14

Noise and Intermodulation

Fundamentally, a receiver is limited in sensitivity by noise that competes with the signal we want. A receiver is also limited in handling strong signals by its nonlinearities, which produce intermodulation products that block reception. *Noise* is a random voltage or current that is present whether a signal is there or not. We distinguish noise from *interference*, which is an unwanted signal coupled into the circuit, and from *fading*, which is a variation in the signal level, caused by interference between radio waves arriving by different paths. There are many different sources of noise. Several forms are caused by bias currents. In diodes, the random arrival times of electrons cause *shot* noise. Another current noise is $1/f$ noise, where power varies inversely with the frequency. This $1/f$ noise is found in contacts, and it is associated with energy states at interfaces called traps. It can often be reduced by improving the fabrication process. However, even in the absence of bias currents there is noise associated with resistors. It is called Johnson noise after John Johnson at the Bell Telephone Laboratories, who first measured it.

14.1 Noise

On an oscilloscope, noise makes a trace appear as a band that evokes the feeling of grass. We can write the noise as a function of time $V(t)$, but we would not be able to predict its value at a future time. We would expect the average over time to be zero, because the voltage will be positive at some times, and negative at others, and these periods cancel each other out. However, the time average of $V^2(t)$ is not zero, because $V^2(t)$ is positive. This means that noise has an rms value, V_{rms}, given by

$$V_{rms} = \sqrt{\frac{1}{\tau}\int_{\tau} V^2(t)\,dt}, \tag{14.1}$$

where τ is an averaging time. Noise also has an average power P_n, given by

$$P_n = V_{rms}^2/R, \tag{14.2}$$

where R is the load resistance. We characterize a receiver's output by the *signal-to-*

noise ratio (SNR), given by

$$\mathrm{SNR} = P/P_n, \tag{14.3}$$

where P is the output signal power. Different applications require very different signal-to-noise ratios, but a good way to compare receivers independently of the application is to ask how much input power is needed to give a 1:1 signal-to-noise ratio at the output. This is called the *minimum detectible signal*, or MDS. The MDS is actually quite an appropriate measure for the NorCal 40A, because a 1:1 SNR is about the lowest that can be used for receiving Morse code. We calculate the MDS by dividing the output noise by the gain G, and write

$$\mathrm{MDS} = P_n/G. \tag{14.4}$$

Another approach is to measure the output noise when no signal is present. Then we find the MDS as the input signal that doubles the output power. In the exercises, we measure voltage instead of power, and so we will find the input signal that raises the output voltage by a factor of $\sqrt{2}$. For typical receivers, the MDS is small, much less than a femtowatt. The next unit prefix down is *atto*, for 10^{-18}. People do talk about attowatts and attofarads, but not often. It is more common to give powers in dB, using one milliwatt as the reference. For example, 10 aW is written as –140 dBm, where "m" denotes a reference power of a milliwatt.

Noise power does not appear at one frequency only; rather it is distributed over all frequencies. This means that we need to talk about noise power density at a particular frequency rather than noise power. We define the *noise power density* N as the noise power per unit bandwidth. The units are W/Hz. If N is constant with frequency, we can write

$$P_n = NB, \tag{14.5}$$

where B is the bandwidth. If N varies with frequency, we need to integrate N over the frequency range we are interested in to find P_n. A receiver's bandwidth is determined primarily by the bandwidths of the IF and audio filters, and a wide range of bandwidths are used in practice. Many receivers allow an operator to switch between narrow- and wide-bandwidth filters. Narrow filters are good for reducing noise, but wide filters make it easier to find signals.

The MDS that we defined depends on the bandwidth, because noise power is usually proportional to bandwidth. It is convenient to have a measure that does not depend on the bandwidth, because the bandwidth is determined for the most part by filters that make only a modest contribution to the receiver noise. Noise is associated primarily with mixers and amplifiers. The *noise-equivalent power*, or NEP, has the same relation to N that the MDS has to P_n. We write

$$\mathrm{NEP} = N/G. \tag{14.6}$$

One way to think about the NEP is that it is the noise density we would need at the input to produce all of the noise that we observe at the output. People say it is the noise *referred* to the input.

14.2 Noise Phasors

We can use phasors for calculating how circuits affect noise just as we do for ordinary AC voltages. We write V_n for a noise phasor. However, noise phasors are different from ordinary phasors because we need to consider the bandwidth. We will use a bandwidth of 1 Hz to make it easy to relate the phasors to the noise power density N. In addition, the noise phasors are random variables, and we need probability theory to describe them. We will state our results in terms of expected values. If this terminology is not familiar to you, it is reasonable to think of it as an average. We indicate an expected value with an overline. For example, for a noise voltage V_n with a probability density function p, we write the expected value of $|V_n|^2$ as

$$\overline{|V_n|^2} = \int |V_n|^2 p \, dA, \tag{14.7}$$

where dA is an element of area in the complex V_n plane. We can relate this to the power density N by writing

$$N = \frac{\overline{|V_n|^2}}{2R} \text{ W/Hz}, \tag{14.8}$$

where R is the resistance. The units of V_n are a little strange, V/$\sqrt{\text{Hz}}$. We can take this as a reminder that the noise voltage increases as the square root of the bandwidth, in contrast to the noise power, which is proportional to the bandwidth.

We will use several arithmetic properties of expected values in our calculations. These follow from Equation 14.7. For a constant α, we write

$$\overline{\alpha |V_n|^2} = \alpha \overline{|V_n|^2}, \tag{14.9}$$

because we can bring a scalar multiple out of an integral. Now consider that we want to add two noise voltages V_1 and V_2. We can expand the expected value of $|V_1 + V_2|^2$ as

$$\overline{|V_1 + V_2|^2} = \overline{|V_1|^2} + \overline{|V_2|^2} + \overline{V_1 V_2^*} + \overline{V_1^* V_2}. \tag{14.10}$$

The last two terms are called *correlations*. Notice that the correlations are complex conjugates, so that the sum is real. The correlations indicate when part of each noise voltage comes from the same physical source. If the noise voltages come from two different sources, such as two different resistors, the sources are *independent*, and the correlation is zero:

$$\overline{V_1 V_2^*} = 0. \tag{14.11}$$

We can therefore write the power density of the sum, N, as

$$N = N_1 + N_2, \tag{14.12}$$

where N_1 and N_2 are the noise power densities for V_1 and V_2.

14.3 Nyquist's Formula

The formula for noise in resistors was first derived by Harry Nyquist, who worked at Bell Labs with Johnson. Nyquist used a statistical physics argument similar to the derivation of Planck's formula for blackbody radiation. In fact, you can think of Johnson noise as blackbody radiation in a circuit. First we need to understand why resistors have noise. You can make a resistor hot by applying a voltage or a current, and this means that the thermal energy associated with the vibrations of atoms couples to the voltages and currents. However, even if you do not apply a voltage, the thermal vibrations produce noise voltages and currents through this coupling. By this logic you would not expect a capacitor or inductor to produce noise, because they do not get hot when you apply a voltage or current. For these elements, the energy transfer and storage are electric and magnetic, and there is no coupling between thermal vibrations and voltages.

Nyquist used transmission-line theory to derive his formula, but it is easier for us to use an RLC circuit. We consider a resistor R at an absolute temperature T that is connected to a series resonant circuit (Figure 14.1a). The connecting wires couple the LC resonator to the vibrations of the atoms inside the resistor. Our calculation takes several steps. First we use circuit theory to find the capacitor voltage in terms of the resistor voltage. Then we integrate over frequency to find the energy stored in the capacitor. In Figure 14.1a, V_n is the resistor noise voltage phasor. We write the capacitor voltage V_c by a potential-divider formula:

$$V_c = \frac{1}{j\omega C}\,\frac{V_n}{R + j\omega L + 1/(j\omega C)} = \frac{V_n}{-\omega^2 LC + j\omega RC + 1}. \tag{14.13}$$

We can write the expected value of $|V_c|^2$ as

$$\overline{|V_c|^2} = \frac{\overline{|V_n|^2}}{|1 - \omega^2 LC + j\omega RC|^2}. \tag{14.14}$$

In thermal equilibrium, the energy that is stored in the inductor and capacitor is given by the Equipartition Theorem from classical thermodynamics, which specifies that the expected value of the energy associated with a resonance is the thermal energy kT. We last saw the thermal energy in Chapter 9 in connection with diode and transistor currents. Here k is Boltzmann's constant, 1.38×10^{-23} J/K. We can find the stored energy kT by multiplying by $C/2$ and integrating over frequency.

Figure 14.1. (a) Deriving the Nyquist noise-voltage formula from an RLC circuit. (b) Calculating the available noise power density from a resistor with a matched load.

This gives us

$$kT = \frac{C}{2}\int_0^\infty \overline{|V_c|^2}\,df = \frac{C}{2}\int_0^\infty \frac{\overline{|V_n|^2}\,df}{|1-\omega^2 LC + j\omega RC|^2}. \tag{14.15}$$

Now we assume that the LC circuit has a very high Q. This gives the integrand a large peak at the resonant frequency that dominates the integral. We will let the Q be high enough that we can assume that $\overline{|V_n|^2}$ is constant over the frequency range that is important for the integral. Later on, we will see that $\overline{|V_n|^2}$ is independent of frequency anyway. The high-Q assumption lets us bring $\overline{|V_n|^2}$ out from under the integral sign. We can write

$$kT = \frac{C\,\overline{|V_n|^2}}{2}\int_0^\infty \frac{df}{|1-\omega^2 LC + j\omega RC|^2}. \tag{14.16}$$

This integral looks difficult, but it is one of a family of integrals that can be attacked through the calculus of residues. This is an elegant technique in complex analysis that lets one turn truly awful looking integrals into simple expressions. This one is given as integral #3.1123 in *Table of Integrals, Series, and Products* by Gradshteyn and Ryzhik, published by Academic Press:

$$\int_0^\infty \frac{df}{|1-\omega^2 LC + j\omega RC|^2} = \frac{1}{4RC}. \tag{14.17}$$

If we substitute for this integral in the previous equation, we get

$$kT = \frac{\overline{|V_n|^2}}{8R}, \tag{14.18}$$

which gives

$$\overline{|V_n|^2} = 8kTR. \tag{14.19}$$

This is the Nyquist noise formula. Notice that the noise voltage is independent of frequency. Because equipment for measuring noise invariably gives the rms voltage, it is more common to see this formula as

$$V_{rms} = \sqrt{4kTR}\ \ \mathrm{V}/\sqrt{\mathrm{Hz}}. \tag{14.20}$$

We can write an elegant alternative statement of the Nyquist noise formula if we consider the available noise power from a resistor. We can calculate this as the power dissipated in a matching load (Figure 14.1b). The load voltage is $V_n/2$, and the available power density N is given by

$$N = \frac{\overline{|V_n/2|^2}}{2R} = kT. \tag{14.21}$$

In words, the available power density from a resistor is kT, independent of the resistance. This is so convenient that people commonly use temperature as a measure of noise power density, even when it is not Johnson noise. We call this the *effective noise temperature* T_e, and we write it as

$$T_e = N/k. \tag{14.22}$$

People also define a noise temperature T_n for receivers, amplifiers, mixers, and attenuators by dividing the NEP by k to get

$$T_n = \frac{\text{NEP}}{k} = \frac{N}{Gk}. \tag{14.23}$$

We have given the noise temperature a simple definition here, but there are complications. The formulas depend on whether a receiver amplifies one sideband or both. Furthermore, there are matching issues that we are neglecting, and at very high frequencies, there are quantum-mechanical corrections. We will not worry about these things, but you should realize that there is a lot more to this than we cover here.

A particularly interesting example is the noise temperature of an antenna. Antennas are not ordinarily made with resistors and thus they produce very little noise by themselves. However, they pick up natural radio waves. A plot of antenna noise for a wide range of frequencies is shown in Figure 14.2. At the operating frequency of the NorCal 40A, 7 MHz, the noise temperature is extremely high, millions of kelvins. This comes primarily from lightning in tropical thunderstorms. At frequencies from 30 MHz to 1 GHz, the temperature is lower but still

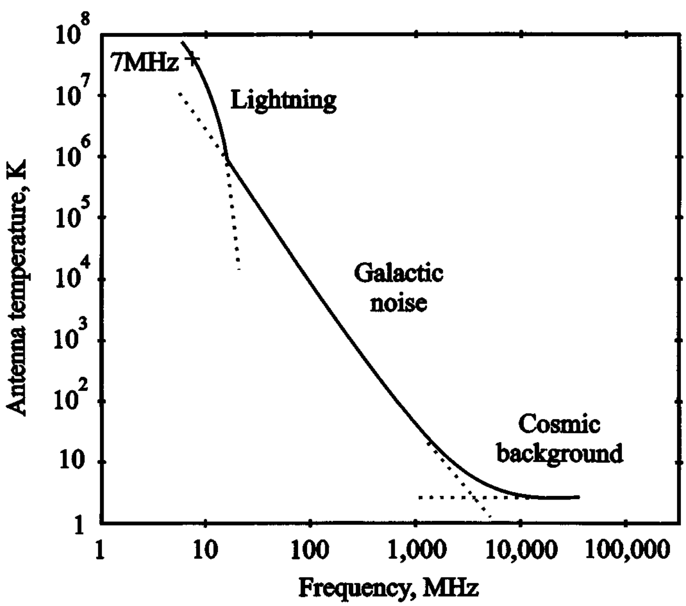

Figure 14.2. Antenna noise temperature versus frequency. For the frequency range from 30 MHz to 1 GHz, this is the noise temperature for a directive antenna pointed at the center of our galaxy. For frequencies above 1 GHz, this is the noise temperature for an antenna at a high, dry site pointed straight up. This plot is adapted with permission from Figure 8.6 in an extremely interesting book, *Radio Astronomy*, 2nd edition, by John Kraus, published by Cygnus-Quasar. For radio astronomers, this noise *is* the signal. This book has an extensive discussion of astronomical radio sources and excellent coverage of receivers and antennas for radio astronomy.

large, and the dominant source is the central region of our galaxy. At higher frequencies, the noise is quite small, and carefully designed antennas that point out to space receive the cosmic background radiation that is the dying embers of the primordial fireball. The cosmic background radiation has a noise temperature of 3 K.

A receiver designer tries to make the receiver noise lower than the antenna noise, so that the sensitivity is limited by the antenna rather than the receiver. It is a lot easier to do this at 7 MHz, where the antenna noise is enormous, than at 3 GHz, where the antenna noise is very low. However, the frequency range from 1 GHz to 10 GHz presents incredible opportunities for long-distance communication. For example, the *Voyager* spacecraft, now beyond the orbit of Pluto, speaks to us with a puny 10-W transmitter that has only slightly more power than the NorCal 40A.

14.4 Attenuator Noise

Now we find the noise from a resistive attenuator. We can use Nyquist's formula for this. We let L be the loss factor and N_a be the output noise power density (Figure 14.3a). Attenuators are commonly designed so that if they are terminated with a particular resistance R at the input, the resistance looking into the output port is also R. We will assume that this is the situation, and further assume that the resistor and attenuator have the same temperature T. We let the output available noise power density with a resistor R at the input be N' (Figure 14.3b). The noise N' includes noise from both the attenuator and the resistor. We can apply Equation 14.21 directly and write the available noise power density N' as

$$N' = kT. \tag{14.24}$$

This noise is in two parts. There is noise from the resistor that passes through the attenuator. This is given by kT/L. The rest is produced by the attenuator. We write

$$N' = kT/L + kT(1 - 1/L). \tag{14.25}$$

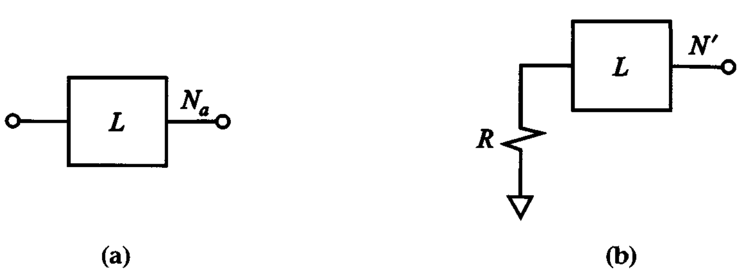

Figure 14.3. Attenuator with a loss factor L and an output noise density N_a (a). Attenuator with an input resistance R (b).

We can identify the right term as attenuator noise, and write

$$N_a = kT(1 - 1/L). \tag{14.26}$$

An attenuator with little loss has little noise. However, as the loss increases, the noise density approaches kT. We can use Equation 14.23 to write the attenuator noise temperature T_a as

$$T_a = N_a L/k = T(L - 1). \tag{14.27}$$

In this formula, we have multiplied by the loss L. This is equivalent to dividing by the gain in Equation 14.23, because loss is the reciprocal of the gain.

Although these formulas have been developed for attenuators, we can apply them to filters in the pass band if the loss in a filter is dominated by resistance in the inductors and capacitors.

14.5 Cascading Components

We calculate the total noise in receivers by adding the noise powers from the antenna and the different receiver stages. We can do this because the noise that comes from different parts of a receiver will usually be uncorrelated. There are exceptions, such as fluctuations in power-supply voltages that affect many components simultaneously. We also have to be careful to refer all the noise components to the same place in the system before we add them. We consider an amplifier chain with three amplifiers that are each characterized by a gain G_i, an output noise density N_i, and a noise temperature T_i (Figure 14.4). We include an antenna noise temperature T_a.

We write the output noise density N as

$$N = G_3 G_2 G_1 k T_a + G_3 G_2 N_1 + G_3 N_2 + N_3. \tag{14.28}$$

Notice that the noise from the antenna is amplified by the entire chain, but noise from the last amplifier appears directly. Usually this means that the noise in the early stages dominates the noise performance of a receiver. One way to see this in the NorCal 40A is to turn up the AGC. This attenuates the mixer and filter noise, so that only the noise from the Audio Amplifier is left. If you try this, the speaker sound simply goes away. We can rewrite this formula to give the receiver noise

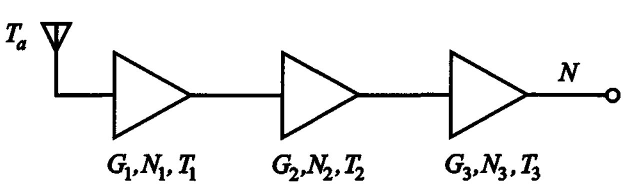

Figure 14.4. Finding the noise for an amplifier chain.

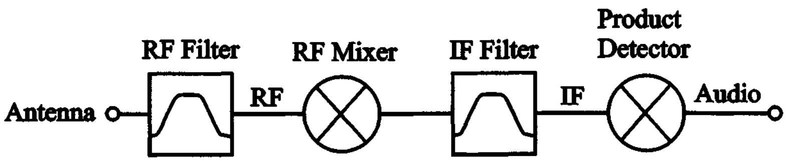

Figure 14.5. Input filters and mixers in the NorCal 40A receiver.

temperature T_r as

$$T_r = T_a + T_1 + T_2/G_1 + T_3/(G_1G_2). \tag{14.29}$$

The noise-temperature contributions of the later stages are reduced by the gain of the earlier stages.

There is an alternative to noise temperature called *noise figure* that is often quoted by manufacturers. The noise figure F is related to the noise temperature T_n by the formula

$$T_n/T_0 = F - 1, \tag{14.30}$$

where T_0 is a reference temperature, usually 290 K. For example, Philips specifies that the SA602AN mixers in the NorCal 40A have a noise figure of 5 dB. This corresponds to a mixer noise temperature of 630 K.

As an example, let us predict the noise temperature of the NorCal 40A. We consider the first four elements of the receiver shown in Figure 14.5. We assume that each filter is at a physical temperature of 290 K and has a loss factor of $L = 3.2$ (5 dB). Assuming that the loss in the filter is due to resistance, we write the noise temperature of each filter using Equation 14.27:

$$T_f = 290(L - 1) = 630\,\mathrm{K}. \tag{14.31}$$

For the mixers, we take numbers from the data sheets, which specify a gain of 18 dB ($G = 63$) and a noise figure of 5 dB ($T_m = 630$ K).

We use Equation 14.29 to write the noise temperature T_r of the receiver as

$$\begin{aligned} T_r &= T_f + T_mL + T_fL/G + T_mL^2/G \\ &= 630 + 2{,}020 + 30 + 100 = 2{,}780\,\mathrm{K}. \end{aligned} \tag{14.32}$$

The terms represent the contribution, in order, of the RF Filter, the RF Mixer, the IF Filter, and the Product Detector. This corresponds to a noise figure of 10 dB, which is 4 dB less than the measured value. The largest component is the RF Mixer at 2,020 K. The IF Filter and the Product Detector contribute much less because their noise temperatures are divided by the large gain of the RF Mixer. Even though this noise temperature sounds high, antenna temperatures at 7 MHz are much higher than this, so that the receiver noise is usually not a problem.

14.6 Measuring Noise

We can measure the MDS of a receiver in several ways. The most direct is to measure the output noise power and the gain, and divide. Some care is needed to make sure the AGC is off when you measure the gain. Another approach is find the input signal power that gives an output that is twice the original output noise power. Here the challenge is to introduce a very small signal with a known power level. Many function generators do not provide small signals, and an adjustable attenuator is needed. Care must be taken to prevent the signal from leaking around the attenuator.

For measuring the receiver noise temperature T_r, we need a noise source with a known power density. Some function generators provide this feature. We can adjust the noise density until the output power doubles. If it is not convenient to vary a noise source continuously, we can use two different sources with known effective temperatures. If a receiver has good noise performance, one can use a resistor at two temperatures for this purpose. It is common to use room temperature and the temperature of liquid nitrogen, 77 K, because the resistor can be immersed in liquid nitrogen. We assume that two different source temperatures, T_1 and T_2, are available, and that we measure the output power in each case. We write the results as

$$P_1 = \alpha(T_r + T_1), \tag{14.33}$$

$$P_2 = \alpha(T_r + T_2), \tag{14.34}$$

where α is a proportionality constant. The quotient of P_1 and P_2 is called the Y factor. Since the Y factor is usually quoted as a number greater than one, we will assume $T_1 > T_2$ and write

$$Y = \frac{P_1}{P_2} = \frac{T_r + T_1}{T_r + T_2}. \tag{14.35}$$

Now we can solve for the receiver noise temperature T_r to get

$$T_r = \frac{T_1 - YT_2}{Y - 1}. \tag{14.36}$$

14.7 Intermodulation

Earlier we studied spurious responses that result from signals that give an output at the same audio frequency as the signal we want, even though they are at the wrong frequency. These responses are suppressed by filters. There is a different spurious component that is generated when there is more than one strong signal at the input. This is illustrated in Figure 14.6, where two input signals at closely spaced frequencies f_1 and f_2 are present. A nonlinear response in a mixer or amplifier produces signals at harmonic combinations of f_1 and f_2. These frequencies are

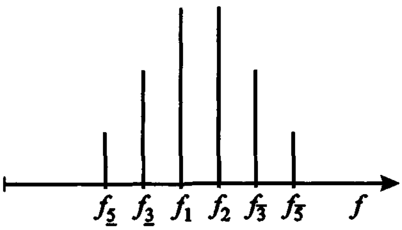

Figure 14.6. Intermodulation products that are close to the input frequencies f_1 and f_2.

called *intermodulation products*. Many intermodulation products are at quite different frequencies from the input signals, and this means that the RF Filter can block the inputs. However, there are four products that are quite close in frequency to the input signals, and this means that the RF Filter cannot stop them. These are the third-order products

$$f_{\underline{3}} = 2f_1 - f_2, \tag{14.37}$$
$$f_{\overline{3}} = 2f_2 - f_1 \tag{14.38}$$

and the fifth-order products

$$f_{\underline{5}} = 3f_1 - 2f_2, \tag{14.39}$$
$$f_{\overline{5}} = 3f_2 - 2f_1. \tag{14.40}$$

These are also shown in Figure 14.6. If the receiver is tuned to these frequencies, we may hear an interfering tone.

Now let us consider how the intermodulation products come about. We represent the response by a Taylor series

$$V = G_v V_i + G_2 V_i^2 + G_3 V_i^3 + G_4 V_i^4 + G_5 V_i^5 + \cdots, \tag{14.41}$$

where G_v is the voltage gain, and the other coefficients show the non linear behavior. Assume that the input voltage V_i contains two frequency components at f_1 and f_2:

$$V_i = V_1 \cos(2\pi f_1 t) + V_2 \cos(2\pi f_2 t). \tag{14.42}$$

The higher-order terms, $V_i^2, V_i^3, \ldots,$ generate intermodulation products. At low input levels these components are below the receiver noise. However, at higher levels, the intermodulation products increase rapidly, producing spurious tones.

The most important intermodulation products are the third- and fifth-order ones, because the second- and fourth-order products are relatively far away. We will calculate third-order coefficients. For this, we need to find a large number of cosine products. To make things simpler, we assume that $V_1 = V_2 = V$ and $f_1 < f_2$. We can interpret the product as being made up of a sum frequency and a difference frequency. We write

$$V_1 \cos(2\pi f_1 t) \cdot V_2 \cos(2\pi f_2 t) = \frac{V^2}{2}[\cos(2\pi f_2 t + 2\pi f_1 t) + \cos(2\pi f_2 t - 2\pi f_1 t)]. \tag{14.43}$$

When we expand the product

$$V_i^3 = (V\cos(2\pi f_1 t) + V\cos(2\pi f_2 t))^3 \tag{14.44}$$

we get all the possible sum and difference combinations of three frequencies chosen from f_1 and f_2. There is a common coefficient of $V^3/4$. The sum and difference frequencies often repeat, and our job is to count the number of repetitions, rather like dice and card combinations. You should work through each of these carefully so that you will be ready to find the fifth-order products in Problem 35. We consider the frequencies in two groups. First are the sum frequencies. There are four of these: $3f_1$, $2f_1 + f_2$, $2f_2 + f_1$, and $3f_2$. The third harmonics $3f_1$ and $3f_2$ appear once, and the mixed sums $2f_1 + f_2$ and $2f_2 + f_1$ appear three times each. Now consider the difference frequencies. There also four of these: $2f_1 - f_2$, f_1, f_2, and $2f_2 - f_1$. It may be hard to see the original frequencies f_1 and f_2 as arising from a difference, but we can write

$$f_1 = f_1 + f_2 - f_2 \tag{14.45}$$

and

$$f_1 = f_1 + f_1 - f_1. \tag{14.46}$$

The differences $2f_1 - f_2$ and $2f_2 - f_1$ each appear three times. The original frequencies f_1 and f_2 appear nine times each. The count for each frequency is shown in Figure 14.7 on top of each line. The presence of the original frequencies indicates that intermodulation will modulate the original signals in addition to producing products at other frequencies.

Notice that the coefficients for the sum frequencies form a line from Pascal's triangle (Figure 14.8). The coefficients in the difference-frequency group are also derived from the same line of the triangle except that they are multiplied by a factor of three, which is itself a coefficient from the same line.

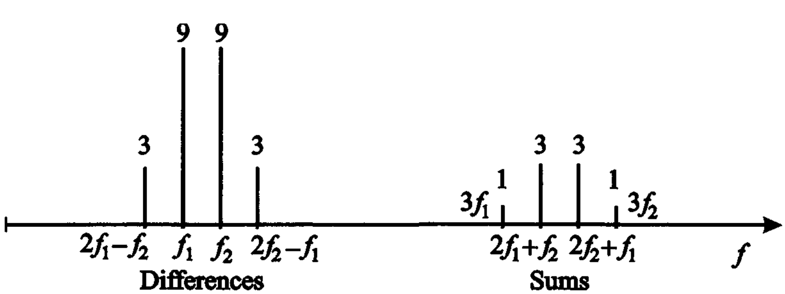

Figure 14.7. Third-order coefficients for the intermodulation products for $V_i = V[\cos(2\pi f_1 t) + \cos(2\pi f_2 t)]$. There is a common factor of $V^3/4$ for each coefficient.

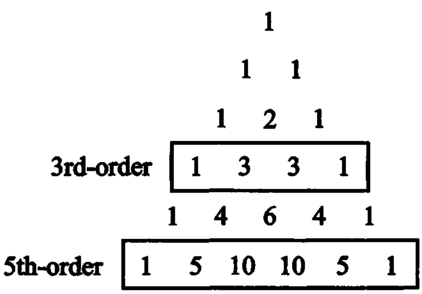

Figure 14.8. Pascal's triangle. The numbers in each row are obtained by adding the pair of numbers above. The coefficients for third- and fifth-order products are boxed.

14.8 Dynamic Range

To see how intermodulation products affect reception, we plot output power versus input power for an intermodulation product and an ordinary signal (Figure 14.9). The input for the signal is a single carrier, whereas the inputs for the intermodulation product comprise two carriers of equal power. It is traditional to consider the input power to be the power of one of the input carriers, rather than the total. On a log scale, the slope gives the order of product. A signal with linear gain has a 1:1 slope, whereas intermodulation products have steeper slopes. In theory, third-order intermodulation products have a slope of three, and fifth-order products have a slope of five. In practice, the situation is more complicated, because both third- and fifth-order products appear at $f_{\underline{3}}$ and $f_{\overline{3}}$. Often the slope becomes steeper at higher power levels, because the fifth-order products overtake the

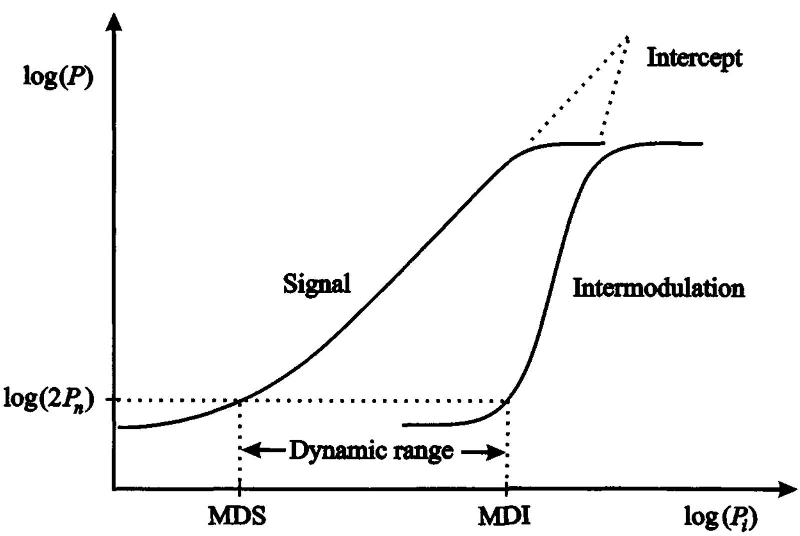

Figure 14.9. Finding the dynamic range for a receiver. Plot of the output signal and intermodulation product P versus the input power P_i on log scales. The outputs saturate at high levels because of the AGC. People often extrapolate the linear portion of the curves until they intersect. Manufacturers often quote the input or output powers associated with the intercept as a measure of the quality of the amplifier or mixer.

third-order products. Other amplifiers and mixers are not adequately described by a small number of Taylor-series terms, and the slope of the intermodulation products may be lower than three. Computer programs that simulate intermodulation products often assume a slope of three. It is therefore a good idea to be cautious in interpreting the simulations and to measure the products yourself.

The MDS is the input signal power that gives an output SNR of 1:1. We can identify it as the signal that gives a total output power, signal plus noise, of $2P_n$, where P_n is output noise. In the same way, we can identify the *minimum detectible intermodulation input*. This is labeled MDI on the plot. This is the input power level that gives a total output, tone plus noise, of $2P_n$. The difference between the MDS and the MDI is called the *dynamic range*. We write it as

$$\text{Dynamic range} = \text{MDI} - \text{MDS}. \tag{14.47}$$

The dynamic range is invariably quoted in dB. It is a measure of the range of useful signals for the receiver. It is the difference between signals that are just strong enough to be heard and signals that are just strong enough to cause interfering intermodulation products. Good receivers have a dynamic range of 100 dB. If the noise power increases, the two curves approach each other, and the dynamic range decreases. For example, in the NorCal 40A, antenna noise is usually 30 dB above receiver noise, and this reduces the dynamic range considerably. Assuming that the slope of the signal is 1:1, and that the slope of the intermodulation product is 3:1, the MDS will increase by 30 dB, while the MDI only increases by 10 dB. Thus the dynamic range drops by 20 dB. We can get some of this dynamic range back by adding an attenuator. For example, if we add 15 dB of attenuation, the MDS drops by 15 dB, but the MDI falls by only 5 dB, giving us a 10-dB improvement in dynamic range. The NorCal 40A includes an RF Gain pot to help improve the dynamic range.

A more fundamental solution to the intermodulation problem is to use a different mixer. In the SA602AN, dynamic range is limited by the exponential relationship between base voltage and collector current in the bipolar transistors of a long-tailed pair. This causes intermodulation products when the input signal levels approach the thermal voltage, 25 mV. Diode and FET mixers have better intermodulation performance, but they make the receiver more complex.

FURTHER READING

A good introduction to probability and random variables is given in *Probability, Random Variables, and Random Signal Principles*, by Peyton Peebles, published by McGraw-Hill. Residue calculus is covered in *Theory of Functions of a Complex Variable*, by A. I. Markushevich, published by Chelsea Publishing Company.

PROBLEM 34 - RECEIVER RESPONSE

First we make a plot of the audio frequency response of the receiver. This response is affected by both the IF Filter and the Audio Amplifier. Make the connections shown in

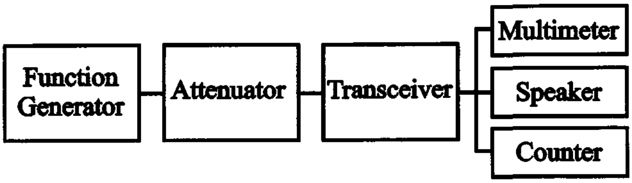

Figure 14.10. Connections for measuring the audio frequency response.

Figure 14.10, and set the function generator and attenuator for an input power of 100 fW at a frequency between 7,020 and 7,030 kHz. If you work in a lab where others also make measurements, you should set the kilohertz digit to the bench number to avoid interference. Tune the receiver so that the output audio frequency is 620 Hz. The output audio voltage should be near 100 mVrms. If it is appreciably lower, check the receiver components, particularly the filter capacitors C1 and C2.

A. Readjust the attenuator and the function generator to give an audio output voltage of 100 mVrms. Now plot the audio voltage on a log scale as the audio frequency varies from zero to 1,200 Hz. Find the 3-dB bandwidth. This plot is a good check of the BFO setting. If the peak of the plot is outside of the frequency range from 600 Hz to 650 Hz, you should readjust the BFO.

For this part of the lab, you will need to work with a partner. We measure the MDS using weak input signals from another transceiver. Function generators often have a very limited power range, and it may be difficult to isolate a function generator from the receiver at low power levels. To get lower input signal levels, we use another NorCal 40A as the signal source, and run it off a battery to keep signals from coupling back through the wall outlets.

B. Select one of the two transceivers to be the transmitter. Set the VFO Tune pot to mid range, and plug in a battery. You will probably find it convenient to plug a switch into the Key jack to turn the transmitter on and off. We need to reduce the power of the transmitter. What is the peak-to-peak voltage needed to deliver −40 dBm to a 50-Ω load? To get this voltage, first set the Drive pot R13 to minimum gain. Then mistune the Transmit Filter capacitor C39 to reduce the output power to −40 dBm. Make sure you have the 50-Ω load in parallel with the scope input, or else the settings will be off.

The other transceiver will act as the receiver. Make the connections shown in Figure 14.11, with the key switch off. The RF Gain control on the receiver should be fully clockwise for minimum attenuation. Measure the output noise voltage when the transmitter is off with a multimeter. Set the attenuator to give an input signal of −100 dBm,

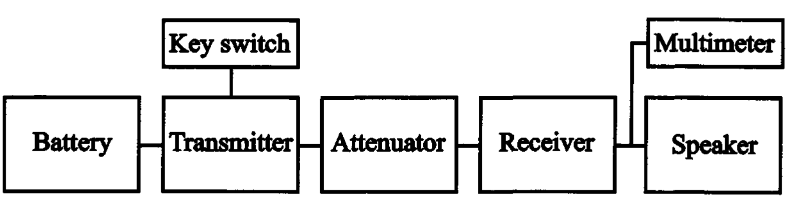

Figure 14.11. Measuring the receiver response.

and tune the receiver for maximum audio output. Once the receiver is tuned, it is a good idea to put the mouth of the speaker face down on the table to preserve the ears of others.

Now check the signal level at –150 dBm. This is well below the MDS, and you should not hear any tone in the noise. The multimeter reading should be the same as it was without any signal. Larger voltages indicate that the signal is leaking through somewhere. Sometimes the cable between the transmitter and the attenuator is the source of this leakage, and you may be able to reduce it by making a direct connection between the attenuator and the transmitter. Also try moving the transmitter and battery far away from the receiver.

C. Plot the output voltage on a log scale as the input power varies from −150 dBm to –50 dBm.

D. What is the MDS? This would be the signal that gives an output power of $2P_n$, or a multimeter reading of $\sqrt{2}V_{rms}$, where V_{rms} is the rms output noise voltage.

E. What is the weakest input signal that you can still hear? What is the signal-to-noise ratio at this level?

F. A function generator that can produce noise is useful for finding the NEP. For example, the HP33120A produces noise that is spread over a bandwidth of 10 MHz. With a function-generator setting of −30 dBm and an attenuator setting of 60 dB, what is the input noise power density to the receiver? Find the NEP as the input noise density that gives an output of $2P_n$. Divide the MDS by the NEP to find the bandwidth. People call this the *noise bandwidth*, because it is usually close to but not the same as the 3-dB bandwidth.

G. Now connect your receiver to an antenna. Tune to a part of the band where you do not hear a signal. What is the output voltage? Antenna noise at 7 MHz is primarily due to lightning, and this gives it a boom and crash sound that is different from the steady roar of receiver noise.

H. Use the plot you made in Part C to find the MDS for antenna noise.

I. What is the antenna noise temperature?

You should retune the transmitter to full power. Make a note of your MDS for receiver and antenna noise to use in the next problem.

PROBLEM 35 – INTERMODULATION

This problem is best done in groups of three, because two transmitters are needed. If function generators are used instead, you need to be careful, because function generators can also produce intermodulation products. Unfortunately these products are at the same frequencies we are interested in, and so they interfere with an intermodulation measurement.

A. Find the coefficients and frequencies for $[\cos(2\pi f_1 t) + \cos(2\pi f_2 t)]^5$. You may assume that f_2 is larger than f_1.

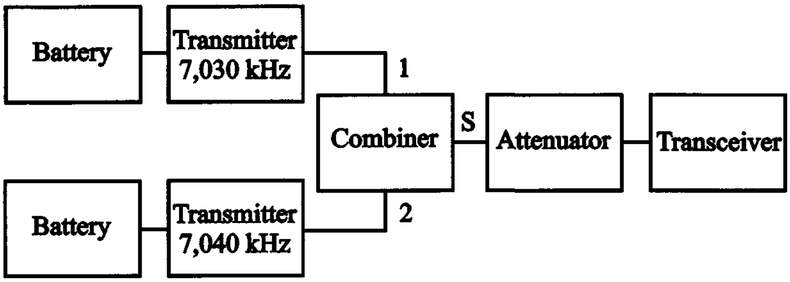

Figure 14.12. Measuring intermodulation products.

Choose two of your three transceivers to be transmitters and one to be the receiver. Set one transmitter for 7,030 kHz, and set the other for 7,040 kHz. Set the output power to a 50-Ω load for each to 2 μW. The two transmitters should be connected to a power combiner as shown in Figure 14.12. The connector labeled "S" is the sum port. A power combiner is different from an ordinary BNC tee. With a tee, power couples from one transmitter to the other, causing intermodulation in each transmitter. The power combiner isolates each transmitter from the other to prevent this. Power combiners have a combining loss of 3 dB, because half the power is dissipated in a resistor inside the combiner. This means that the power into the attenuator at each frequency is 1 μW. The isolation is not perfect, but the power coupled between transmitters is usually more than 20 dB below the power that goes to the sum port. This makes the input to the other transmitter less than 10 nW, which is small enough to prevent transmitter intermodulation products.

B. To hear the tone at f_3, set the attenuator so that the input power for each component is −40 dBm, and tune the receiver for a signal near 7,020 kHz. Now vary the input power with the attenuator. Plot the output audio voltage on a log scale versus the input power in dBm. Use a wide enough input range that the output extends from the noise floor up to 200 mVrms.

C. Now find the tone at f_5, and make a plot for it on the same graph.

D. From the graph and your measurement of the receiver MDS in the previous problem, find the dynamic range of your receiver.

E. From your measurement of the antenna MDS in the previous problem, find the antenna-limited dynamic range.

PROBLEM 36 – DEMONSTRATION

Present the transceiver that you built for inspection. The construction should be complete, and the solder connections should be neat.

A. Find a weak signal in the frequency range from 7,000 to 7,040 kHz. The receiver filters, the VFO, and the BFO will need to be properly adjusted to receive the signal.

B. Transmit a signal with at least 2 W of power within 200 Hz of the received signal. The sidetone should match the tone of the received signal.