with Equation 7.14 is that pressure appears twice, once in the denominator and once in a derivative. This means that the equation is nonlinear. However, at ordinary sound levels, the pressure changes are small enough that we can consider the P in the denominator to be constant. We can then see that the quantity that corresponds to distributed capacitance is

$$C \Longleftrightarrow \frac{1}{\gamma P}. \tag{7.16}$$

We also need to understand that U is the average velocity of the gas molecules at some position. The gas molecules have additional random motion that may be much larger than the average velocity. U is different from the velocity of sound, which is a constant. The average velocity U actually oscillates at the frequency of the sound.

By analogy with transmission-line equations, we write the sound velocity v as

$$v = \sqrt{\gamma P/\rho}. \tag{7.17}$$

The units of pressure are pascals (Pa), where a pressure of 1 Pa exerts a force of 1 newton per square meter. Atmospheric pressure at room temperature and sea level is 101 kPa. The density ρ has units of kg/m^3. At room temperature and sea level, the density of air is $\rho = 1.20$ kg/m^3. If we substitute for γ, P, and ρ in the formula, we find that v is 344 m/s. This is a million times slower than the speed of radio waves, which propagate at the speed of light. We relate frequency and wavelength in the same way as before, except that sound wavelengths are much shorter. Our NorCal 40A filters are tuned for an output audio frequency of 620 Hz. On a piano keyboard, this is near D♯ (622 Hz) in the second octave above middle C. At 620 Hz, the wavelength λ is given by

$$\lambda = v/f = 55 \text{ cm}. \tag{7.18}$$

There are other analogies between sound and transmission-line waves. We define the characteristic impedance Z_0, given by

$$Z_0 = \sqrt{\rho \gamma P} = v\rho. \tag{7.19}$$

Sound obeys the same reflection formulas as transmission-line waves. The product of the pressure P and velocity U is power density, with units of W/m^2. We write the power density as S:

$$S = PU. \tag{7.20}$$

The intensity of sound is given by the *sound pressure level*, which is written as L_p, according to the formula

$$L_p = 20 \log(P/P_0)\,\mathrm{dB}, \tag{7.21}$$

where P is the pressure amplitude and P_0 is a reference pressure. These are not for the background atmospheric pressure, just the sound pressure itself. The reference pressure P_0 is 20 μPa, which is close to the lowest sound level that we can hear. We can also write P_0 in terms of the power density S as

$$S = P_0^2/Z_0 = 970\ \mathrm{fW/m^2}. \tag{7.22}$$

This means that a sound pressure level $L_p = 0$ dB gives a power density quite close to 1 pW/m^2.

7.2 Hearing

Our ears are remarkable for the range of sound pressure levels they can accommodate. The ratio between the minimum pressure level that we can hear and the pressure level that causes pain is 1 million, or 120 dB. Table 7.1 shows the pressure levels for various sounds. Our perception of loudness is not linear. A pressure level that is 10 dB greater than another is perceived as about twice as loud. We say that our ears have a *logarithmic* response, which means that differences in loudness depend on the ratio of pressures. This makes a dB scale convenient for discussing hearing. We are not particularly sensitive to differences in loudness. It is difficult to perceive changes in L_p that are less than 0.5 dB. This corresponds to a power change of 10%, which is easy to measure with instruments.

We can hear sounds with frequencies over a wide range from 20 Hz to 15 kHz. We perceive a frequency change as a pitch change, and we can distinguish pitch differences of about 3 Hz. Our perception of loudness depends both on the sound pressure level and frequency. This is shown in Figure 7.2. Our ears are most sensitive in the frequency range from 500 to 6,000 Hz. As the frequency drops below 500 Hz, our ears become considerably less sensitive. Hearing acuity decreases with increasing age, particularly for frequencies above 1 kHz. The drop is about 1 dB per year, starting around age 40.

The solid lines in Figure 7.2 show contours of constant loudness. The loudness level is expressed in *phons* (pronounced to rhyme with "johns"), which are marked

Table 7.1. Pressure Levels and Power Densities for Typical Sounds.

Sound	Loudness	L_p	Power Density
		0 dB	1 pW/m^2
rustling leaves	barely audible	10 dB	10 pW/m^2
broadcast studio		20 dB	100 pW/m^2
bedroom at night	quiet	30 dB	1 nW/m^2
living room		40 dB	10 nW/m^2
classroom	moderate	50 dB	100 nW/m^2
conversation at 1 m		60 dB	1 μW/m^2
truck interior	noisy	70 dB	10 μW/m^2
city street		80 dB	100 μW/m^2
heavy truck	very noisy	90 dB	1 mW/m^2
shout at 1 m		100 dB	10 mW/m^2
jackhammer	intolerable	110 dB	100 mW/m^2
jet takeoff at 50 m		120 dB	1 W/m^2

Source: Adapted from Table 6.1 in *The Science of Sound*, 2nd Edition, by Thomas D. Rossing, Copyright 1990, by Addison-Wesley Publishing Company, Inc. Reprinted by permission.

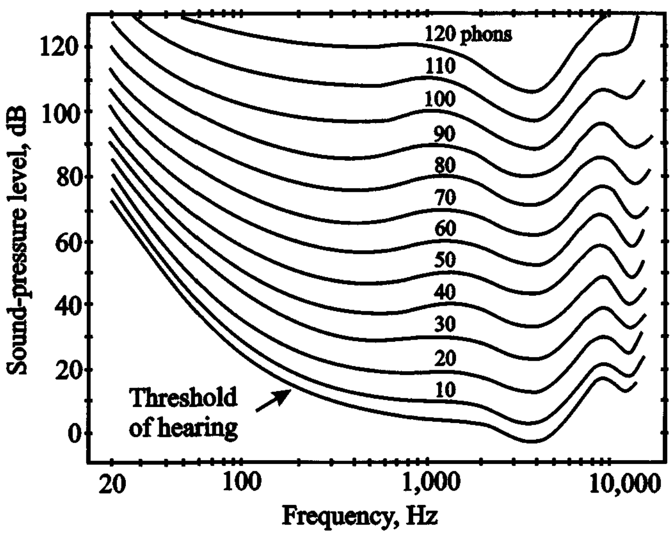

Figure 7.2. Loudness contours for people in the age range from 18 to 25. The bottom curve is the threshold of hearing. From Robinson and Dadson, *British Journal of Applied Physics*, volume 7, p. 166, 1956.

on each contour. The contours indicate combinations of pressure and frequency that are equally loud to the ear. The phon unit uses the sound pressure levels at 1 kHz as a reference. The top contour at 120 phons is the level where the sound begins to become painful. One interesting feature about the loudness contours is that they bunch together at low frequencies. This presents challenges in recording music, because we usually listen to music at a lower level than it is recorded at. In particular, you may need to boost the bass to compensate for this bunching.

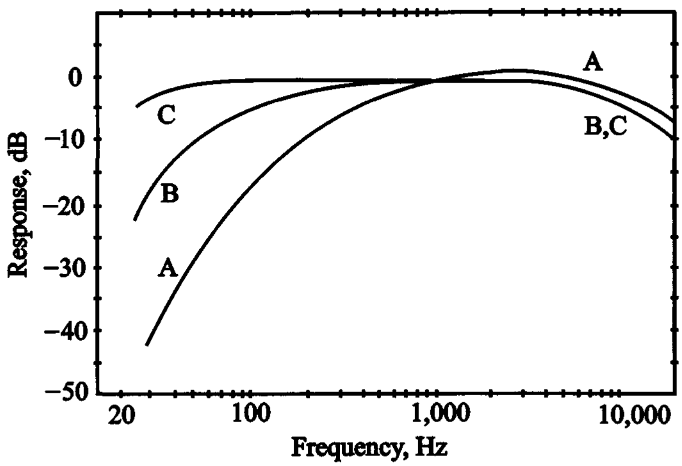

Figure 7.3. Weighting curves for sound level meters. Sound level meters usually allow a choice of A and C weighting. From Figure 1.11 in *Handbook of Recording Engineering,* by John Eargle, published by Van Nostrand Reinhold. Used by permission.

The loudness contours are also taken into account in designing offices and lecture halls. Greater sound pressure levels are allowed at lower frequencies than at high frequencies, because we are less sensitive to noise at low frequencies.

Sound-level meters take these loudness contours into account (Figure 7.3). The A curve is like the 40-phon loudness contour, but turned upside down. With A-weighting, a meter reading gives an indication of loudness. This is useful in evaluating background noise levels for offices and classrooms. Federal agencies also use the A-weighting to set workplace noise limits. For example, in the United States, OSHA (Occupational Safety and Health Act) regulations set the maximum continuous exposure level at 90 dB with A-weighting. In contrast, the C curve is quite flat. This is useful for comparing pressure levels at different frequencies in the lab.

7.3 Masking

How well a message is understood depends on the ratio of signal power to noise power. This power ratio is called the *signal-to-noise ratio,* and it is traditionally given in dB. Your transceiver is designed for receiving Morse Code at low power levels. Morse Code transmissions have a series of short and long pulses (*dits* and *dahs*). The receiver converts these pulses to 620-Hz audio tones. Your ear is quite good at rejecting noise and other interfering signals, as long as they fall outside of a 150-Hz band around 620 Hz. If the interfering signal is closer than this, your ears tend to perceive the two signals as a single tone. The bandwidth over which this fusing occurs is called the *critical* bandwidth. Noise within the critical bandwidth has the effect of hiding the signal. This is called *masking*. Masking occurs if the

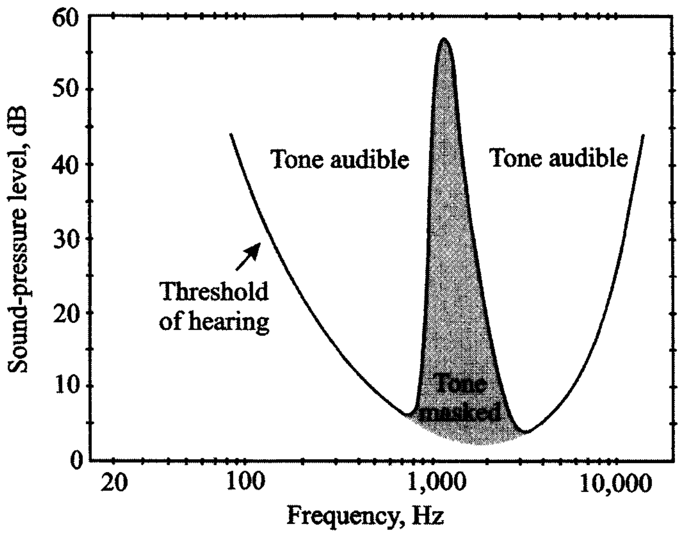

Figure 7.4. Masking of a signal tone by 1,200-Hz noise at L_p = 60 dB. Adapted from Figure 3.6 in *Acoustic Noise Measurements* by Jens Broch, published by B & K Instrument Company.

noise power within the critical band is comparable to the signal power. The plot in Figure 7.4 shows the masking effect of 1,200-Hz noise at a pressure level of 60 dB. At frequencies that are far away from the noise, we can detect the signal tone right down to the normal threshold of hearing. As the frequency comes closer to 1,200 Hz, stronger and stronger tones are masked and cannot be heard. Right at 1,200 Hz, signal tones can be detected if they have sound pressure levels of 57 dB or larger. This is 3 dB below the noise. This means that Morse Code can be received when the signal power is comparable to the noise power within the critical bandwidth. By contrast, an AM or TV broadcaster aims at 50-dB signal-to-noise ratios over much larger bandwidths to provide a clear sound and picture. This means that the NorCal 40A can communicate at distances of thousands of miles with a transmitter power of 2 W, whereas a broadcaster might use 50 kW for coverage in a single city.

7.4 rms Voltages

In the next problem, you measure an AC voltage with a multimeter. The multimeter measures the *root-mean-square*, or rms, voltage rather than peak-to-peak voltage. The root-mean-square voltage V_{rms} is defined as the square root of the time-averaged value of $V^2(t)$. Mathematically, we write this as

$$V_{rms} = \sqrt{\overline{V^2(t)}}, \tag{7.23}$$

where the bar indicates the average value. For example, consider finding the rms voltage for the voltage given by $V = V_p \cos(\omega t)$, where V_p is the peak voltage. We can write V^2 in the form

$$V^2(t) = V_p^2 \cos^2(\omega t) = \frac{V_p^2}{2} + \frac{V_p^2}{2}\cos(2\omega t). \tag{7.24}$$

Because the average value of a cosine function is zero, we can write the average value of $V^2(t)$ as

$$\overline{V^2(t)} = \frac{V_p^2}{2}. \tag{7.25}$$

The rms voltage is given by

$$V_{rms} = \sqrt{\overline{V^2(t)}} = V_p/\sqrt{2}. \tag{7.26}$$

For a cosine voltage, the rms voltage is lower than the peak voltage by a factor of $\sqrt{2}$. This means that the rms voltage is lower than the peak-to-peak voltage by a factor of $2\sqrt{2}$. It is also convenient to use rms voltages for waveforms that do not have well-defined peak voltages, such as noise. Multimeters say "true rms" if they can also measure noise voltages. We can relate the rms voltage in a simple way to power. We write the average power P_a as

$$P_a = \overline{V^2(t)}/R = V_{rms}^2/R. \tag{7.27}$$

When we write power in terms of the rms voltage, we do not need to worry about remembering a factor of two. People often use rms voltages just because it makes power formulas simpler. This looks just like the formula for power for a DC voltage, and for this reason, the rms voltage is sometimes called the *effective* voltage.

FURTHER READING

Richard Feynman has an excellent discussion of the ideal gas law and the propagation of sound in the first volume of *The Feynman Lectures on Physics* (Addison-Wesley) in Chapters 39 and 47. The book, *Handbook of Recording Engineering*, by John Eargle, published by Van Nostrand Reinhold, is an excellent reference for recording music. A book with good coverage of the acoustics of musical instruments is *The Science of Sound* by Thomas Rossing, published by Addison-Wesley Longmans. A good book on the mathematical theory of sound is *Vibration and Sound*, by Philip Morse (McGraw-Hill).

PROBLEM 17 – TUNED SPEAKER

The NorCal 40A was originally designed for use with headphones, and for this reason the sound level is rather low for a loudspeaker. However, the transceiver is designed for output audio tones in the frequency range from 600 Hz to 650 Hz, and we can make a resonant speaker system that generates excellent sound levels in this frequency range. The response at other frequencies is reduced, and this helps suppress interference and noise. Figure 7.5 shows the construction.

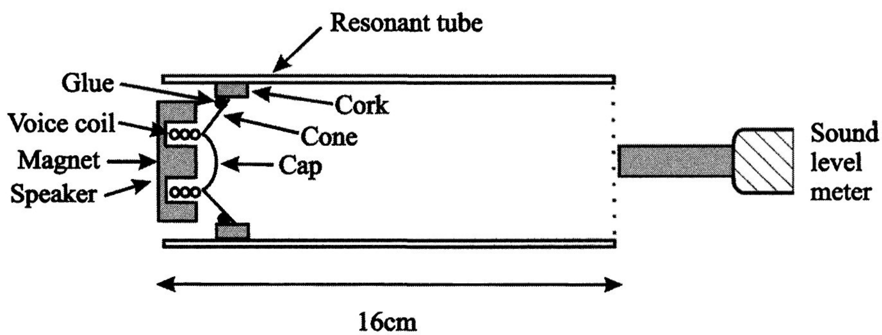

Figure 7.5. Tuned-speaker cross section. A speaker is mounted inside a 16-cm section of a cardboard mailing tube. The length is chosen to give a resonance in the 600-to-650 Hz range.

The speaker itself consists of a black paper cone that is attached to a coil of wire (the voice coil) that is mounted inside a ring magnet. When we put a current through the voice coil, the interaction between the coil and the magnetic field of the magnet produces a force on the paper cone. This gives the air next to the cone a velocity, which causes the sound. Our speaker is a small one, 2.25 inches in diameter, with a power rating of only a quarter watt.

To build the tuned speaker, slip a cork liner strip inside the tube about 1 cm from the end. Use a glue gun to attach the speaker to the cork liner. Start by plugging the glue gun into an outlet to heat up the glue. When the glue is hot, you can push it out of the tip with the trigger. Please be careful to get the glue at the joint between the speaker and the cork, and not on the tube. You should make a solid bead of glue completely around the edge of the speaker. Wait a few minutes for the glue to dry. The cork and speaker should still be able to slide inside the tube. Try to be neat with the glue.

Solder a pair of wires to the speaker, and add a stereo mini plug (Figure 7.6). You should do this carefully, because poor plug connections are a common cause of problems, and they are not easy to diagnose. One lead should be soldered to the outer ground strip, and the other should be soldered to one of the inner-conductor lugs. The inner-conductor contacts connect to leads that are shorted together on the circuit board, so that you do not have to connect to both. Take care that the pressure from the plastic hood does not cause the wires to short out. Also, do not crimp the plug ground strip until you have finished soldering the plug. Otherwise the insulation of the wires may melt through and

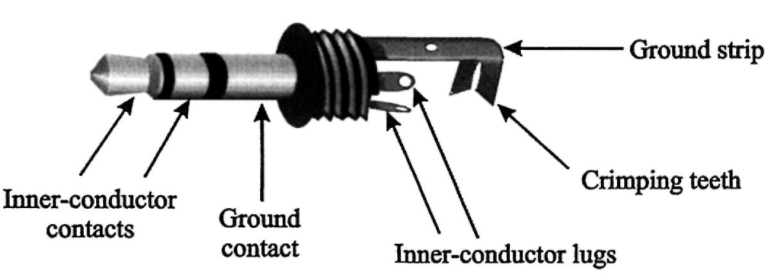

Figure 7.6. Stereo mini-plug connections.

short out the connection. Check your connections by measuring the resistance between the connector contacts with a multimeter.

The sound pressure level can be measured with a *sound level meter*. For the Tenma #72–860, the settings should be Lo, S, and C. The Lo (for low) position sets the measurement range from 35 dB to 100 dB. The S (for slow) setting gives a sound pressure level that is averaged over 1.5 seconds. By comparison, the F (fast) setting averages over 0.2 seconds. With the "slow" setting, the reading is more stable than with the "fast" setting, but the meter cannot respond to changes as quickly. The C weighting curve treats different frequencies the same, so that it is easy to make comparisons at different frequencies.

Mount the speaker and the meter on foam blocks so that the end of the meter with the microphone points into the tube, with the microphone centered in the mouth (Figure 7.5). The foam blocks should be oriented at angles so that sound reflections off the sides do not interfere with the measurement.

A. Connect the function generator to the speaker. For these measurements set the function generator initially to a 600-Hz sine wave. The amplitude setting should be 25-mVrms. Do not use peak-to-peak voltages here, because we will be making a comparison with the AC voltage on a multimeter, which gives an rms voltage. The tube should have a resonance in the frequency range from 600 Hz to 650 Hz. The sound will peak at the resonant frequency. If you need to, you can lower the resonant frequency slightly by moving the speaker closer to the end, and you can raise it somewhat by pushing the speaker further into the tube. Find the resonant frequency and record L_p.

B. Now measure f_l and f_u, where L_p drops by 3 dB from the maximum. Use these frequencies to calculate the Q.

C. The speaker impedance is nominally 8 Ω, but it actually varies with frequency. Use an AC voltmeter reading at resonance to calculate the impedance. For this calculation, you may assume that the speaker is resistive.

D. Now let us calculate the resonant frequency we would expect from a transmission-line equivalent circuit (Figure 7.7). In the circuit, pressure appears as voltage and velocity as current. The speaker controls the velocity of air, and so it is represented by a current source. The tube is a section of transmission line. The length is the distance from the cap in the center of the speaker to the mouth of the tube. At the open end of the tube, the sound pressure drops suddenly, because the air can spread out easily. For this reason, the open end is represented as a short circuit in the transmission-line model. There is a parallel resonance when the length

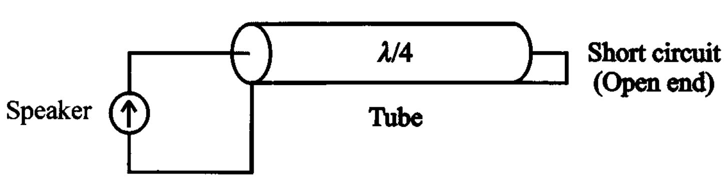

Figure 7.7. Transmission-line equivalent circuit for the tuned speaker.

is a quarter wavelength. Calculate this frequency, using 344 m/s for the speed of sound.

E. There is also a series resonance where the tube is a half wavelength long and the sound pressure is a minimum. Calculate the frequency where the tube length is a half wavelength long. Now measure the series resonant frequency, and note L_p. Measure the speaker impedance at this frequency.

F. There is another resonant frequency below 600 Hz that gives a pressure maximum. This is the resonant frequency of the speaker itself. Measure this frequency.

PROBLEM 18 – ACOUSTIC STANDING-WAVE RATIO

A. For this measurement, a section needs to be added to the tube to extend its length to more than a half wavelength. In addition, slip on a phenolic rod with ruler markings over the microphone of a sound level meter, so that sound-level measurements can be made down inside the tube. Insert the phenolic rod down the tube and measure the sound pressure level at intervals of 1 cm down the tube as far as you can go. The function generator should be set for a sine wave at 620 Hz, with the amplitude adjusted so that the maximum pressure level inside the tube is 114 dB, or 10 Pa. You should search carefully to make sure that you find the minimum pressure level and its position. Make a linear plot of the pressure in pascals versus the distance from the end in cm.

B. What is the ratio of the maximum pressure to the minimum pressure? This is the standing-wave ratio (SWR).

C. In the book, *Vibrations and Sound,* Philip Morse gives an approximate value for the SWR as

$$\mathrm{SWR} = \frac{\lambda^2}{2\pi A} \tag{7.28}$$

where λ is the wavelength and A is the inside area of the tube. Calculate Morse's SWR value and compare it with your measurement.

D. The point of minimum pressure is actually less than a half wavelength from the mouth. It is as if the reflection actually occurs at a point beyond the end of the tube. The difference is written as δ (the Greek letter *delta*). Morse gives the formula for δ as

$$\delta = \frac{4d}{3\pi} \tag{7.29}$$

where d is the inner diameter of the tube. Calculate Morse's δ and compare with your measurement.