Lexicase Selection

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Background and Motivation

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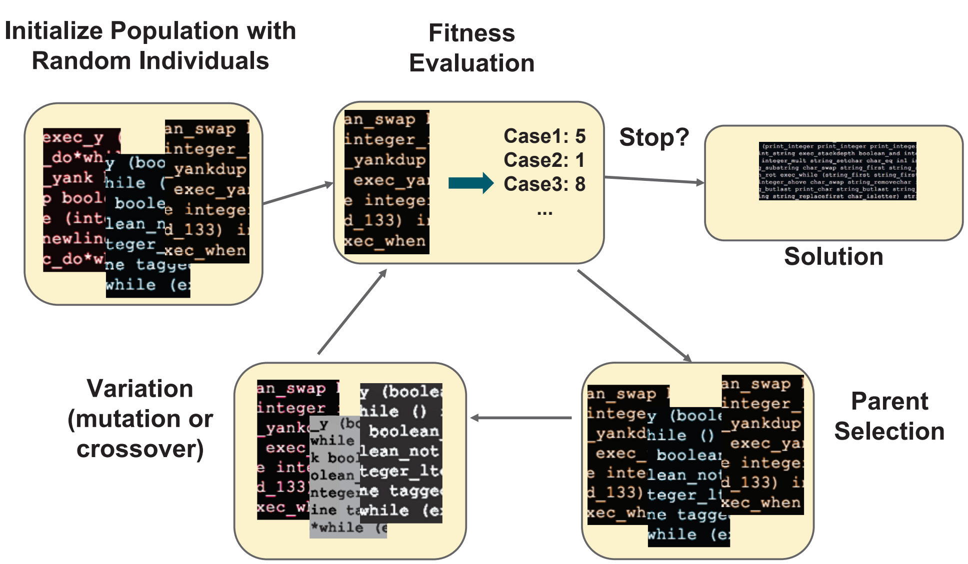
Parent Selection in Evolutionary Computation



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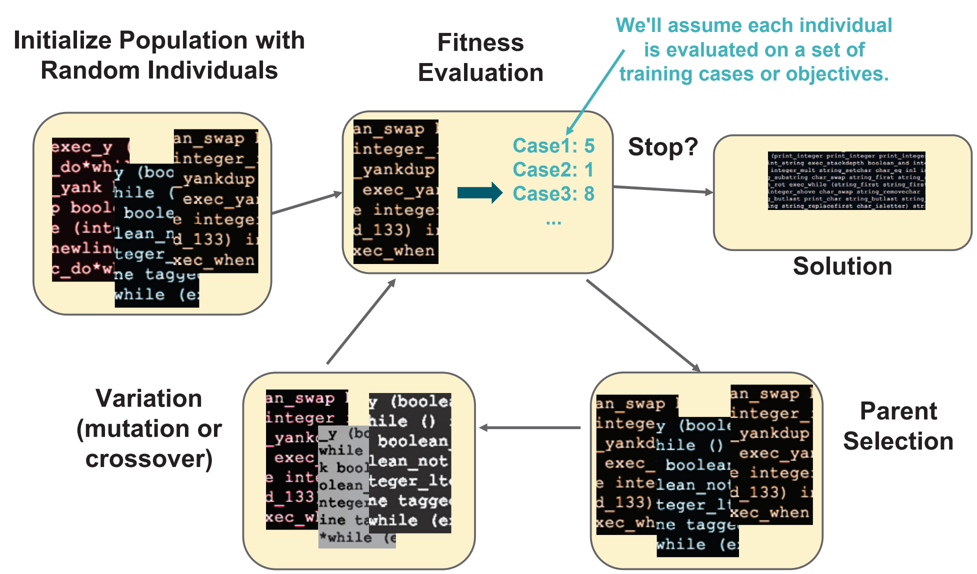

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Nomenclature

- (training) **Cases**:
  - Samples of training data
  - Sometimes referred to as “test cases”
- Semantics:
  - The behavior of a GP program on the training cases
  - The genome of a GA
- Errors:
  - The (absolute, squared etc.) difference between an individual’s semantics and the desired semantics on the training cases

	Training Data				
Cases	x1	x2	x3	x4	Target
1	1	0	86	7.5	6
2	0	1	3	6.9	3
3	1	3	45	12.3	8
4	1	6	-6	0.78	9
5	0	5	29	1.2	2

Individual A	
Case	Semantics
1	16
2	8
3	13
4	-6
5	12

	Individual Errors				
Case	A	B	C	D	E
1	10	8	73	15	15
2	5	7	60	12	12
3	5	8	0	14	0
4	15	8	0	15	106
5	10	7	1	1	1
Total Error:	45	38	134	57	134

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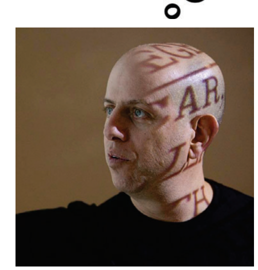
Origin Story

- Late one night...
- How can we evolve a calculator?
  - Modal: Multiple unrelated functions
    - Different training cases
  - How to maintain in the population behaviors that are good at parts of problem?

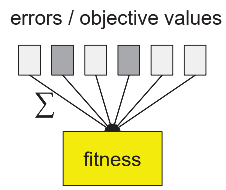


Lee Spector

Spector, Lee. (2012). Assessment of problem modality by differential performance of lexicase selection in genetic programming: a preliminary report. *GECCO*.

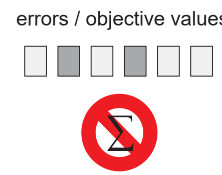
Motivation

- Most parent selection methods use a single *aggregated* fitness value
  - Ex: total error across set of training cases
  - Even multi-objective methods (e.g. NSGA-II) and quality diversity methods aggregate errors
- Obscures useful info
  - Ex: Individual Q performs well on some cases and poorly on others
    - perhaps Q has genetic material worth propagating!
    - Q has poor total error
    - Q not likely selected by tournament selection
      - The skill Q is good at may be lost in the population
- Generalists vs. Specialists



Motivation: Semantic-Aware Selection

- De-aggregating fitness
- Aggregating creates an "Information Bottleneck"
  - a rich amount of information in errors reduced to a single value
  - see: Krawiec
- Semantic-aware selection methods make use of individual semantics/errors



- Krawiec, K., et al. (2015). Behavioral Program Synthesis: Insights and Prospects. *GPTP*
- Krawiec, K., & O'Reilly, U.-M. (2014). Behavioral Programming: A Broader and More Detailed Take on Semantic GP. *GECCO*.

GECCO

Areas Where Lexicase Selection has been Beneficial

GP Program Synthesis

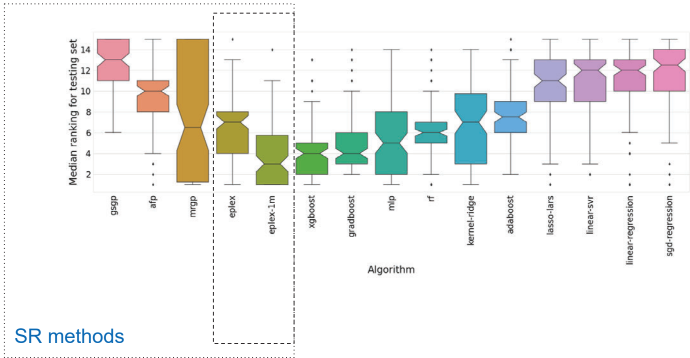
- Program synthesis: generating programs with multiple data types and control flow structures
- Lexicase selection has outperformed tournament selection and other selection methods across many benchmark problems

Problem	Tourn	IFS	Lex
Number IO	68	72	98
Small Or Large	3	3	5
For Loop Index	0	0	1
Compare String Lengths	3	6	7
Double Letters	0	0	6
Collatz Numbers	0	0	0
Replace Space with Newline	8	16	51
String Differences	0	0	0
Even Squares	0	0	2
Wallis Pi	0	0	0
String Lengths Backwards	7	10	66
Last Index of Zero	8	4	21
Vector Average	14	13	16
Count Odds	0	0	8
Mirror Image	46	64	78
Super Anagrams	0	0	0
Sum of Squares	2	0	6
Vectors Summed	0	0	1
X-Word Lines	0	0	8
Pig Latin	0	0	0
Negative To Zero	10	8	45
Scrabble Score	0	0	2
Word Stats	0	0	0
Checksum	0	0	0
Digits	0	1	7
Grade	0	0	4
Median	7	43	45
Smallest	75	98	81
Syllables	1	7	18
Problems Solved	13	13	22

- Thomas Helmuth and Lee Spector. (2015) General program synthesis benchmark suite. *GECCO*
- Forstenlechner, S. et al. (2017). A Grammar Design Pattern for Arbitrary Program Synthesis Problems in Genetic Programming. *EuroGP*.

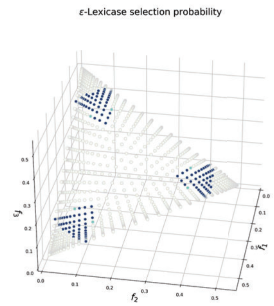
Regression

- Epsilon-lexicase selection has been shown to outperform many state-of-the-art GP and ML methods for regression

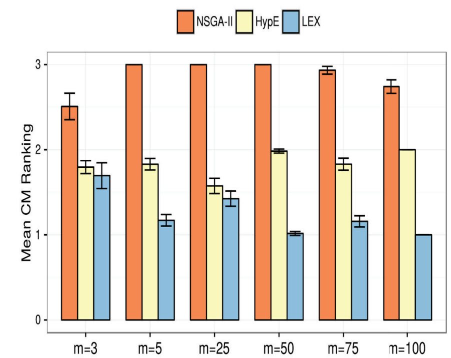


- La Cava, W. et al (2016). Epsilon-Lexicase Selection for Regression. *GECCO*
- Orzechowski, P. et al. (2018) Where Are We Now? A Large Benchmark Study of Recent Symbolic Regression Methods. *GECCO*

Many objective optimization



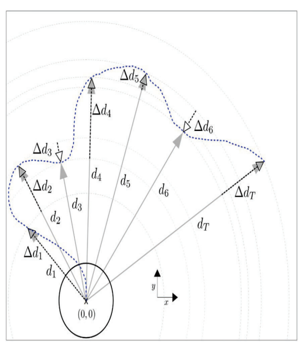
Convergence Measure Rankings, DTLZ problems, for increasing numbers of objectives (m)



La Cava, W. & Moore, J. H. (2018) An Analysis of ϵ-Lexicase Selection for Large-Scale Many-Objective Optimization. *GECCO*

Evolutionary Robotics

- Quadruped animat application, lexicase selection outperformed other selection methods
- Works well for soft robotics evolution of locomotion



Moore, J. M., & Stanton, A. (2018). Tiebreaks and Diversity: Isolating Effects in Lexicase Selection. *ALIFE*.

La Cava, W., & Moore, J. H. (2018). Behavioral search drivers and the role of elitism in soft robotics. *Artificial Life*, 206–213.

Other Evolutionary Computation Results

- Boolean logic and finite algebras problems using GP
  - Liskowski, P. et al. (2015) Comparison of semantic-aware selection methods in genetic programming. *GECCO*.
- Learning Classifier Systems
  - Aenugu, S., & Spector, L. (2019). Lexicase Selection in Learning Classifier Systems. *GECCO*.
- Boolean constraint satisfaction using GA
  - Metevier, B. et al. (2019) Lexicase selection beyond genetic programming. *GPTP*.

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The Lexicase Selection Algorithm

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Lexicase Selection Algorithm:

To Pick One Parent

```
1. pool ← population
2. cases ← list of training cases, shuffled
3. while |pool| > 1 and |cases| > 0:
     a. t ← first case in cases
     b. best ← the best error value of any individual in pool on case t
     c. pool ← filter pool to include only individuals with error of best
        on t
     d. pop t from cases
4. if |pool| = 1:
     a. return the one individual in pool
5. else:
     a. return random individual from pool
```

Thomas Helmuth, et al. (2015) Solving uncompromising problems with lexicase selection. *IEEE Transactions on Evolutionary Computation.*

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Lexicase Selection: Example 1

Case order: 5, 2, 1, 3, 4

- 5: best is 1, pool = {C, D, E}
- 2: best is 12, pool = {D, E}
  - *Note: best is always relative to pool, not full population*
- 1: best is 15, pool = {D, E}
- 3: best is 0, pool = {E}
- return E

	Individual				
Case	~~A~~	~~B~~	~~C~~	~~D~~	E
1	10	8	73	15	15
2	5	7	60	12	12
3	5	8	0	14	0
4	15	8	0	15	106
5	10	7	1	1	1
Total Error:	**45**	**38**	**134**	**57**	**134**

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Lexicase Selection: Example 2

Case order: 1, 2, 5, 4, 3

- 1: best is 8, pool = {B}
- return B

	Individual				
Case	~~A~~	B	~~C~~	~~D~~	~~E~~
1	10	8	73	15	15
2	5	7	60	12	12
3	5	8	0	14	0
4	15	8	0	15	106
5	10	7	1	1	1
Total Error:	**45**	**38**	**134**	**57**	**134**

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Lexicase Selection: Example 3

Case order: 3, 5, 4, 1, 2

- 3: best is 0, pool = {C, E}
- 5: best is 1, pool = {C, E}
- 4: best is 0, pool = {C}
- return C

	Individual				
Case	A	B	C	D	E
1	10	8	73	15	15
2	5	7	60	12	12
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When it's applicable

- When *fitness* can be decomposed into component parts.
  - Ex: summations / averages over cases (mean squared error, etc)
- Places it doesn't apply:
  - Single output, black-box function optimization
- How many fitness components?
  - There are `factorial(n)` different shufflings of `n` cases
  - Lexicase can select from at most that number of different error vectors
  - 4! = 24 isn't much if you have a population such as 1000
  - 6! = 720 is often reasonable

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Epsilon Lexicase

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Working with floating point semantics

- When program semantics/errors are floating point, it is much less likely to have ties.
  - This leads to very shallow selection events using lexicase selection
- Epsilon-lexicase selection
  - Relaxes the lexicase filtering step
  - Only individuals who fall outside of some epsilon of best are filtered each step

- La Cava, W. et al (2016). Epsilon-Lexicase Selection for Regression. *GECCO*
- La Cava, W. et al. (2019). A Probabilistic and Multi-Objective Analysis of Lexicase Selection and Epsilon-Lexicase Selection. *Evolutionary Computation.*

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epsilon-Lexicase Selection Algorithm:

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```
1. pool ← population
2. cases ← list of training cases, shuffled
3. while |pool| > 1 and |cases| > 0:
   a. t ← first case in cases
   b. best ← the best error value of any individual in pool on case t
   c. epsilon ← median absolute deviation of population on case t
   d. pool ← filter pool to include only individuals within epsilon of
      best
   e. pop t from cases
4. if |pool| = 1:
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5. else:
   a. return random individual from pool
```

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Optimizations and Tricks

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Pre-Selection Filtering: Motivation

- In GP, programs often produce the same error vector as other programs
  - Call these *equivalent*
- If 2 or more equivalent programs would make it to the end of lexicase, we would need to look at every case to find this out
  - This is inefficient
  - If only one such individual existed, we could stop lexicase earlier

	Individual	
Case	**A**	**B**
1	17	17
2	0	0
3	4	4
4	12	12
5	1	1

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Pre-Selection Filtering: Algorithm

- Group individuals into *equivalence classes* based on their error vectors
  - once per generation
- Run lexicase selection on *error vectors*, *one from each equivalence class*
  - instead of individuals
- After picking an error vector with lexicase selection, select a random individual from its equivalence class as a parent

- This has no *functional* effect on the results of lexicase - same probability of selection for every individual
- Can provide substantial speedup of running times
- Note: is not functionally equivalent for dynamic Epsilon Lexicase

Thomas Helmuth, et al. (2020) On the importance of specialists for lexicase selection. *GPEM*

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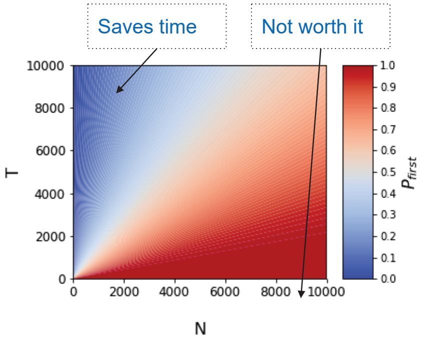
Lazy Evaluation

- Some training cases may not get used for selection
- Computational savings depend on the ratio of training cases (T) to number of selections (N).
- Every case probably comes first in selection when

$$T \leq \frac{1}{1-(0.5)^{1/N}}$$

- Otherwise, lazy evaluation may see significant gains in performance.



The probability of a case appearing first.

La Cava, W. et al. (2019). A Probabilistic and Multi-Objective Analysis of Lexicase Selection and Epsilon-Lexicase Selection. *Evolutionary Computation*

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Why does Lexicase Selection Work?

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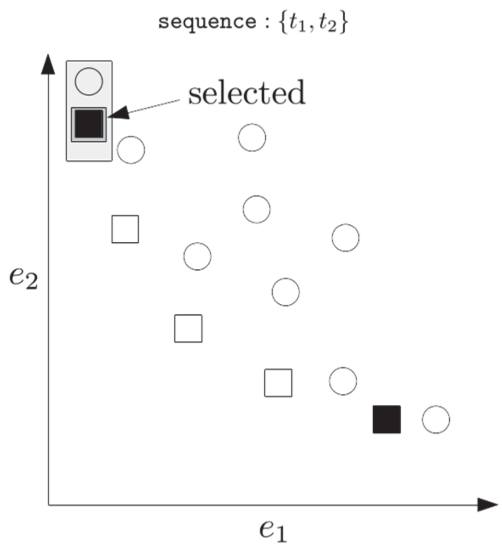
Lexicase Selections are Pareto Optimal

- Individuals who are selected are on the Pareto front defined by the cases

Lexicase Selection



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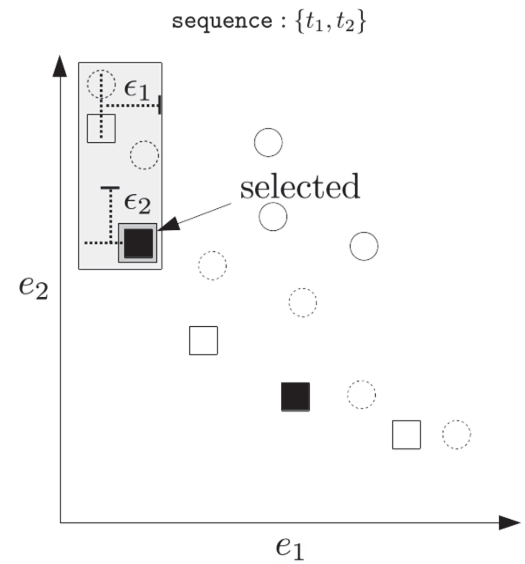
epsilon-Lexicase Selections are epsilon-Pareto Optimal

- Epsilon Lexicase selects individuals that are *epsilon*-Pareto Optimal
- Within epsilon of the Pareto Optimal points
- It does *not* necessarily select the Pareto Optimal points

epsilon-Lexicase Selection



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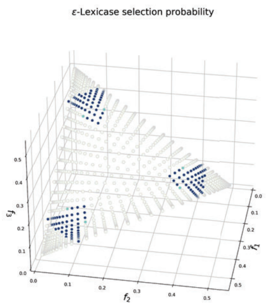

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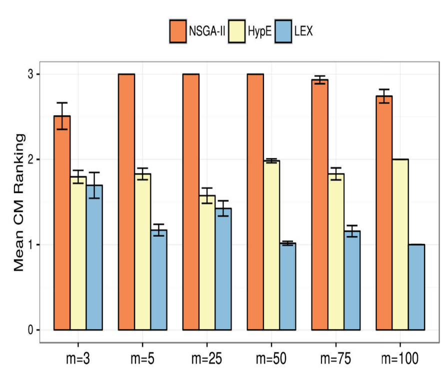
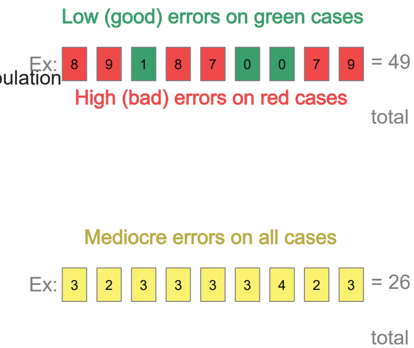


La Cava, W. & Moore, J. H. (2018) An Analysis of ε-Lexicase Selection for Large-Scale Many-Objective Optimization. *GECCO*

Specialists vs. Generalists

❖ **Specialists**:
- relatively low errors on a subset of training cases
- relatively high errors on other training cases
- poor total error (aggregate fitness) relative to population

❖ **Generalists**:
- similar errors on all training cases
- not particularly low errors on any training cases
- good total error relative to population

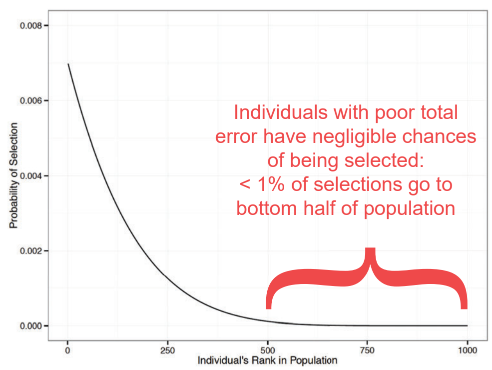


Thomas Helmuth et al. (2019) Lexicase selection of specialists. *GECCO*

Specialists vs. Generalists

❖ Which are better to select?
- Aggregating errors emphasizes generalists
- Lexicase selection emphasizes specialists

❖ Empirical answer is specialists in most cases



Ex: Tournament size = 7

Thomas Helmuth et al. (2019) Lexicase selection of specialists. *GECCO*

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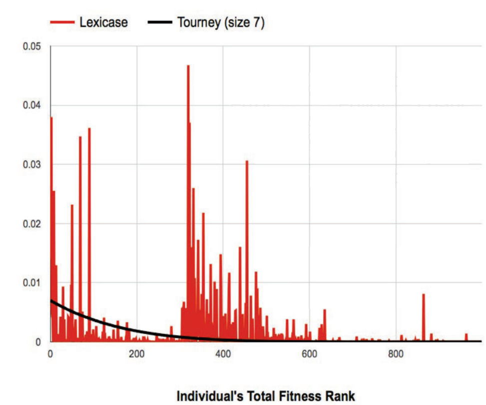

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Getting from Specialists to Generalists

- In the end, we want program that performs well on all cases
  - solution = generalist?
- How to go from specialists to generalists?
- Specialists gain additional specialties in more cases, leading to generalization
  - lexicase likely to select
  - as opposed to selecting generalists and hoping to get better on all cases at once

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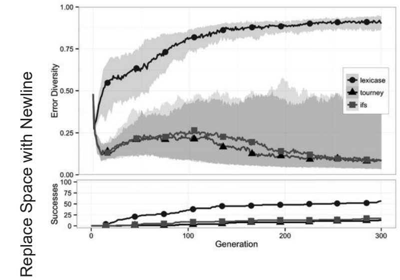
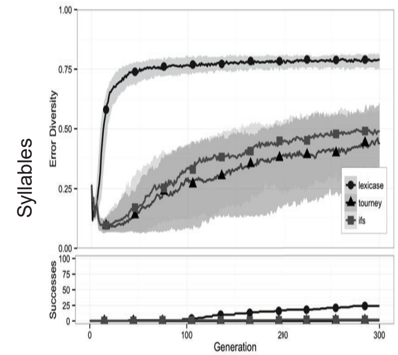
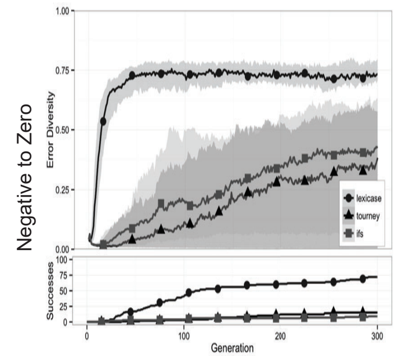
Population Diversity in GP

- Lexicase selection produces and maintains higher levels of behavioral diversity across full GP runs
- Why?
  - it selects individuals that perform well in different parts of the search space







Thomas Helmuth et al. (2015) Lexicase selection for program synthesis: A diversity analysis. *GPTP*

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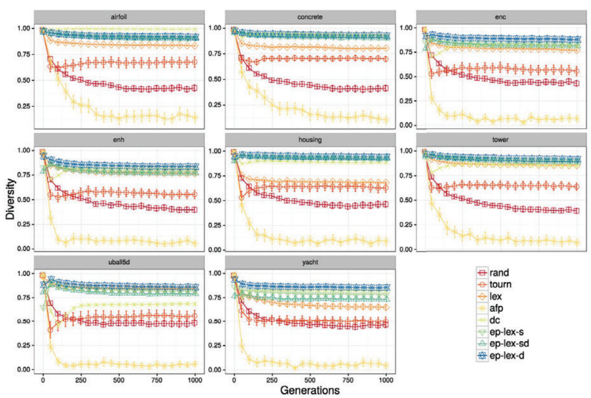
Diversity in GP for Symbolic Regression

- Also maintains high behavioral diversity in symbolic regression



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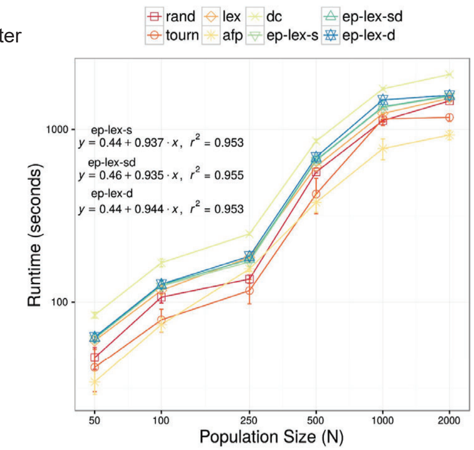
Running Time

Worst case running time

- Population of *N* individuals, *T* training cases
- Worst-case running time:
  - single selection event: O(NT)
  - Per generation: O(N^2T)
- Occurs when all individuals are identical
  - In other words, doesn't occur with pre-selection filtering
- Rarely observed
- Tournament selection worst-case O(NT) per generation

Experimental Running Time

- Observed running time is much better than the worst-case
- Closer to linear in population size



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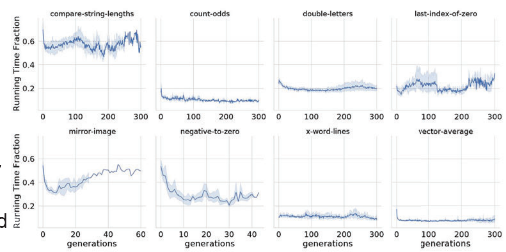
Expected Running Time

- Define a new similarity metric: ε-Cluster Similarity
  - Similar to 'clique' number from graph theory
- When populations have *low Cluster Similarity*, running time is O(N + T) instead O(N*T)



T. Helmuth, J. Lengler, W. La Cava (2022). Population Diversity Leads to Short Running Times of Lexicase Selection. *PPSN*

Extensions

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Extensions

- ❖ Alternate definitions of epsilon
  - User-defined thresholds
    - Moore & McKinley (2016) A Comparison of Multiobjective Algorithms in Evolving Quadrupedal Gaits. *SAB*
    - La Cava et al (2016) Epsilon lexicase selection for regression. *GECCO*
  - MADCAP epsilon lexicase
    - Spector, L. et al. (2018) Relaxations of Lexicase Parent Selection. *GPTP XV*
- ❖ epsilon-lexicase survival
  - La Cava, W.; Moore, J. (2017) A General Feature Engineering Wrapper for Machine Learning Using epsilon-Lexicase Survival. *EuroGP*
- ❖ Combinations with other methods
  - Novelty search: Knobelty and novelty-lexicase
  - DOCLEX
    - Liskowski, P.; Krawiec, K. (2017) Discovery of Search Objectives in Continuous Domains. *GECCO*
- ❖ Using smaller pools / islands
  - Works when less selection pressure is desirable

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Discovery of Objectives + Lexicase Selection

- Apply clustering to population semantics to identify sub-tasks
- Feed these into lexicase selection

	Individual Errors				
Case	**A**	**B**	**C**	**D**	**E**
1	10	8	73	15	15
2	5	7	60	12	12
3	5	8	0	14	0
4	15	8	0	15	106
5	10	7	1	1	1

	Clustered Errors				
Cluster	**A**	**B**	**C**	**D**	**E**
1	-	-	-	-	-
2	-	-	-	-	-
3	-	-	-	-	-

Lexicase selection

Liskowski, P.; Krawiec, K. (2017) Discovery of Search Objectives in Continuous Domains. *GECCO 17*

T

Down-sampled Lexicase Selection

- ❖ Each generation, use a subsample of the training cases to evaluate individuals
  - Similar to mini-batches used in gradient descent
- ❖ Fewer program evaluations → longer evolution for the same computational cost
- ❖ Works very well, even using small portions (5-10%) of the training set
- ❖ *This has given the best performance on program synthesis problems of any lexicase selection variant*

- Hernandez, J. G. et al. (2019). Random subsampling improves performance in lexicase selection. *GECCO*.
- Ferguson, A. J. et al. (2019). Characterizing the Effects of Random Subsampling on Lexicase Selection. *GPTP*.
- Thomas Helmuth and Lee Spector. (2020) Explaining and exploiting the advantages of down-sampled lexicase selection. *ALife*.

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Weighted Case Shuffling

- ❖ Natural question: is there a better way to shuffle cases than uniformly random?
- ❖ Tested:
  - 3 different weighted shuffle algorithms
  - 9 different bias metrics for weighting cases
- ❖ None of these outperform uniform shuffle!
- ❖ Why? Hypotheses:
  - Lower diversity because of less even emphasis on the search space
  - Fewer selections of specialists that perform well on cases that receive less emphasis

Sarah Anne Troise, Thomas Helmuth. (2017) Lexicase selection with weighted shuffle. *GPTP*.

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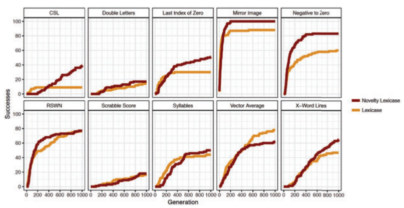
Combining Lexicase and Novelty Search

Novelty Lexicase Selection

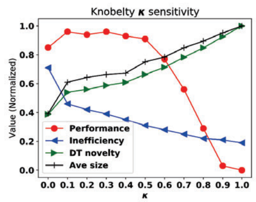
- Combines novelty scores on each case and errors into one set of cases
- Produces more diversity and higher successes in long GP runs



Lia Jundt, Thomas Helmuth. (2019). Comparing and combining lexicase selection and novelty search. *GECCO.*

Knobelty

- Uses novelty search selection K proportion of the time and lexicase selection (1 - K) proportion of the time



Kelly, J. at al. (2019). Improving Genetic Programming with Novel Exploration-Exploitation Control. *EuroGP*.

B

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B

Conclusions

- Lexicase selection is:
  - easy to implement
  - effective at improving performance and diversity
  - applicable to many areas of evolutionary computation

- Contact us with questions / comments!
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