6

Transformers

Previously we have made inductors by wrapping a coil on a toroidal core. We make a *transformer* by adding another coil. Transformers are valuable in radio circuits because they can do several different jobs at once. A transformer blocks DC voltages but transmits AC. It can step up a voltage or current and change the impedance levels of loads to eliminate reflections. Finally, a coil on a transformer has an inductance that can be used in a resonant circuit as a filter.

6.1 Inductance Formulas

To start, let us see how an inductor works. Consider a toroidal core with a single loop of wire (Figure 6.1). We let the current in the wire be I and the voltage across the ends be V. In a magnetic material like iron or ferrite, the current produces a large magnetic flux Ψ (the Greek capital letter *psi*) around the toroid. The units of flux are volt·seconds (Vs). We will assume that the flux is proportional to the current. This is reasonable as long as the current is not too large and the toroid is not permanently magnetized. We write

$$\Psi = A_l I, \tag{6.1}$$

where A_l is called the *inductance constant*. A_l is the inductance of a single turn, but we can also use it to calculate the inductance of larger coils. In addition, Faraday's law says that the voltage V is the time derivative of the flux,

$$V = \frac{d\Psi}{dt}. \tag{6.2}$$

This formula takes getting used to, because it says that in a loop of wire, where we would ordinarily expect no voltage between the ends because the wire is continuous, there will be a voltage if the magnetic field through the loop changes with time. We can rewrite this formula in terms of phasors:

$$V = j\omega\Psi. \tag{6.3}$$

We can combine the two formulas to give the relation

$$V = j\omega A_l I. \tag{6.4}$$

This is the relation between voltage and current for an inductor.

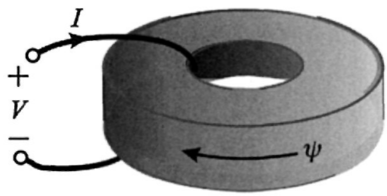


Figure 6.1. A toroidal core with one turn.

Consider what happens if we add more loops. Each additional loop will act like the first in producing flux. We can write the total flux as

$$\Psi = NA_l I. \tag{6.5}$$

Moreover, the flux produces a voltage in each loop. Because the loops are connected in series, the total voltage is proportional to the number of loops, and we can write

$$V = Nj\omega\Psi. \tag{6.6}$$

We combine these two formulas and get

$$V = j\omega N^2 A_l I. \tag{6.7}$$

The inductance is given by

$$L = N^2 A_l. \tag{6.8}$$

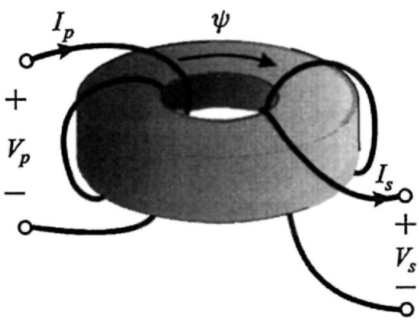
This is the formula that we have been using. The inductance is proportional to the square of the number of turns. The inductance constant A_l is given in manufacturer's data sheets for different core sizes and materials. Larger cores have larger inductance constants. However, it is important to realize that A_l can change drastically with frequency. It is also somewhat dependent on the number of turns and how they are spread. Data for the cores in the NorCal 40A are given in Appendix D. There are two iron-powder cores and two nickel–zinc ferrite cores. The iron-powder cores have lower inductance constants that are suitable for inductors. The nickel–zinc ferrites have higher inductance constants that are better for transformers.

6.2 Transformers

Now consider a toroid with two coils (Figure 6.2a). This is called a *transformer*. The coils are called *primary* and *secondary* to distinguish them. It is not always clear which is which, although usually you can think of the primary coil as the input coil and the secondary as the output. We denote the primary coil by the subscript p and the secondary by the subscript s. A voltage will be induced in each coil by the changing flux. We can calculate the voltages using Faraday's law again. We write

$$V_p = N_p j\omega\Psi, \tag{6.9}$$

$$V_s = N_s j\omega\Psi, \tag{6.10}$$



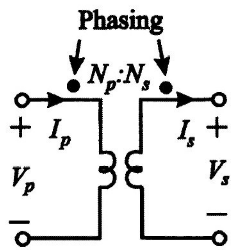


Figure 6.2. Adding a second coil to make a transformer (a), and schematic symbol (b).

where N_p is the number of turns in the primary and N_s the number of turns in the secondary. We can write V_s in terms of V_p if we take the ratio of these two formulas:

$$V_s = \frac{N_s}{N_p} \cdot V_p. \tag{6.11}$$

This says that the voltage ratio is the same as the turns ratio. The schematic symbol for a transformer is a pair of facing coils (Figure 6.2b). There is a choice in how the coils are wound – clockwise or counterclockwise. This determines the sign of the secondary voltage. If the sign is important, people add black phasing dots at the positive voltage terminals.

Calculating currents is more complicated than calculating voltages. We have to be careful about the directions. The usual convention is that the primary current I_p is positive if it is *entering* the transformer, and the secondary current I_s is positive if it is *leaving*. The total flux is the sum of the fluxes produced by each coil. We can write this as

$$\Psi = N_p A_l I_p - N_s A_l I_s. \tag{6.12}$$

There is a minus sign because the directions of I_p and I_s are different. We can rewrite this relation as

$$I_p = \frac{\Psi}{N_p A_l} + \frac{N_s}{N_p} I_s. \tag{6.13}$$

We rewrite the flux term with Equation 6.9 to give

$$I_p = \frac{V_p}{j\omega L_p} + \frac{N_s}{N_p} \cdot I_s, \tag{6.14}$$

where L_p is the inductance of the primary coil. The term $V_p/(j\omega L_p)$ is the current that the primary coil would draw if there were no secondary. It is called the *magnetizing current*. The term $(N_s/N_p)I_s$ is the *transformer current*, and it is controlled by the turns ratio. We now consider the limit where the magnetizing current is small.

6.3 Ideal Transformers

Assume that the primary inductance L_p is quite large so that we can neglect the magnetizing current. We can rewrite the voltage and current equations (Equations 6.11 and 6.14) as

$$V_s = \frac{N_s}{N_p} \cdot V_p \tag{6.15}$$

and

$$I_s = \frac{N_p}{N_s} \cdot I_p. \tag{6.16}$$

These are the voltage and current relations for an *ideal transformer*. These equations mean that we can step up the voltage or current, depending on the turns ratio. If $N_s > N_p$, the transformer will step up the voltage. If $N_s < N_p$, the transformer will step up the current. You might think about why we cannot step up both the voltage and current at the same time. If we take the product of the two equations, we get

$$V_s I_s = V_p I_p, \tag{6.17}$$

which says that the power that comes out of the secondary coil is the same as the power we put in the primary coil. If both voltage and current increased, the output power would be bigger than the input power, which is not possible, because the transformer has no additional source of power. In the NorCal 40A, we use the transformer T1 to step up the current from the Driver Amplifier to the Power Amplifier. The two coils have 14 turns and 4 turns, and so the current increases by a factor of 3.5.

Since the voltage steps up when the current goes down, and vice versa, a transformer changes the impedance of a load. To see this, consider Figure 6.3, which shows a transformer together with a load. If we take the ratio of Equation 6.15 and Equation 6.16, we get

$$\frac{V_s}{I_s} = \left(\frac{N_s}{N_p}\right)^2 \cdot \frac{V_p}{I_p}. \tag{6.18}$$

We can rewrite this in terms of the load impedance $Z_s = V_s/I_s$ and the primary impedance $Z_p = V_p/I_p$ as

$$Z_p = \left(\frac{N_p}{N_s}\right)^2 Z_s. \tag{6.19}$$

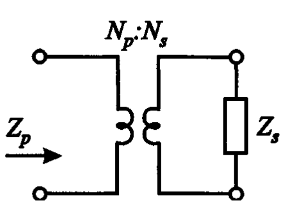


Figure 6.3. Ideal transformer with a load.

In words, the impedance changes by the square of the turns ratio. Transformers are commonly used in matching for maximum power transfer. For example, in the NorCal 40A, the RF Mixer, which has a source resistance of 3 kΩ, is the source for the IF Filter, which has an impedance of 200 Ω. This means that there will be a large mismatch loss without a transformer. We use the transformer T3 with a primary of 23 turns and a secondary of 6 turns. This gives a primary impedance Z_p given by

$$Z_p = \left(\frac{23}{6}\right)^2 \cdot 200\,\Omega = 2.9\,\mathrm{k}\Omega, \tag{6.20}$$

which is quite close to 3 kΩ.

6.4 Magnetizing Current

Now we consider the effect of the magnetizing current. Rewriting the voltage and current equations (Equations 6.11 and 6.14) with the magnetizing current included, we get

$$V_p = \frac{N_p}{N_s} \cdot V_s, \tag{6.21}$$

$$I_p = \frac{V_p}{j\omega L_p} + \frac{N_s}{N_p} \cdot I_s. \tag{6.22}$$

Figure 6.4a shows an equivalent circuit, which includes an inductor L_p in parallel with an ideal N_p:N_s transformer. It is important to think through why this equivalent circuit works. In Equation 6.22, the primary current I_p is made up of two parts, the magnetizing current and the transformer current. The magnetizing current is the current for an inductor L_p with voltage V_p. We can calculate L_p from the inductance constant A_l and the number of turns N_p by the usual formula:

$$L_p = N_p^2 A_l. \tag{6.23}$$

The transformer current is that for an ideal N_p:N_s transformer with a primary voltage V_p. These relations, where the voltages of the two components are the same, but the current is the sum of the currents for each component, are those

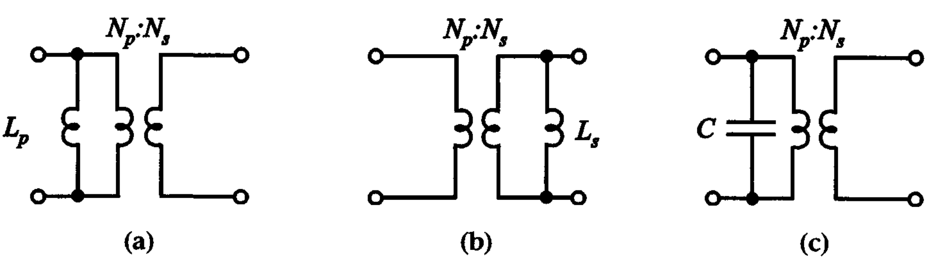


Figure 6.4. The equivalent circuit for a transformer, including the magnetizing current (a). Alternative equivalent circuit for a transformer with a shunt inductor L_s on the secondary side (b). A tuned transformer (c).

for components in parallel. I have put the magnetizing current on the primary side, but we could also have started from the secondary side, and ended up with a magnetizing current on the secondary side. The easy way to do this is to pretend that we are using the secondary as the input, and swap the subscripts. In the equivalent circuit (Figure 6.4b), we use the secondary inductance L_s, given by

$$L_s = N_s^2 A_l, \tag{6.24}$$

and put it on the secondary side.

Including the magnetizing current in the equivalent circuit raises several issues. First, at low frequencies, the shunt inductor will short out the ideal transformer. This means that the transformer acts as a high-pass filter and that it will isolate DC circuits. Second, we can make a band-pass filter by adding a capacitor to resonate the shunt inductance. A transformer with a tuning capacitor is called a *tuned transformer*. For example, the NorCal 40A uses a shunt capacitor (C6) with the matching transformer T3 to resonate out the transformer inductance. A tuned transformer acts as a band-pass filter for removing interference at the same time that it changes impedance levels for matching. In other situations, we may not need the band-pass filter, but it may be difficult to get a large enough inductance to achieve the full current or voltage step up. The tuning capacitor allows us to resonate the inductance at the frequency we are interested in, so that the ideal transformer equations apply.

FURTHER READING

A good source of practical information on inductors and transformers is *Secrets of RF Circuit Design*, by Joseph Carr, published by McGraw-Hill. Carr also discusses the iron and ferrite materials used for the cores.

PROBLEM 15 - DRIVER TRANSFORMER

In the NorCal 40A, the transformer T1 couples the Driver Amplifier to the final Power Amplifier. The Power Amplifier requires a substantial drive current, and we use the transformer to provide a current step-up. In addition, the transformer isolates the DC voltages and currents in the Driver Amplifier from the Power Amplifier. This makes it possible to connect the output collector of the Driver Amplifier to the supply and the input base of the Power Amplifier to ground at the same time.

The core we use for this transformer is an FT37–43. It has an orange dot to identify the core material as #43 ferrite. The inductance constant for the #43 mix varies greatly with frequency, dropping by almost a factor of three as the frequency increases from 100 kHz to 7 MHz.

Carefully construct the transformer T1 as shown in Figure 6.5. Start by cutting 10-cm and 25-cm lengths of #26 wire for the coils. It is better to wrap the long coil first and to check the turns before you start the short coil. Count carefully as you wrap the second coil, because it is not easy to check the count when there are two coils. Transformers cause more problems in construction than any other component. Make sure that each

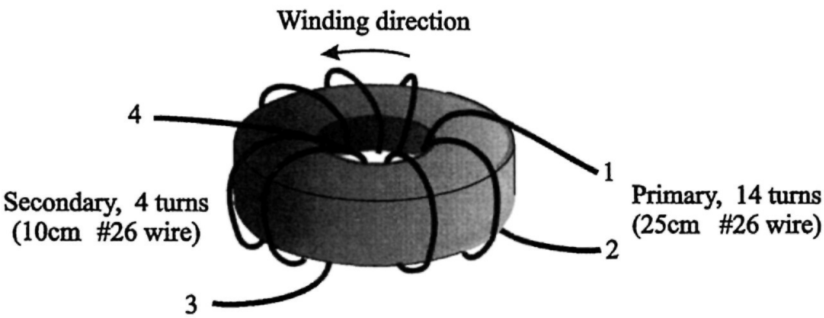


Figure 6.5. Wiring for T1. Not all turns are shown. The numbers match holes in the printed-circuit board.

lead goes to the correct hole and that the wires are properly stripped before you solder them in. One way to check that the insulation is off the wires is to coat the wires with a thin layer of solder before you insert the transformer into the circuit. If the insulation is still there, the solder will not coat the wire smoothly but will form a bead. The transformer should lie flat after it is soldered in.

Install R14 (100 Ω). Leave the resistor a few millimeters above the board surface so that you can attach leads from the oscilloscope. Make oscilloscope connections to R14 with a 50-Ω termination. Solder one end of a 1 k-Ω resistor to the large middle hole in the S1 outline and one end of a 200-Ω resistor into the Q6 hole that is closest to the transformer (Figure 6.6). The other ends of these resistors are the input leads for the function generator. The function generator should be set for a 5-Vpp, 7-MHz sine wave. Figure 6.7 shows the circuit.

A. Measure the output voltage *V*.

B. Now calculate *V* from the circuit diagram.

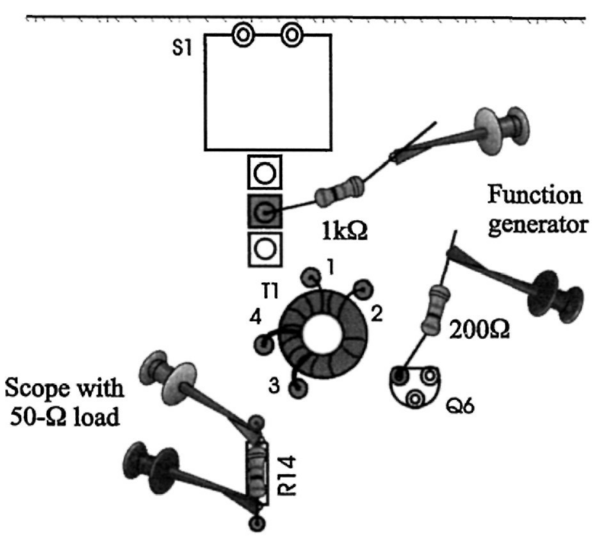


Figure 6.6. Connections for measurements on T1.

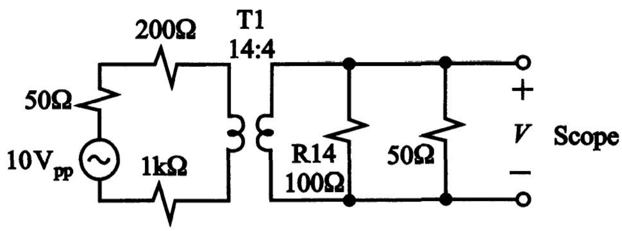


Figure 6.7. Circuit for measurements on T1.

C. Measure the 3-dB low-frequency cut-off f_C.

D. Use your measurement of f_C to deduce the inductance constant A_l, taking all the resistances into account. Note that it may be considerably greater than the 7-MHz value of 160 nH/turn^2 given in Appendix D. After you finish, remove the 1-kΩ and 200-Ω resistors and clean the holes with solder wick.

PROBLEM 16 – TUNED TRANSFORMERS

The NorCal 40A has two tuned transformers, T2 and T3, to match impedances at the input and output of the RF Mixer. Study the endpaper to see how these transformers fit into the circuit. Both transformers use the ferrite core FT37–61, with $A_l = 66$ nH/turn^2. These cores are not painted. T2 combines with the series resonant circuit we studied in Problem 8 to make a 2-element Butterworth band-pass filter at 7 MHz. This is the RF Filter. In addition, the transformer steps up the 50-Ω cable impedance to 1.5 kΩ to match the input of the RF Mixer. T3 is at the output of the RF Mixer. It steps down the 3-kΩ output resistance of the RF Mixer to match the 200-Ω input impedance of the IF Filter at 4.9 MHz.

For T2, start with a 35-cm section of #26 wire and wrap 20 turns (Figure 6.8). For the primary, it is convenient to use a single loop of bare #22 wire. Install T2, the variable capacitor C2, and C4 (5 pF). We also need to connect a temporary 1.5-kΩ resistor to act as a load in place of the RF Mixer U1. The resistor should be soldered to the #1 and #3 holes in U1 (Figure 6.9). These holes are numbered starting at the round solder pad in the lower corner, and proceeding counterclockwise. Attach a 10:1 scope probe across the resistor. Note that the #3 hole is the ground. For an input connection, solder a short piece

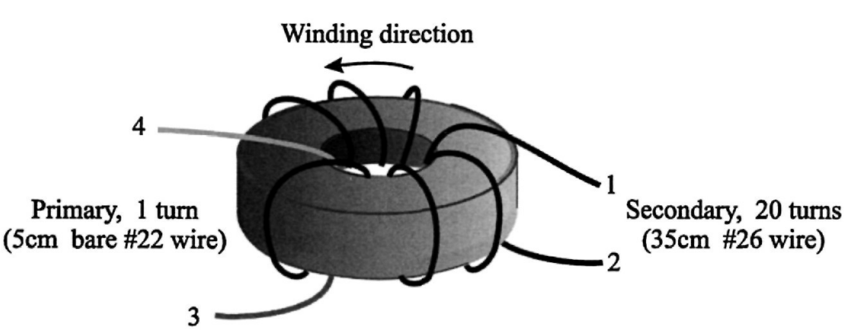


Figure 6.8. Wiring for T2. Not all turns are shown. The numbers match holes in the printed-circuit board.

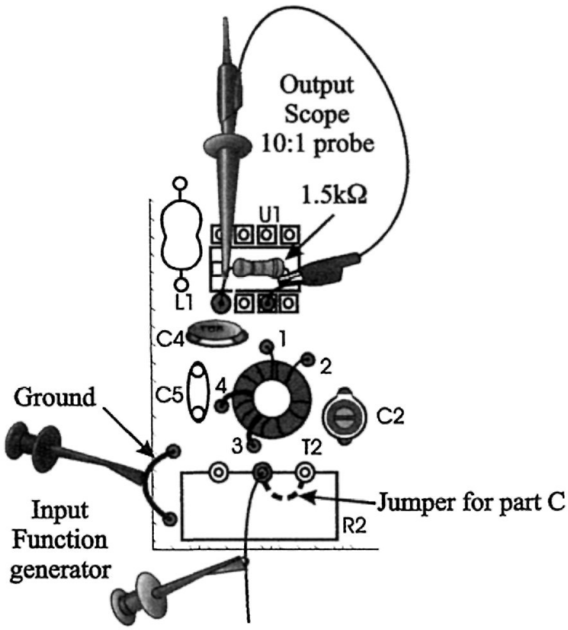


Figure 6.9. Connections for measurements on T2.

of bare #22 wire to the center hole of R2 to attach a lead from the function generator. For a ground connection, solder a loop of bare #22 wire to the two small holes on the edge of the board next to the R2 outline.

A. Set the function generator for a 0.5-Vpp, 7-MHz sine wave. Adjust C2 for maximum output. Find the ratio of the power absorbed by the load P to the available power P_+, and express the loss in dB. Measure the 3-dB bandwidth.

Now we will make a model for the transformer to use in *Puff* (Figure 6.10). The model includes a shunt inductor and an ideal 20:1 transformer. In *Puff*, the ideal transformer would be listed in the Parts Window as `x 20`, where `x` indicates a transformer and `20` is the turns ratio expressed as a decimal number. It is important to draw the transformer in the correct direction. It is in the shape of a trapezoid. For a turns ratio greater than one, the wide side is the high-voltage side.

Make sure you include C4, C2, and the capacitance of the 10:1 probe C_p. The output is tricky, because *Puff*'s ports have an impedance equal to `zd`, which is 50 Ω in this

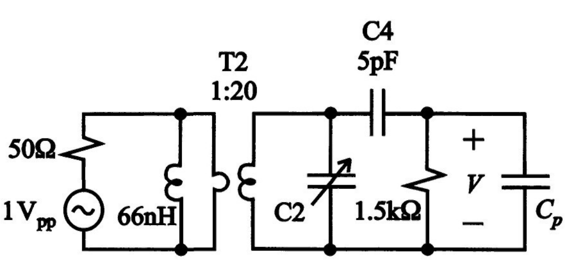


Figure 6.10. Circuit model for T2.

problem. However, if you include an ideal transformer part `x 5.5`, where 5.5 is the square root of the impedance ratio 1500/50, you can connect it between the filter output and Port 2 to give an impedance of 1.5 kΩ.

B. In the computer model, adjust C2 to give the maximum value of $|s_{21}|$ at 7 MHz. Compute the 3-dB bandwidth from the computer model. Make a screen dump of $|s_{21}|$.

C. Return to your circuit board. Join the tuned transformer to the series resonant circuit, L1 and C1. You can do this by connecting your input wire as a jumper between the center and right holes in R2 (Figure 6.9). The function generator should be connected through the Antenna jack J1. Adjust C1 and C2 to give a maximum output voltage V at 7 MHz. What is the combined loss of the Harmonic Filter and the RF Filter in dB? You should make a note of this loss for the future. We will need it to analyze the receiver performance. If the loss is greater than 7 dB, something is likely to be wrong. You might try tuning C1 and C2 again carefully. If this does not work, you might check the solder joints, and make sure that the coil leads are in the correct holes.

D. A major purpose of the RF Filter is to remove the VFO image. Without the RF Filter, this frequency would be received just as well as the desired signal at 7 MHz. The image frequency f_{vi} is given by Equation 1.38 as

$$f_{vi} = f_{if} - f_{vfo} = 4.9\,\text{MHz} - 2.1\,\text{MHz} = 2.8\,\text{MHz}. \qquad (6.25)$$

Find the image rejection ratio R_i in dB, using the formula

$$R_i = 20\log(V_{rf}/V_{vi})\,\text{dB}. \qquad (6.26)$$

The image response will be small, and you should increase the function generator setting to 10 Vpp for this measurement. In addition, you will need to switch in the oscilloscope's low-pass filter. You might notice that at the image frequency, the output may not be a pure sine wave because of harmonic content. Your filter rejects the image at 2.8 MHz much better than the harmonics at 5.6 MHz and 8.4 MHz. These components are usually present at a low level in the output of a function generator, but they are made more prominent by the filter.

Remove the temporary 1.5-kΩ load resistor and the jumper in R2 and clean the holes with solder wick. Turn off the low-pass filter on the scope so that it will not throw off later measurements.

E. Extend your *Puff* model to include the series resonant circuit L1 and C1. What does the model predict for R_i?

The NorCal 40A has one more transformer, T3, that connects the RF Mixer to the IF Filter. Like T2, this core is an FT37–61. The primary has 23 turns of #28 wire and the secondary has 6 turns of #26 wire (Figure 6.11). You should start by cutting a 40-cm section of #28 wire and a 15-cm section of #26 wire for the coils. Construct and install T3.

F. At the IF frequency, 4.9 MHz, what capacitance would be needed to tune the transformer on the primary side? on the secondary side?

Install the 47-pF tuning capacitor C6. Do not be concerned if C6 is different from what you calculated. The designer said he chose 47 pF because the radio sounds best with that value!

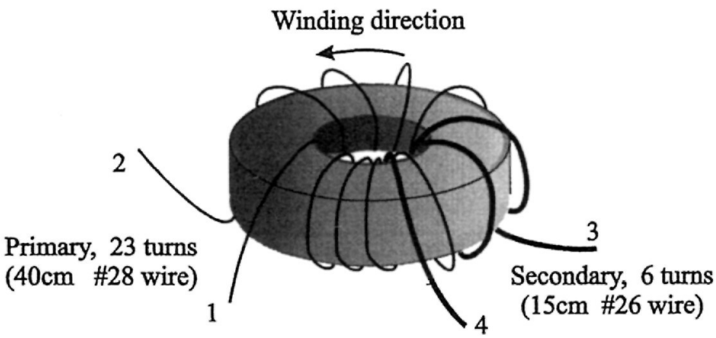


Figure 6.11. Wiring for transformer T3. Not all turns are shown. The numbers match holes in the printed-circuit board.

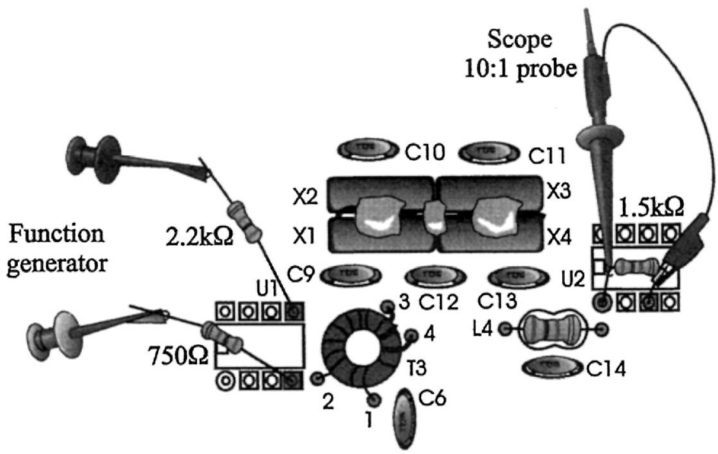


Figure 6.12. Input and output connections for measuring the loss of the complete IF-Filter network.

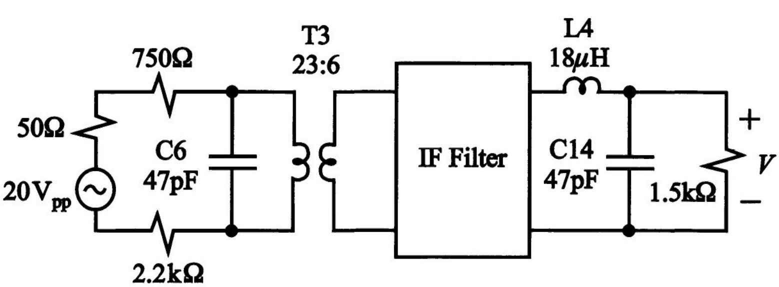


Figure 6.13. Circuit for measuring the loss of the complete IF-Filter network.

For the input connections, solder a 750-Ω resistor in the #4 hole in U1 and a 2.2-kΩ resistor in the #5 hole (Figure 6.12). Connect the function generator to the resistors. The combination of these resistors and the 50-Ω generator resistance gives us 3 kΩ to match the RF Mixer (Figure 6.13). The output connections will be like those for the RF Filter. Solder a 1.5-kΩ resistor between the #1 and #3 holes in U2. Attach the scope across the resistor with a 10:1 probe.

G. Use a 10-Vpp setting on the function generator, and adjust the frequency for maximum scope voltage. Calculate the loss as

$$L = 10\log(P_+/P)\,\mathrm{dB}, \tag{6.27}$$

where P_+ is the power available from the 3-kΩ source and P is the power delivered to the 1.5-kΩ load. Save this number for analyzing the receiver performance later. If the loss is greater than 10 dB, something is likely to be wrong. You might check the solder joints, and make sure that the coil leads are in the correct holes. Remove the resistors and clean the holes with solder wick when you are finished.