9

Transistor Amplifiers

When a transistor is active, the current gain β is large, in the range of 100 or more. This means that we can use the transistor as an amplifier to increase the power of a signal. The amplifier may be considered the single most important device in communications electronics, and it is key to both receivers and transmitters. Developing amplifiers has been a central focus of electrical engineering from the days of the first vacuum tubes, and it is just as important today. There are many issues to consider in designing an amplifier. In transmitters, we are very interested in efficiency. High efficiency makes it easier to dissipate the heat and allows long battery life in portable transmitters. In receivers, it is important to add as little noise as possible to the signal. In this chapter, we study *linear* amplifiers, where the amplitude of the output tracks the amplitude of the input. In the next chapter, we consider saturating amplifiers, where only the frequency of the output follows the input.

9.1 Common-Emitter Amplifier

The basic transistor amplifier is shown in Figure 9.1a. It uses an npn transistor with a load resistor R at the collector. The supply voltage is written as V_{cc}. It is traditional to double the subscript of a supply voltage to distinguish it from an AC voltage. This circuit is called a *common-emitter* amplifier. You do have to be on your guard with amplifier names. In this example, the emitter is grounded, and so the name *common emitter* is straightforward enough. The idea is that the input is made up of the base and emitter, and the output is made up of the collector and emitter, so that the emitter is *common* to both the input and the output. However, there will usually be other components connected to the emitter, so that it may not be easy to make a distinction. Later we will study a common-collector amplifier, the emitter follower.

At the top of Figure 9.1b is the base-voltage waveform. The positive voltage is limited to the forward voltage V_f of the base–emitter diode. Base current will only flow when the source voltage V_o is sufficiently positive to turn on the base–emitter diode. When V_o goes negative, the base voltage also goes negative, and the base current will cease. The collector-voltage waveform is shown beneath the base voltage. When current flows in the base, the transistor is active, and there will be a large collector current. This collector current causes a voltage across the

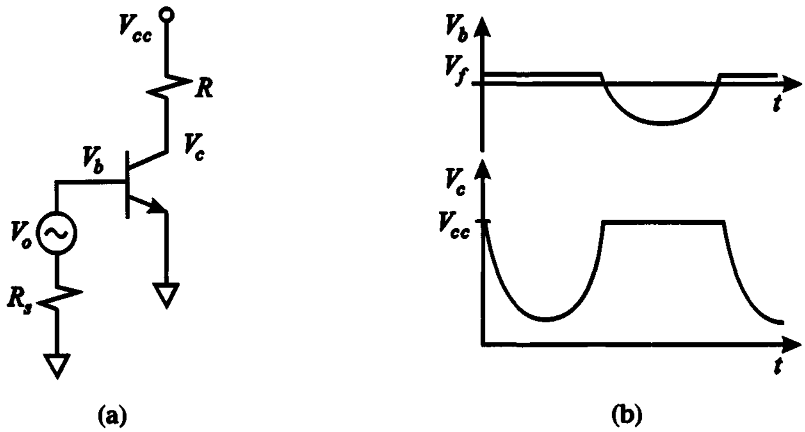

Figure 9.1. (a) Common-emitter amplifier. The triangles indicate ground connections. (b) Base voltage V_b (top) and collector voltage V_c (bottom).

load resistor, and the collector voltage drops. If the source voltage is large enough, we will turn the transistor completely on, and drive the collector voltage to the saturation level, as in the Receiver Switch in Problem 19.

This amplifier has a drawback. The transistor is off half the time, and the output is only half a cosine. To get the entire cosine, we must offset the base voltage to keep the transistor from shutting off. We should note, however, that this half-cosine circuit is not as dumb as it looks, and we will consider it again when we discuss Class-B amplifiers in the next chapter. For now, we add a DC voltage V_{bb} to the base circuit (Figure 9.2a). V_{bb} is called a *bias*, and it is usually provided by a resistive voltage divider between the supply and ground. In the lab, function generators often allow us to add this offset voltage. The idea of the bias voltage is to keep the base conducting continuously. Thus the collector voltage will be a full cosine wave (Figure 9.2b). The largest collector voltage we can get is a swing from V_{cc} when the collector current I_c is zero to near ground when the transistor is on the edge of saturation. We can get this large swing by adjusting the bias for a collector voltage near $V_{cc}/2$.

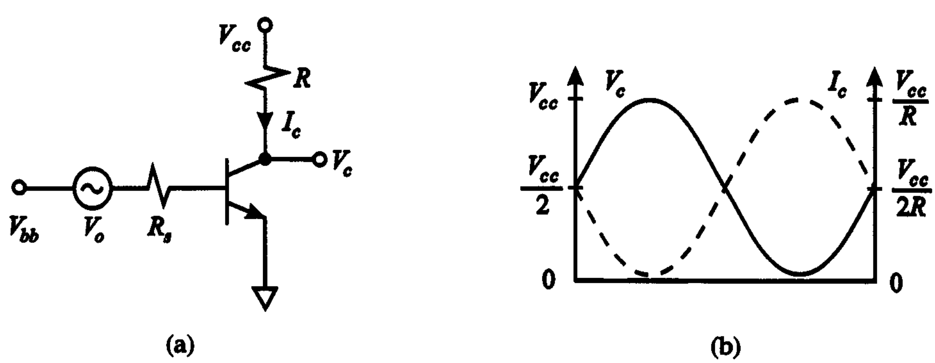

Figure 9.2. Common-emitter amplifier with a bias voltage V_{bb} (a), and collector waveforms for a maximum peak-to-peak voltage (b).

We distinguish between different classes of amplifiers by the bias. If we bias so that the base–emitter diode is always on, it is a *Class-A* amplifier. In the NorCal 40A, the Driver Amplifier is Class A. In the next chapter, we define other letter classes of amplifiers where the transistor is off during part of the cycle. The advantage of Class A is that the output is a good replica of the input with little distortion. The disadvantage of Class A is poor efficiency. The other classes distort the waveform but are much more efficient than Class A.

9.2 Maximum Efficiency of Class-A Amplifiers

We can calculate the maximum efficiency for a Class-A amplifier, assuming that the output voltage varies from zero to V_{cc}. The collector current will vary from zero to V_{cc}/R. We define the efficiency η (the Greek letter *eta*) as

$$\eta = P/P_o, \tag{9.1}$$

where P is the AC load power and P_o is the DC supply power. We can write P_o as

$$P_o = V_{cc} I_o, \tag{9.2}$$

where I_o is the average collector current, given by

$$I_o = \frac{V_{cc}}{2R}. \tag{9.3}$$

This gives us the supply power

$$P_o = \frac{V_{cc}^2}{2R}. \tag{9.4}$$

Now we can write the AC load power P as

$$P = \frac{V_{pp} I_{pp}}{8} = \frac{V_{cc}^2}{8R}. \tag{9.5}$$

We can see that $P = P_o/4$, so that the efficiency η is 25%. This is the maximum efficiency for a Class-A amplifier with a resistive load.

It is interesting to track the power flow. In addition to the AC load power P, there is a DC load power P_{rdc}. We can write this as

$$P_{rdc} = \frac{V_{cc}^2}{4R}. \tag{9.6}$$

This means that half the power from the supply is lost as DC power in the resistor. There is also DC power in the transistor. Because the average voltage and current across the transistor are the same as for the resistor, the DC power is also the same. We write this as P_{tdc}, given by

$$P_{tdc} = \frac{V_{cc}^2}{4R}. \tag{9.7}$$

However, this is not the end of the story. The AC peak-to-peak voltage and current for the transistor are also the same as for the resistor, except Figure 9.2b shows

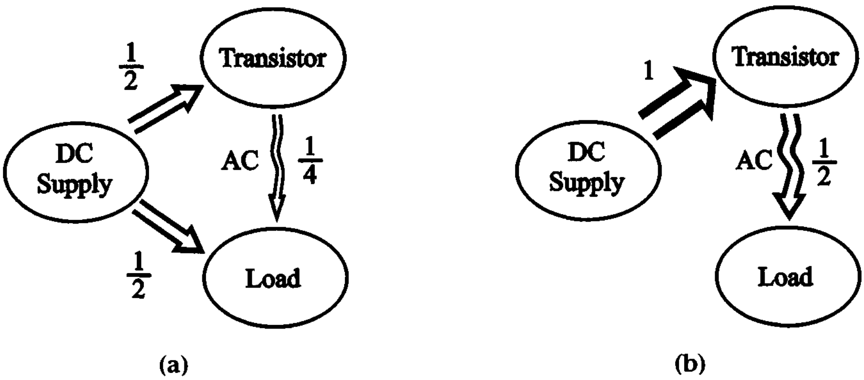

Figure 9.3. Power flow in a Class-A amplifier with maximum efficiency for a resistive load (a) and for a transformer-coupled load (b).

that they are 180° out of phase. Thus we can write the transistor AC power P_{tac} as

$$P_{tac} = -\frac{V_{cc}^2}{8R}, \tag{9.8}$$

where there is a minus sign because the voltage and current are out of phase. The interesting part about this power is that it is negative, indicating that the transistor produces AC power rather than absorbing it like a resistor. Half the DC power delivered to the transistor is converted to AC power, which is consumed in the load. Figure 9.3a summarizes this power flow. The DC power from the supply splits between resistor and the transistor. The transistor power further splits between the power that is absorbed and the AC power that is delivered to the load.

There are two major disadvantages of this Class-A circuit. First, half the power is lost as DC power in the resistor. Second, many loads cannot be connected directly to the supply. If the load is the base of another npn transistor, for example, we will want a voltage that is near ground. We can use a transformer to solve these problems (Figure 9.4a). Because DC power does not couple through a transformer, this eliminates the DC power in the load. In addition, the DC resistance between the supply and the transistor is zero, and so the average collector voltage is V_{cc}

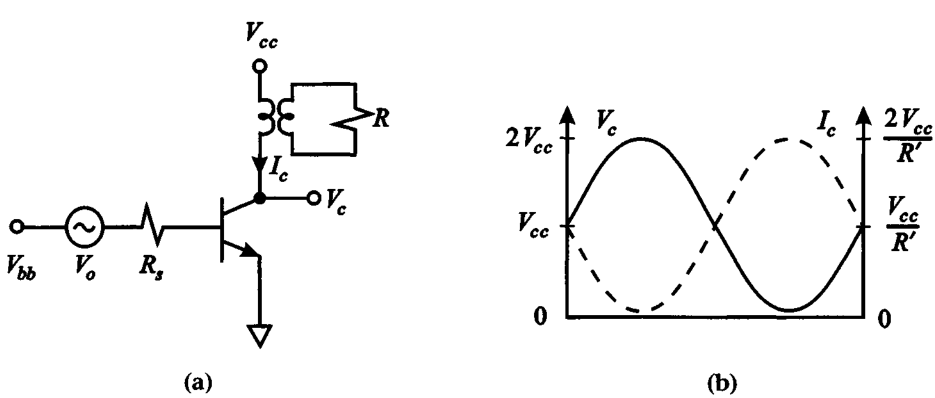

Figure 9.4. Class-A amplifier with a transformer-coupled load (a), and the maximum peak-to-peak voltage and current waveforms (b).

instead of half V_{cc} (Figure 9.4b). This means that the peak-to-peak voltages and currents can be twice as large as before. The Driver Amplifier in Problem 21 uses a transformer. We can write the supply power as

$$P_o = V_{cc}I_o = V_{cc}^2/R', \tag{9.9}$$

where R' is the effective load resistance, given by

$$R' = n^2R, \tag{9.10}$$

where n is the turns ratio. The average output power P can be written as

$$P = \frac{V_{pp}I_{pp}}{8} = \frac{V_{cc}^2}{2R'}. \tag{9.11}$$

We can see that $P = P_o/2$, and thus the maximum efficiency is 50%. The maximum efficiency is twice as high as for a resistive load because the transformer prevents DC power from being lost in the resistor. Figure 9.3b shows the power flow. The efficiency in practical Class-A amplifiers is typically between 30 and 40%. One important point is that a Class-A amplifier dissipates even more power when there is no output. This is a major disadvantage, and it is in sharp contrast to the other classes of amplifiers, where power dissipation is small when there is no output.

The transformer turns ratio controls the peak-to-peak current. Manufacturers specify a collector current limit in their data sheets. We would aim for a maximum current I_m under this limit. From Figure 9.4b, we can write

$$I_m = \frac{2V_{cc}}{R'} = \frac{2V_{cc}}{n^2R}. \tag{9.12}$$

This lets us choose n. To get the full current swing, the bias current should be set to half this maximum.

9.3 Amplifier Gain

In addition to efficiency, we characterize an amplifier by its gain, which is the ratio of output power to input power. Figure 9.5 shows a circuit with an amplifier,

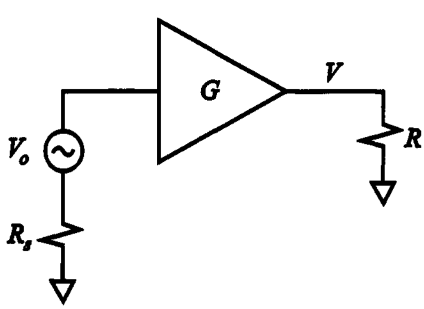

Figure 9.5. Amplifier with a source and a load.

a source, and a load. We write the gain G in dB as

$$G = 10\log(P/P_+)\,\mathrm{dB}. \tag{9.13}$$

In the lab, we will calculate the output power P from a measurement of the peak-to-peak voltage across the load. We write

$$P = \frac{V_{pp}^2}{8R}. \tag{9.14}$$

The available power P_+ is the maximum power that would be delivered by the source to a matched load. For a matched load, the voltage is half the open-circuit voltage of the source. We write the matched-load voltage as V_+, given by

$$V_+ = V_o/2. \tag{9.15}$$

We then write P_+ as

$$P_+ = \frac{V_{+pp}^2}{8R_s}. \tag{9.16}$$

In the lab, V_{+pp} is the amplitude setting on a properly calibrated function generator. These formulas show that the gain depends on both voltages and resistances.

9.4 IV Curves

One thing that you may notice when you build the Driver Amplifier in Problem 21 is that the output looks somewhat distorted on the oscilloscope (Figure 9.6a). The collector voltage V_c is wider on the bottom and narrower on the top. We can understand what is happening if we check the base voltage V_b. It is distorted in the same way as the collector voltage, except that the widening is on the top and the narrowing is on the bottom. If we take into account the fact that the

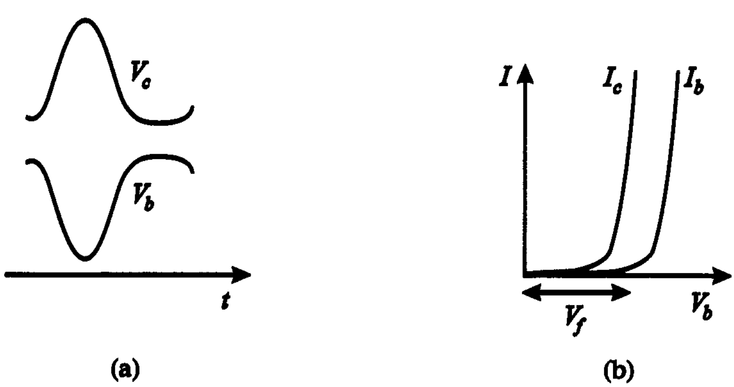

Figure 9.6. (a) Distortion in the Driver Amplifier collector (V_c) and base (V_b) voltage waveforms, as seen on an oscilloscope. The scales are adjusted to compensate for gain. (b) IV curves for the base–emitter diode when the transistor is active.

amplifier inverts the output, it means that at least to the eye, the circuit is acting as a good voltage amplifier, in that the output voltage is a faithful replica of the input voltage. It is interesting to check this on an oscilloscope by inverting one channel and adjusting the voltage scale to compensate for the gain of the amplifier. If you do this, you will see that the input and output waveforms match quite well.

We can understand the distortion in the base voltage curve if we consider IV curves for the base–emitter diode (Figure 9.6b). There are two currents to consider, the base current I_b and the collector current I_c. They have a similar shape, except that the base current is lower by a factor of β. The effect is to shift the base-current curve to the right of the collector-current curve. We leave the details to a book on solid-state devices, but we can say that when the base voltage is near V_f, the currents are given to a close approximation by

$$I_b = I_{bs} \exp(V_b/V_t), \tag{9.17}$$

$$I_c = I_{cs} \exp(V_b/V_t), \tag{9.18}$$

where I_b is the base current, I_c is the collector current, and V_b is the voltage between the base and emitter. A similar expression can be used for the current and voltage in a pn diode. V_t is called the *thermal voltage*, and it is given by

$$V_t = kT/q, \tag{9.19}$$

where k is Boltzmann's constant, 1.38×10^{-23} J/K, T is the absolute temperature in kelvins, and q is the electronic charge, 1.60×10^{-19} C. At a typical room temperature of 295 K, or 22°C, we have

$$V_t = 25\,\text{mV}. \tag{9.20}$$

This formula gives a hint that the underlying physics of current flow in diodes and transistors is thermal excitation. The currents I_{bs} and I_{cs} are proportionality constants. They are called *saturation currents*, and they are related by the current gain β via

$$I_{cs} = \beta I_{bs}. \tag{9.21}$$

Both I_{cs} and I_{bs} increase strongly with temperature, and β varies with both temperature and current level. Hence the saturation currents are not really constants.

Another thing that you should notice is that in Equations 9.17 and 9.18, I_b and I_c are always positive, even when the applied voltage is negative. This must be incorrect. Otherwise, we would have a perpetual power source, because the power for negative voltages would be negative. Often you will see a term I_{bs} and I_{cs} subtracted from these equations to make the currents go to zero when the voltage is zero, and to make the current negative when the voltage is negative. The actual currents at low and negative voltages are more complicated. In particular, the effective value of V_t changes. For this reason I prefer to give a simpler formula that holds where we apply it, at voltages near V_f.

The equations tell us that diode and transistor currents increase by a factor of e for every 25-mV increase in voltage. This is a steep increase. For a factor of ten, we need only change the voltage by

$$\Delta V = V_t \ln(10) \approx 60 \text{ mV}. \tag{9.22}$$

This means that in a device with a β of 100, the I_b curve is shifted to the right of the I_c curve by 120 mV. This is the reason that the forward voltages for the base–emitter diodes tend to be between 700 mV and 800 mV, rather than the 600 mV to 700 mV for ordinary diodes.

9.5 Base Resistance

The slope of the base current gives us the conductance of the base–emitter diode. This is a small-signal conductance, and we write it as g_b. Because Equation 9.17 is exponential, this has a simple form:

$$g_b = \frac{dI_b}{dV_b} = \frac{I_b}{V_t}. \tag{9.23}$$

You should go through the details to verify this formula. It is more common to use the base resistance r_b, which is the inverse of g_b. At room temperature, we write

$$r_b = \frac{25 \text{ mV}}{I_b}. \tag{9.24}$$

This expression can also be used for a diode. The formula is similar to the one we found for the resistance of a switch in saturation, except that the voltage constant is different (Equation 8.7). As an example, a typical base bias current for the Driver Amplifier in Problem 21 might be 250 μA. This gives us $r_b = 100\ \Omega$. We can use this base resistance to draw a small-signal AC model of an active transistor (Figure 9.7a) and to write the base voltage as

$$v = i_b r_b. \tag{9.25}$$

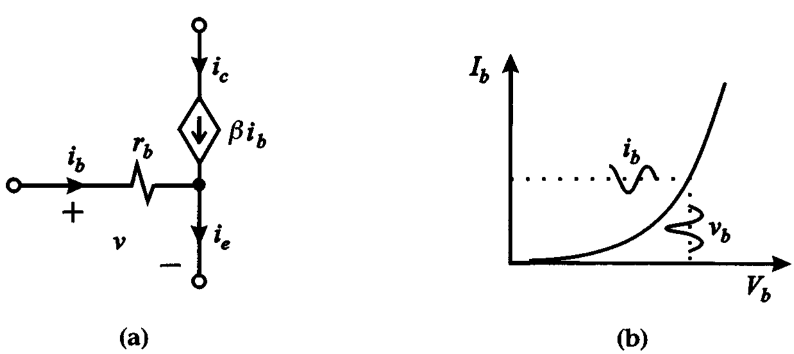

Figure 9.7. Small-signal model of an active transistor, including the base resistance r_b (a). Understanding the distortion in the base voltage waveform (b).

We will use lower-case letters to distinguish this signal voltage and current from the DC bias voltages and currents.

The slope of the collector-current curve is written in a similar way as

$$g_m = \frac{dI_c}{dV_b} = \frac{I_c}{V_t}, \tag{9.26}$$

where g_m is called the *transconductance* to indicate that the current and voltage are for different terminals. We will not use the transconductance in our calculations for bipolar transistors, but we will need it for field-effect transistors.

The IV curves explain the distortion in the base waveforms (Figure 9.6a). Consider applying a sine-wave current i_b in addition to the bias current (Figure 9.7b). The resulting voltage v_b has bigger negative peaks than positive peaks because the graph is curved. This accounts for the distortion we see. We can improve the distortion by lowering the source impedance. The source impedance for the Driver Amplifier in Problem 21 is rather high, around 600 Ω. In the complete transceiver, the Buffer Amplifier provides a much lower source impedance for the Driver Amplifier, and this reduces the distortion.

There is another thing that you might think about from the current plots. A small change in voltage causes a large change in current, and this can make it difficult to set the base current with a voltage source. Since β is usually not well known, we will have even more uncertainty in the collector current. Moreover, the forward voltage V_f shifts downward by 2 mV for every degree that the temperature rises. We will not dwell on this, but it serves as a warning that biasing can be a tricky business. We can make biasing much easier if we add an emitter resistor.

9.6 Emitter Degeneration

In Figure 9.8a, we add an emitter resistor R_e. This is called *emitter degeneration*. This resistance makes it easier to set the bias and control the gain. We consider the bias first. Figure 9.8b shows a simplified model for the amplifier input. The base–emitter diode is represented by a voltage source. This is because a diode has a low resistance when it is on and a voltage that normally varies over only a small range. Since this is just what a voltage source does, it is convenient to use it in the equivalent circuit. In addition, we have shown the emitter resistor R_e, together with the collector current I_c. The base current is much smaller than the collector current, so that $I_c \approx I_e$. If we assume that we have a particular collector current goal in mind for the bias, we can write the required base bias voltage V_{bb} as

$$V_{bb} \approx V_f + I_c R_e. \tag{9.27}$$

Notice that this voltage does not depend on β as long as it is large. This makes the voltage much easier to set.

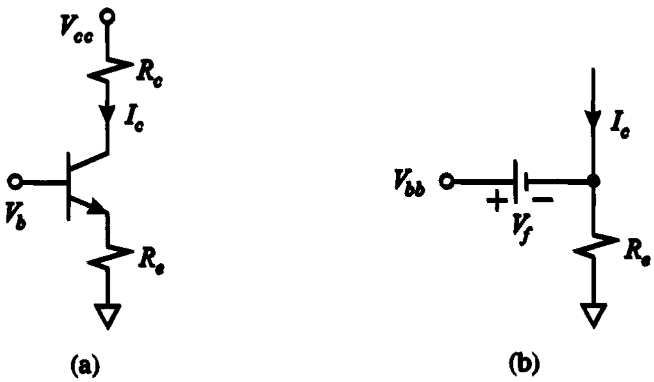

Figure 9.8. Adding an emitter resistor to the common-emitter amplifier (a), and equivalent circuit for calculating the bias voltage (b).

The emitter resistor also allows us to set a voltage gain that is determined by resistances that are under much better control than β. To see how this works, let us define the voltage gain G_v as the ratio

$$G_v = v/v_i. \tag{9.28}$$

In the formula, v is the output AC voltage and v_i is the input AC voltage. Working in terms of AC voltages and currents simplifies things, because the bias voltages have no effect on AC signals. We can replace the bias voltages by shorts in the equivalent circuit (Figure 9.9a). We assume that β is large so that $i_b \ll i_e$ and $i_e \approx i_c$, and we write the input voltage v_i as

$$v_i = i_b r_b + i_e R_e \approx i_c R_e. \tag{9.29}$$

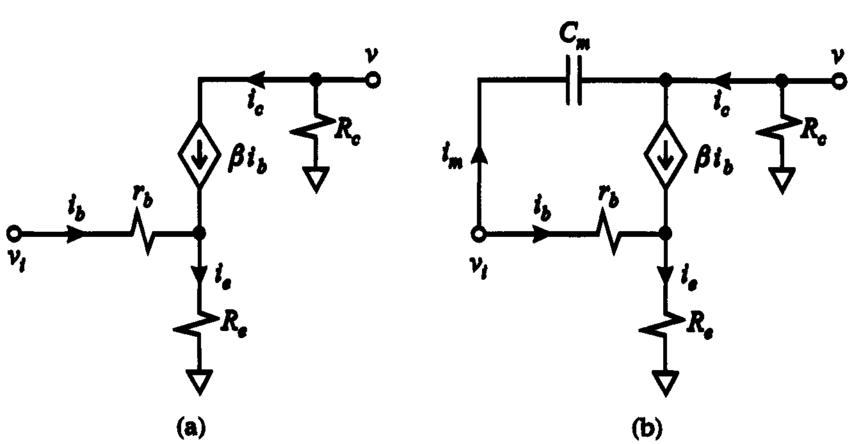

Figure 9.9. (a) Equivalent circuit for finding the voltage gain G_v and the input impedance Z_i of a common-emitter amplifier with an emitter resistor. (b) Adding the Miller capacitance.

The output voltage is a little trickier, because the current flows up through R_c. This gives the output voltage a minus sign. We can write the output voltage v as

$$v = -i_c R_c. \tag{9.30}$$

Notice that since v has the opposite sign from v_i, the amplifier inverts. We can divide these formulas to get the gain as

$$G_v = -R_c/R_e. \tag{9.31}$$

Now we have a gain that does not depend on the value of β, but rather on resistors that have precise values. Note that the gain can be less than 1 if $R_e > R_c$.

We can also use Figure 9.9a to find the AC input impedance. This time, we cannot neglect the base current, because it is the input. We write the input impedance as

$$Z_i = v_i/i_b. \tag{9.32}$$

From Equation 9.29, we have

$$v_i \approx i_c R_e = \beta i_b R_e. \tag{9.33}$$

This gives us our input impedance:

$$Z_i = \beta R_e. \tag{9.34}$$

Here the factor of β enters directly to increase the input impedance. This helps if we want to see the full open-circuit voltage of the source. In Problem 22, however, we will find that we may not actually achieve this high impedance, because the input is shunted by capacitance from the base to the collector. This capacitance is called the *Miller capacitance*. It arises because the layers in a transistor lie on top of one other, so that a voltage in one layer induces charges in another layer. The capacitance is small, usually only a few picofarads. However, the effect is magnified by the gain of the amplifier. To see this, consider the circuit in Figure 9.9b, where we have added a capacitor at the input. The voltage on one side of the capacitor is v_i, and the voltage on the other side is the collector output voltage $v = G_v v_i$. We can write the capacitor current i_m in terms of the total voltage across the capacitor:

$$i_m = j\omega C_m(v_i - v) = j\omega C_m(v_i + |G_v|v_i) = j\omega C_m(|G_v| + 1)v_i. \tag{9.35}$$

The effect of the capacitance is multiplied by a factor of $|G_v| + 1$. We can write the input impedance in the form

$$Z_i = \beta R_e \parallel (|G_v| + 1)C_m. \tag{9.36}$$

Now we calculate the output impedance that we would use in writing a Thevenin or Norton equivalent circuit for our amplifier. So far we have assumed that the collector voltage does not affect the current of an active transistor. However, there is a tilt to the IV curves (Figure 9.10). The reason the lines slope is that the effective thickness of the base becomes smaller at high voltages, and this increases β. An

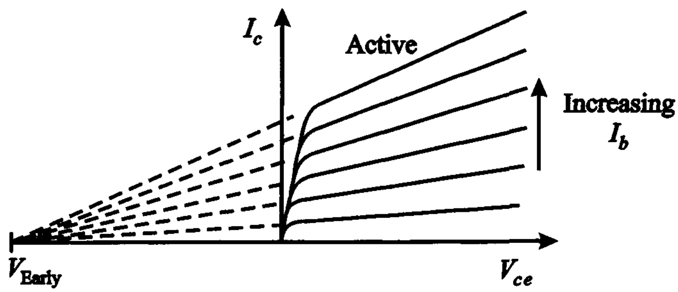

Figure 9.10. Collector current and voltage, showing the slope in the active region. If we extend these lines backwards, they intersect at the Early voltage.

interesting thing about the lines is that they intersect at one point on the voltage axis. This point is called the *Early voltage*. The Early voltage is usually not quoted in manufacturer's data sheets, and it varies greatly among transistors, because it depends strongly on the thickness of the base. For the 2N2222A transistor that we use in the Driver Amplifier in Problems 21 and 22, the Early voltage is 145 V. Because this is much larger than any voltage we apply to the collector, we can write the collector resistance r_c approximately as

$$r_c \approx V_{\text{Early}}/I_c. \tag{9.37}$$

Like the base resistance r_b, the collector resistance varies inversely with the current. For a collector bias current of 50 mA, we get a collector resistance r_c of 3 kΩ. We can draw a small-signal model for the transistor that includes a shunt collector resistance r_c (Figure 9.11a).

In Figure 9.11b, we show a small-signal model for the common-emitter amplifier. In the model, we use a combined source resistance R'_s, given by

$$R'_s = R_s + r_b. \tag{9.38}$$

The collector also has capacitance associated with it, and this should be included, because its reactance is often comparable to the collector resistance. We define a

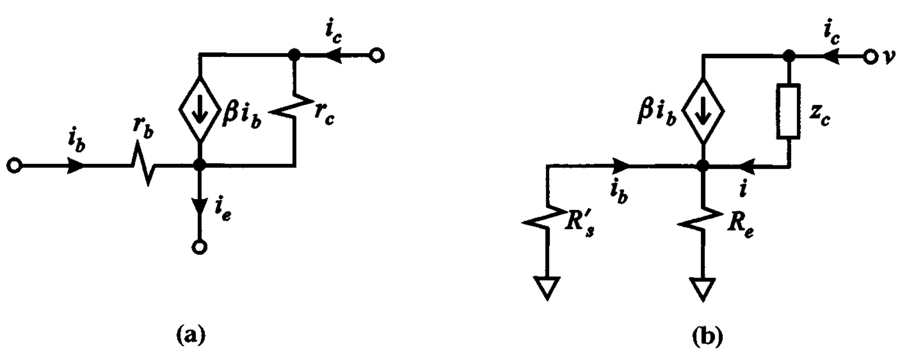

Figure 9.11. Small-signal transistor model that includes the collector resistance r_c (a). Small-signal model for the common-emitter amplifier for calculating the output impedance Z_o (b).

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collector impedance z_c given by

$$z_c = r_c \parallel C_c, \tag{9.39}$$

where C_c is the output capacitance that is specified in manufacturer's data sheets. For the 2N2222A used in the Driver Amplifier, C_c is 8 pF, which gives a reactance of 2.8 kΩ at 7 MHz.

We write the output impedance of the amplifier as

$$Z_o = v/i_c. \tag{9.40}$$

The key to the calculation is finding the current i through the collector impedance z_c. We use Kirchhoff's current law to write it as

$$i = i_c - \beta i_b. \tag{9.41}$$

We can use the current-divider formula to write i_b as

$$i_b = -\frac{i_c R_e}{R_s' + R_e}. \tag{9.42}$$

When we substitute into the previous formula, we get

$$i = i_c\left(1 + \frac{\beta R_e}{R_s' + R_e}\right). \tag{9.43}$$

Now we write the voltage v as

$$v = i z_c + i_c R_s' \parallel R_e \tag{9.44}$$

and the output impedance as

$$Z_o = \frac{v}{i_c} = z_c\left(1 + \frac{\beta R_e}{R_s' + R_e}\right) + R_s' \parallel R_e. \tag{9.45}$$

Usually $|z_c| \gg R_e$, and we can write

$$Z_o \approx z_c\left(1 + \frac{\beta R_e}{R_s' + R_e}\right). \tag{9.46}$$

Ordinarily z_c is multiplied by a large factor, giving a large output impedance. Hence a common-emitter amplifier is an excellent current source.

9.7 Emitter Follower

We have found that the emitter resistor couples the effect of the collector current back to the input to affect the bias, gain, and input impedance. This is called *feedback*. In the emitter follower, we take this to an extreme, eliminating the collector resistor altogether (Figure 9.12a) and using the emitter resistor as the load.

When the base–emitter diode is conducting, the output voltage is the same as the input voltage, except for the base–emitter diode drop V_f. This means that for

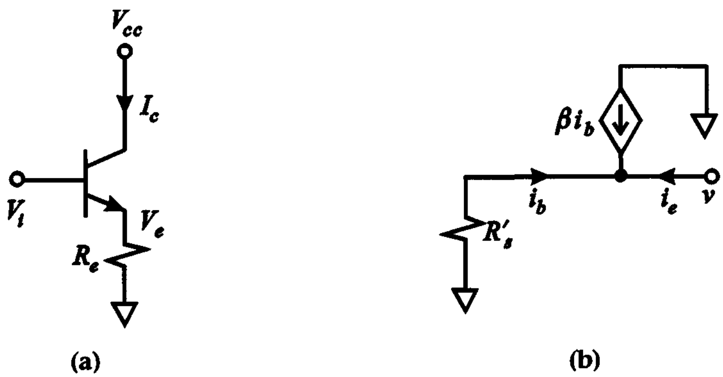


Figure 9.12. Emitter follower (a), and the small-signal circuit model for finding the output impedance Z_o (b).

AC voltages, the voltage gain is 1. Because the output voltage has the same sign as the input voltage, the emitter follower does not invert. A follower does not have any voltage gain, but it does have full current gain, and for this reason it can increase the power. It is useful as a buffer amplifier because the input impedance is very high. We can insert an emitter follower between a source and load when we have a source such as an oscillator or filter that might be affected by load changes. The formulas that we developed for biasing (Equation 9.27) and input impedance (Equation 9.34) for the common-emitter amplifier still hold, because we are using the base for the input as before. The Miller capacitance is not a problem because the collector voltage is constant. However, the output impedance is quite different, because the output is taken from the emitter, which is a low-impedance point. The small-signal equivalent circuit is given in Figure 9.12b. We write the output impedance as

$$Z_o = v/i_e. \tag{9.47}$$

We can write the output voltage as

$$v = -i_b R_s', \tag{9.48}$$

where R_s' is given by

$$R_s' = R_s + r_b. \tag{9.49}$$

We write the current i_e as

$$i_e \approx -\beta i_b. \tag{9.50}$$

If we divide v by i_e, we get the output impedance

$$Z_o \approx R_s'/\beta. \tag{9.51}$$

In practical amplifiers, this is quite small, as low as an ohm, making the emitter follower a nearly ideal voltage source. Also notice that in the small-signal model, the collector is an AC ground. The emitter follower is classified as a common-collector amplifier for this reason, even though the collector is actually connected to the supply, not to ground. In the NorCal 40A, there are emitter followers in the Audio Amplifier and in the oscillator circuits in the Product Detector and the Transmit Mixer.

9.8 Differential Amplifier

A differential amplifier amplifies the difference between two signals. Differential amplifiers appear in many places in electrical engineering, because they allow interference to be canceled. In addition, thermal drifts can be reduced. The mixers and the Audio Amplifier in the NorCal 40A are based on differential amplifiers. A representative differential amplifier is shown in Figure 9.13. The actual mixer and Audio Amplifier circuits are considerably more complicated than this, and we will wait until the later chapters for a discussion. The amplifier is made up of a pair of identical common-emitter amplifiers with the emitter resistors tied together. The resistor R_t connects the emitter resistors to ground. This design is called a *long-tailed pair*. The "tail" is the common resistor R_t, and "long" means that $R_t \gg R_e$. We will see why this is useful shortly.

Consider what happens if we apply equal and opposite input voltages,

$$v_{i1} = -v_{i2}. \tag{9.52}$$

This sign change is also reflected in the emitter currents. We write

$$i_{e1} = -i_{e2}. \tag{9.53}$$

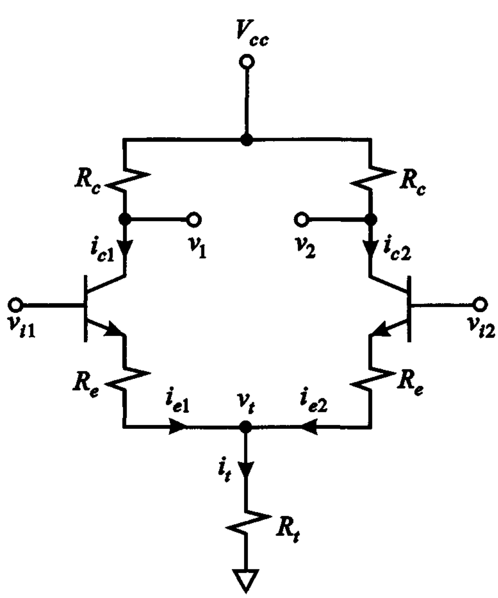


Figure 9.13. The long-tailed pair differential amplifier. The voltages and currents are AC signals. Bias voltages and circuits are not shown.

The tail current i_t is given by

$$i_t = i_{e1} + i_{e2} = 0. \tag{9.54}$$

Thus the tail voltage v_t is zero, and so each amplifier is effectively grounded. We write the collector voltages as

$$v_1 = -(R_c/R_e)v_{i1}, \tag{9.55}$$

$$v_2 = -(R_c/R_e)v_{i2}. \tag{9.56}$$

The output voltage v_d is the difference between the collector voltages. We write

$$v_d = v_1 - v_2 = -(R_c/R_e)(v_{i1} - v_{i2}) = -(R_c/R_e)v_{id}, \tag{9.57}$$

where v_{id} is the differential input voltage given by

$$v_{id} = v_{i1} - v_{i2}. \tag{9.58}$$

The gain for a differential input voltage is the same as the gain of one of the individual amplifiers in the pair. We write the differential gain G_d as

$$G_d = -R_c/R_e. \tag{9.59}$$

In calculating the output resistance, we assume that the internal transistor collector resistance r_c is large and can be neglected. This means that the only path between the two output terminals is the series connection of the two collector resistors. This gives us the differential output impedance

$$Z_d = 2R_c. \tag{9.60}$$

Now consider what happens if we apply the same voltage to each input, so that

$$v_{i1} = v_{i2}. \tag{9.61}$$

We call this a *common-mode* voltage to distinguish it from the differential voltage. By symmetry, the emitter currents are the same, and we write

$$i_{e1} = i_{e2}. \tag{9.62}$$

The tail current i_t is given by

$$i_t = i_{e1} + i_{e2}. \tag{9.63}$$

We can write the input voltages as

$$v_{i1} = R_e i_{e1} + R_t i_t = (R_e + 2R_t)i_{e1}, \tag{9.64}$$

$$v_{i2} = R_e i_{e2} + R_t i_t = (R_e + 2R_t)i_{e2} \tag{9.65}$$

and the output voltages as

$$v_1 = -R_c i_{c1} \approx -R_c i_{e1} = -\frac{R_c v_{i1}}{R_e + 2R_t}, \tag{9.66}$$

$$v_2 = -R_c i_{c2} \approx -R_c i_{e2} = -\frac{R_c v_{i2}}{R_e + 2R_t}. \tag{9.67}$$

The outputs are equal. We call the common-mode input voltage v_{ic} and the common-mode output voltage v_c, and we write

$$v_c = -\frac{R_c v_{ic}}{R_e + 2R_t}. \tag{9.68}$$

The common-mode gain G_c is given by

$$G_c = \frac{v_c}{v_{ic}} = -\frac{R_c}{R_e + 2R_t}. \tag{9.69}$$

In practice, R_t is usually much larger than R_e, which may be small, or even omitted. This means that $G_c \ll G_d$. This is useful because interference often appears on both inputs at the same time, so that it is a common-mode signal.

In the NorCal 40A mixers, we only apply the signal v_i to one input, and the other input is bypassed to ground. In this case, we consider that we have both a differential input voltage v_{id}, given by

$$v_{id} = v_i, \tag{9.70}$$

which is the difference between the two inputs, and a common-mode voltage v_{ic}, which is the average of the two inputs:

$$v_{ic} = \frac{v_i + 0}{2} = \frac{v_i}{2}. \tag{9.71}$$

We can then find the differential output voltage as

$$v_d = G_d v_{id} = G_d v_i \tag{9.72}$$

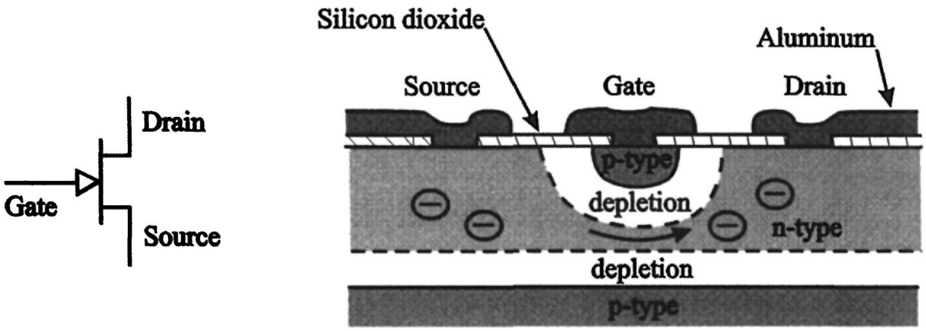
and the common-mode output voltage as

$$v_c = G_c v_{ic} = G_c v_i/2. \tag{9.73}$$

This is just the average of the two output voltages and would be much smaller than the differential output.

9.9 Field-Effect Transistors

In addition to bipolar transistors, there is another major device family called field-effect transistors. We found that bipolar transistors are controlled by an input current. In contrast, field-effect transistors are controlled by an input voltage. There are many types of field-effect transistors. The major use is in computer circuits, but they are important in other areas as well. The field-effect transistor in our transceiver is called a junction field-effect transistor, or JFET for short. JFETs come in two varieties: n-channel, where the charge carriers are electrons, and p-channel, where the carriers are holes. Generally speaking, electrons move faster than holes, and for this reason, n-channel JFETs are more common than p-channel JFETs. In



(a) (b)

Figure 9.14. Schematic symbol for an n-channel JFET (a), and a cross section (b).

the NorCal 40A, there are three JFET circuits: the Buffer Amplifier in Problem 23, the VFO in Problem 26, and the Automatic Gain Control in Problem 32.

The circuit symbol and construction of an n-channel JFET is shown in Figure 9.14. The silicon wafer itself is p-type. The top part of the wafer is converted to n-type by putting it in a furnace with n-type impurities. This forms a diode that is called the body diode. Next a pn diode is made in the n-layer. The connection to this pn diode is called the *gate*. The gate is the input terminal for the device. In addition, two metal contacts are added on each side of the gate. These are called the *source* and *drain*. In a circuit, the drain is usually given a positive bias and electrons flow from the source to the drain. The gate, source, and drain correspond to the base, emitter, and collector in a bipolar transistor, and most people probably wish that the same names had been kept for both.

The voltage on the gate controls the current. A JFET is operated with the gate in reverse bias, so that the gate diode does not conduct. This gives JFET amplifiers a very high input impedance. Associated with the gate diode and body diode are regions where there are no charge carriers and the silicon effectively becomes an insulator. These insulating regions are called *depletion* regions. A depletion region arises because carriers diffuse across the junction between the p and n regions and recombine, leaving charged impurity atoms that expel charge carriers. The electrons flow between the depletion regions of the gate diode and the body diode. This region is called the *channel*. The thickness of the depletion region depends on the diode voltage. The p-type substrate is typically connected to the source, so that the thickness of its depletion region is fixed. As the gate voltage is made more negative, its depletion width increases. This reduces the channel height and decreases the current. This characteristic is shown in Figure 9.15 for the J309 JFET that is used in the NorCal 40A. If we make the gate voltage sufficiently negative, the channel closes off entirely, and no current flows. This voltage is called the *cut-off voltage* V_c.

A field-effect transistor has two different regions of operation, depending on whether the drain voltage is small or large. These correspond to the saturation region and the active region in a bipolar transistor. The low-voltage region in a

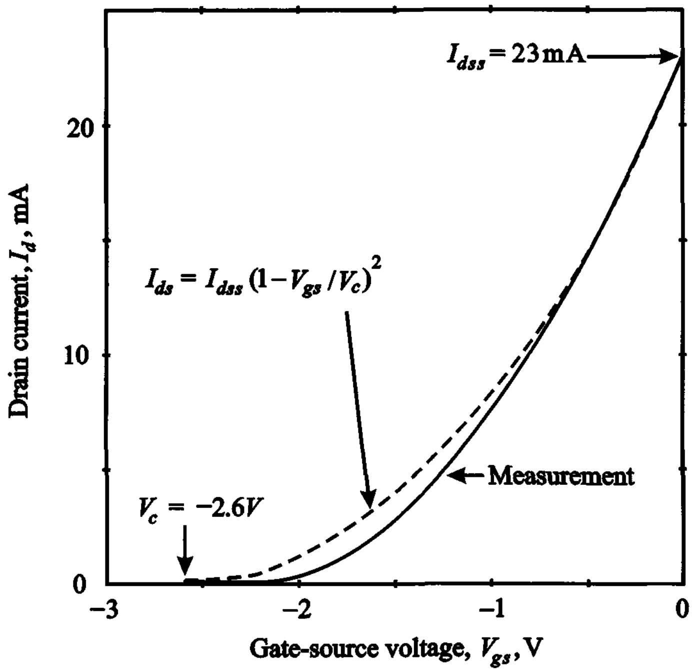


Figure 9.15. Measured drain current I_d as a function of gate-source voltage V_{gs} for the J309 JFET in the NorCal 40A. The drain-source voltage is fixed at 5 V. The dashed line is a plot of Equation 9.74 with $I_{dss} = 23$ mA and $V_c = -2.6$ V.

field-effect transistor is called the linear region, and we will study it in Chapter 13 when we consider the Automatic Gain Control. In this chapter, we consider only the active region where the drain voltage is large. The JFET is relatively difficult to analyze precisely, but we can write the drain current I_d with tolerable accuracy in the active region as

$$I_d = I_{dss}(1 - V_{gs}/V_c)^2, \tag{9.74}$$

where I_{dss} is the current at zero gate-source voltage. The abbreviation *dss* denotes *drain-source* current with the gate *shorted* to the source. This formula is plotted in Figure 9.15 for comparison.

The slope of the drain-current curve dI_d/dV_{gs} is the transconductance g_m. It corresponds to the current gain β in a bipolar transistor, because it tells us how much change in output current we get for a change in the input voltage. We write

$$g_m = \frac{dI_d}{dV_{gs}}. \tag{9.75}$$

Figure 9.16 shows a plot of the transconductance. It is large near zero volts and decreases to zero at the cut-off voltage V_c. This characteristic is used in the VFO start up in Problem 26. The oscillation begins with a zero-voltage bias, where the transconductance is large. This assures that the oscillation starts properly. As the

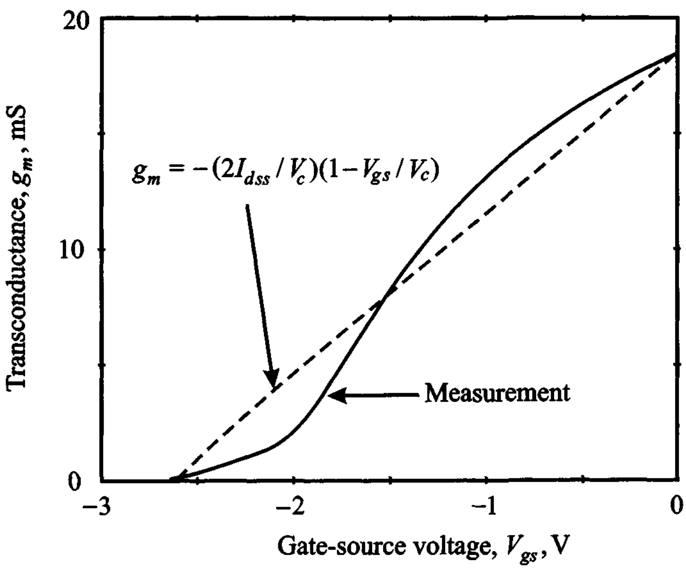


Figure 9.16. Measured transconductance g_m as a function of gate-source voltage V_{gs} for the J309 JFET. The drain-source voltage is fixed at 5 V. The dashed line is a plot of Equation 9.76 with $I_{dss} = 23$ mA and $V_c = -2.6$ V. The data sheets in Appendix D give further information.

oscillation reaches a steady state, the bias shifts to near the cut-off voltage, where g_m is much smaller. This gives a stable operating condition. We can differentiate Equation 9.74 to find an approximate formula for g_m:

$$g_m = -(2I_{dss}/V_c)(1 - V_{gs}/V_c). \tag{9.76}$$

This relation is a straight line, and it is also shown on Figure 9.16.

We can draw a simple small-signal equivalent circuit for the JFET (Figure 9.17a). The gate input draws no current; thus we leave it open-circuited. We represent the

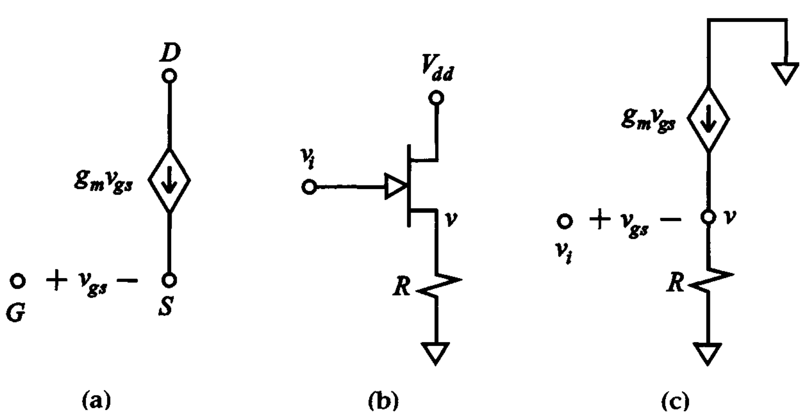


Figure 9.17. Equivalent circuit for a JFET (a), a source-follower circuit with a load R (b), and the equivalent AC circuit for a source follower (c).

control of the drain current by a dependent current source $g_m v_{gs}$. This model is appropriate for the source follower in Problem 23 and the VFO start up in Problem 26. At higher frequencies, capacitances would need to be added to the model. These are given in the data sheets in Appendix D.

9.10 Source Follower

The source-follower circuit (Figure 9.17b) is similar to the bipolar emitter follower. In our transceiver, a source follower is used to isolate the Transmit Mixer from the Driver Amplifier, so that changes in the Driver Amplifier impedance do not affect the Transmit Mixer. One attractive feature of the source-follower circuit is that the load resistor R determines the bias voltage and current – no additional bias components are needed. Let us see how this works. We assume that the gate has a DC connection to ground, so that its DC voltage is zero. For example, in the Buffer Amplifier there is an inductor (L6) that makes this connection. We can write the source voltage as $I_b R$, where I_b is the bias current. This gives us a relation between gate-source bias voltage V_b and the current. We can write

$$V_b = -I_b R. \tag{9.77}$$

This is the equation of a straight line through the origin. It is called a *load line.* In addition, the bias voltage and current must satisfy the relationship given by Figure 9.15. We can find the solution graphically by drawing the load line on this plot and finding the intersection. This is shown in Figure 9.18.

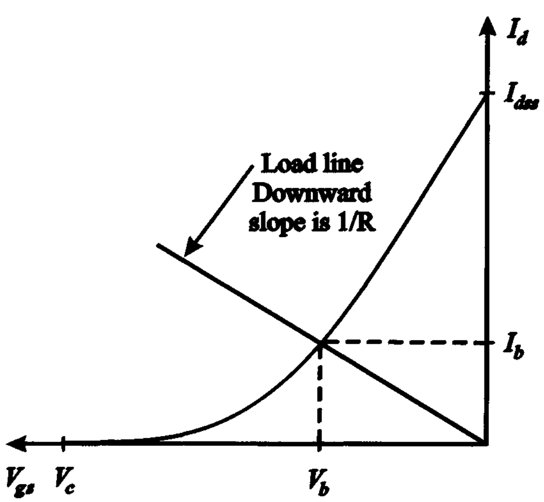


Figure 9.18. Finding the bias voltage V_b and current I_b for the JFET source follower. The bias point is at the intersection of the load line and the drain-current plot.

In the equivalent AC circuit, we replace the drain supply with a short circuit to ground, because the supply does not affect an AC current (Figure 9.17c). From this circuit, we can write an expression for the output voltage v in terms of the gate-source voltage v_{gs}:

$$v = R g_m v_{gs}. \tag{9.78}$$

The gate-source voltage is written as the difference between the input voltage v_i and the output voltage v:

$$v_{gs} = v_i - v. \tag{9.79}$$

Between these two equations, we can eliminate v_{gs}, and write

$$v = \frac{R v_i}{1/g_m + R}. \tag{9.80}$$

This is the same formula we get for a potential-divider circuit with source resistance $1/g_m$ and load resistance R. This means that the output impedance of the follower is

$$Z_o = 1/g_m. \tag{9.81}$$

We can write the voltage gain as

$$G_v = \frac{v}{v_i} = \frac{1}{1 + \frac{1}{g_m R}}. \tag{9.82}$$

The gain is near 1 if $R \gg 1/g_m$.

FURTHER READING

A good introductory textbook for amplifiers is *Microelectronic Circuits and Devices*, by Mark Horenstein, published by Prentice-Hall. The classic advanced textbook is *Analysis and Design of Analog Integrated Circuits*, by Paul Gray and Robert Meyer, published by Wiley. This book emphasizes integrated-circuit amplifiers. *Device Electronics for Integrated Circuits*, by Muller and Kamins, published by Wiley, gives a derivation of the current characteristics for a JFET. *Radio Frequency Design*, by Wes Hayward, published by the American Radio Relay League, has a detailed discussion of JFET amplifiers.

PROBLEM 21 - DRIVER AMPLIFIER

Figure 9.19 shows the Driver Amplifier and the measurement connections. The transformer T1 and the resistor R14 were installed in Problem 15. You should solder in the transistor Q6 and the emitter resistor R12, leaving room on each to attach probes. We use R12 to measure the bias current. Measure the resistance of R12, and make a note of it for later.

Solder in R13. R13 is a variable resistor, or potentiometer ("pot" for short). It is wired so that the resistance can be adjusted from near zero to 500 Ω. R13 controls the gain of

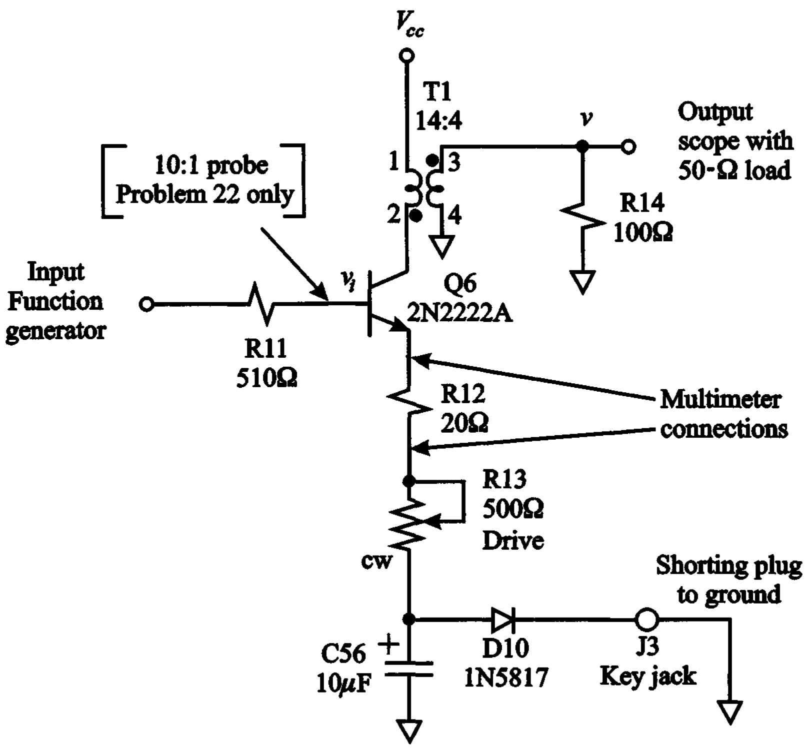


Figure 9.19. Driver Amplifier for the NorCal 40A and the measurement connections.

the amplifier. You should adjust the pot so that it is fully clockwise, leaving it at its lowest resistance. Install C56 and D10, and insert a shorting plug into the Key jack J3. Install the end of R11 that is connected to the base of Q6, and leave the other end free for attaching the function generator. Plug in the power supply and hook up the function generator at the free end of R11 as shown in Figure 9.19. Attach the oscilloscope to R14 with a 50-Ω termination.

The next job is to adjust the amplitude and offset of the function generator. You should work for a large sine-wave output at 7 MHz with high efficiency. A good starting point is an amplitude of 2 V and an offset of 0.5 V. If the offset is too small, the output will clip, because the transistor will turn off during part of the cycle (Figure 9.20a). However, be careful when you increase the offset, because the current grows quickly, reducing the efficiency. It is a good idea not to let the emitter current exceed 50 mA to avoid burning out the transistor. You may feel it getting hot. If the amplitude is too high, the waveform will bottom out when the transistor saturates (Figure 9.20b).

A. Once the amplitude and offset are adjusted, measure the output voltage and calculate the output power *P*. The load resistance is the parallel combination of R14 and the scope termination.

B. Now we calculate the power delivered by the power supply. Record the DC voltage across R12, and calculate the emitter current. Now use the multimeter to measure

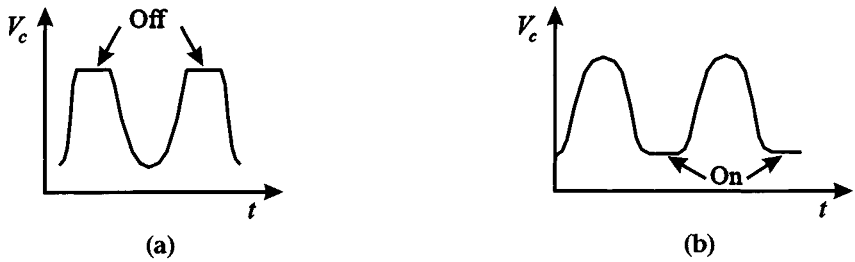


Figure 9.20. Output waveforms for setting the offset and amplitude of the function generator. In (a), the offset is too small, and in (b), the amplitude is too large.

V_{CC}. It is best to measure this voltage at the end of the 1-Ω resistor across S1 that connects to T1. Calculate the supply power P_o and the efficiency η.

C. Next we find the gain. Calculate the available power P_+ from the function generator, using the amplitude setting on the function generator for V_+. Take the source resistance to be the sum of R11 and the function-generator resistance. Calculate the gain G in dB.

PROBLEM 22 - EMITTER DEGENERATION

This is a continuation of the previous problem. This time, we will concentrate on the voltage gain and the input impedance. In the Driver Amplifier, there are two emitter resistors in series, R12 and R13. R12 is a fixed 20-Ω resistor, and R13 is a variable resistor that can be adjusted anywhere from a very low resistance to a resistance of about 500 Ω. This allows the output power to be controlled. The fixed resistance R12 sets the maximum voltage gain.

Make the connections for the previous problem, and add a 10:1 scope probe at the base end of R11. This allows us to measure the AC input voltage v_i. You should use a 7-MHz sine wave with the same offset as before, but use a smaller amplitude of 1 Vpp to reduce the distortion in the voltage waveforms.

A. Measure the voltage gain $G_v = v/v_i$ with R13 set fully clockwise (maximum gain) and fully counterclockwise (minimum gain).

B. Calculate what you expect the voltage gain to be for each setting. You need to take the turns ratio of the transformer into account. Use the multimeter to measure the total emitter resistance for R12 and R13 for each setting. Turn off the function generator and disconnect the power supply when you make resistance measurements. Otherwise there will be errors in the readings.

C. Measure the magnitude of v_i at the maximum gain setting. We can use this to deduce the Miller capacitance, with the equivalent circuit in Figure 9.21 as a guide. In the circuit, we have a 560-Ω source resistance and an open-circuit voltage $v_o = 2$ V, twice the amplitude setting. We will assume that the input impedance of the amplifier is dominated by the Miller capacitance and ignore the rest. Use the equivalent circuit to find an expression for the magnitude of v_i in terms of the Miller capacitance C_m.

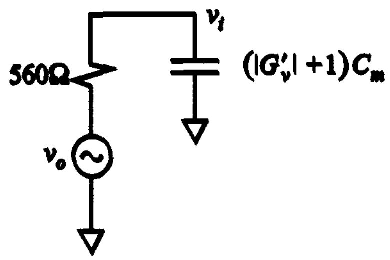


Figure 9.21. Equivalent circuit for extracting the Miller capacitance.

You will need to consider the effect of the transformer carefully. The voltage gain G_v' that is needed is the ratio of the effective collector resistance to the emitter resistance, not the gain you measured at the load. Solve for C_m. When you are finished, solder in the other end of R11, and remove the shorting plug from J3.

PROBLEM 23 – BUFFER AMPLIFIER

The Driver Amplifier has an input impedance that is dominated by Miller capacitance. The effective input capacitance varies over a large range, from 10 pF at low gain to 105 pF at high gain. In the transceiver, the input for the Driver Amplifier comes from the Transmit Mixer, after passing through the Transmit Filter. A large change in load capacitance would upset the filter resonance if we put the filter output into the Driver Amplifier directly. In order to isolate the filter from changes in the driver amplifier, the transceiver has a JFET Buffer Amplifier (Figure 9.22). The Buffer Amplifier is a source-follower circuit, which has a high input impedance but no voltage gain.

To start, solder in the JFET Q5, C36, and R10 for the amplifier. Leave R10 a few millimeters off the board, so that you can get a scope probe on it. C36 is a *bypass* capacitor. It ensures that the AC impedance that the drain sees is low, less than an ohm, to prevent oscillations. The resistor R10 has the same purpose. It is dangerous to put a resonant circuit at the input of a transistor unless there is a resistance to damp out oscillations.

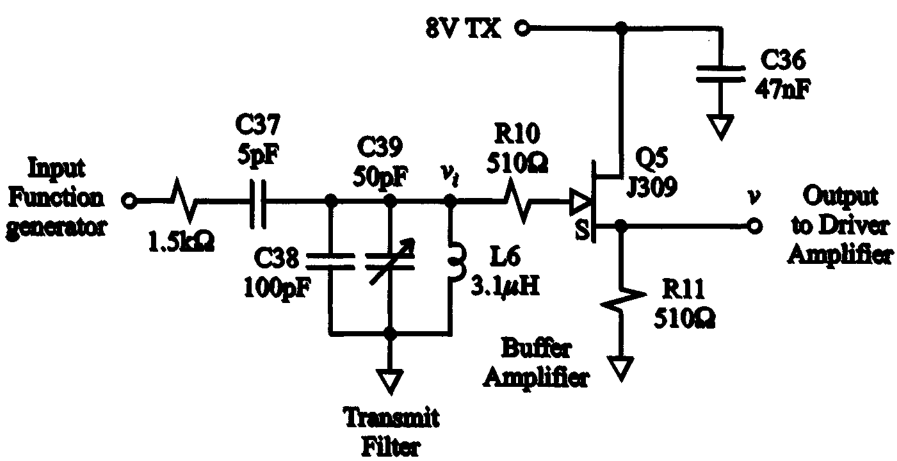


Figure 9.22. Buffer Amplifier for isolating the Driver Amplifier from the Transmit Filter.

In addition, you should solder one end of a 1.5-kΩ resistor into the hole #4 of U4. This provides a connection to C37 for the function generator. We use the 1.5-kΩ resistor to play the role of the Transmit Mixer. The Buffer Amplifier requires 8V TX to operate, and we must put a shorting plug into the Key jack J3 for this.

A. The source resistor R11 determines the bias of both the JFET and the Driver Amplifier. Initially set the gain of the Driver Amplifier to its minimum value, with the R13 pot set fully counterclockwise. This gives the Driver Amplifier a high impedance. Connect the power supply and turn it on. Measure the DC voltage of the source of the JFET with the multimeter. It is convenient to do this measurement by hooking the multimeter lead onto R11. Calculate the drain bias current.

B. For comparison, use Figure 9.15 to calculate the source voltage and drain current that you should expect for a source resistor of 510 Ω.

C. Next we measure the voltage gain of the Buffer Amplifier. This is tricky, because the scope probe affects the resonance of the filter, and the filter must be tuned each time the probe moves. The gain of the Driver Amplifier should be left at its minimum value. Connect the function generator to the circuit through the 1.5-kΩ resistor. Initially the 10:1 scope probe should be attached to the filter end of R10 (Figure 9.22) to measure the input voltage v_i. Set the frequency to 7 MHz and the amplitude to 1 Vpp. The probe capacitance affects the filter resonance, and you should adjust C39 to give a maximum probe voltage. The adjustment is coarse, and the capacitance and pressure of the screwdriver affect the capacitance. You can find the maximum voltage by varying the frequency a few kilohertz. Record the maximum input voltage. Now move the probe to R11 to measure the output voltage v. You will need to adjust C39 again. Find the voltage gain G_v, given by

$$G_v = v/v_i. \tag{9.83}$$

Use this value of G_v to deduce the transconductance g_m.

D. For comparison, use Figure 9.16 to find the transconductance you would expect for the bias voltage you measured.

E. In measuring the input voltage, you attached the 10:1 probe at the filter end of R10, rather than at the gate end. To see why, measure the ratio of these two voltages. Make sure that you retune C39 for these measurements. Now calculate the ratio that you expect, assuming that the impedance of the Buffer Amplifier is very high.

F. Calculate the available power P_+ from the function generator through the 1.5-kΩ resistor and the power P delivered to the 510-Ω load. Make sure that C39 is tuned. Calculate the power gain G in dB.