An Improving Genetic Algorithm for Vehicle Routing Problem with Time Windows

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Abstract—**In this paper, the vehicle routing problem with time windows (VRPTW) was considered, and a mixed integer programming mathematic model of VRPTW was proposed in detail. Meantime, an improved genetic algorithm (IGA) was proposed to overcome the shortcomings of premature convergence and slow convergence of conventional genetic algorithm (GA). The novel crossover-operator、 swapping operator and inversion operator as the core of IGA were constructed to solve VRPTW. The experiment results showed that the IGA can solve VRPTW effectively.**

Keywords-vehicle routing problem, improved genetic algorithm (IGA), integer programming, logistics, optimization

I. INTRODUCTION

The vehicle routing problem (VRP)[1] analyzes efficient routes with minimum total cost for a fleet of vehicles that serve some commodity to a given number of customers. Each customer is visited exactly once by one vehicle, while vehicle activity is bounded by capacity constraints, duration constraints and time constraints. Either each route is a sequence of customers that starts at the depot and finishes at one of the customers to whom goods are delivered, or each route is a sequence of customers that begins at and ends at the distribution depot, where goods are gathered.

With the implementation of JIT (Just in time) strategy recently, many enterprises realize keeping lower inventory level is their very important target. So the enterprises pay more attention to the arrival time of freight, and they realize that the reasonable time windows of freight arrived can help enterprises to reduce inventory cost, improve operation efficiency, and enhance customer satisfaction comprehensively. In this paper, we consider a variant of the VRP where each customer must be serviced within a specified time interval (or time window).The lower and upper bounds of the time window define the earliest and latest time for the beginning of service at the customer. Hence, a vehicle is not allowed to begin service at a customer location after its time window's upper time. Moreover, a waiting time is incurred if a vehicle reaches a customer before the lower bound. Each customer also has a specified service time which is the time spent by the vehicle to load or unload the goods. Hence, the total route time of a vehicle is the sum of travel time (which is proportional to the distance traveled), waiting time and service time. The total route distance should not exceed the maximum route distance of the vehicle.

The classical VRPTW has been the subject of intensive research since the 1980s, Solomon's insertion heuristics [2] is the seminal work behind heuristic construction algorithms. Many efficient heuristic and meta-heuristic approaches have been proposed recently, including the works of Chiang and Russell [3], Potvin and Rousseau [4], Rochat and Taillard [5], Taillard et al. [6]. More recently, Schulze and Fahle [7],proposed new parallel tabu search heuristics that enable large-scale VRPTW instances to be solved. The exact algorithms recently developed for solving the VRPTW can be found in [8-10]. Several works have been carried out advocating the hybrid use of constraint programming and local search. For example, Pesant and Gendreau [11] applied constraint programming to evaluate the local neighborhood to find the best local moves.

The VRPTW is encountered in practice in many contexts, such as home delivery of packages and newspapers. The routes in the VRPTW are Hamiltonian cycles, and the VRPTW is a NP-hard problem. Although the development of modern heuristics has led to considerable progress, the quest for improved performance continues. Genetic algorithm (GA)[12]has drawn great attention from researchers due to its robustness and flexibility and have been used to tackle many combinatorial problems, including certain types of vehicle routing problem. However, GA still appears its weakness in both fields of local search and premature convergence. In our work, we develop a mathematical formulation for VRPTW and further propose an improved genetic algorithm for effective solution of VRPTW.

This paper is organized as follows: In Section 2, we describe the vehicle routing problem with time windows (VRPTW) and present a mixed integer programming mathematic model of VRPTW. In Section 3, we design an improved genetic algorithm (IGA) to solve this model. Then we will give a numerical experiment to test the performance of the improved differential evolution algorithm in Section 4. We conclude the paper in Section 5.

II. FORMULATION FOR VRPTW

We assume that: (a) each vehicle has a container with a physical limitation so that the total loading of each vehicle can not exceed its capacity q ; (b) each vehicle has maximum distance constraints so that the total distance traveled by each vehicle can not exceed L ; (c) a vehicle will be assigned for only one route on which there may be more than one customer; (d) a customer will be visited by one and only one vehicle; (e) each route begins at and ends

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at the distribution depot (0); (f) the demand of each customer is d_i ($i=1,2,\cdots,n$)and $\max d_i \le q$; (g) the distance between customer i and customer j is c_{ij} ; (h) and there are K vehicles in the distribution depot. Each customer must be serviced within a specified time interval (or time window) [a_i , b_i].The lower (a_i) and upper bounds (b_i) of the time window define the earliest and latest time for the beginning of service at the customer, a vehicle is not allowed to begin service at a customer location after b_i. Moreover, a waiting time is incurred if a vehicle reaches a customer before a_i. Each customer also has a specified service time (t_i) which is the time spent by the vehicle to load and unload the goods. Hence, the total route time of a vehicle is the sum of travel time (which is proportional to the distance traveled), waiting time and service time. The total route distance should not exceed the maximum route distance of the vehicle. In each point along its tour each vehicle can not carry a total load greater than its capacity. The goal is to minimize the overall length of the tours. $V=\{1,2,\cdots,K\}$ is the set of vehicles， Let $C=\{1,2,\cdots,n\}$ is the Set of customers ， $N=\{0,1,2,\cdots,n\}$ is the set of customers plus depot (customer 0)，The decision variable $x_{ijk}=1$，if arc (i,j) belongs to the route operated by vehicle k, otherwise is 0. The corresponding mixed integer programming mathematical formulation of VRPTW is given by：

$$\min \sum_{k\in V}\sum_{i\in N}\sum_{j\in N} c_{ij}x_{ijk} \tag{1}$$

s.t.

$$\sum_{k\in V}\sum_{j\in N} x_{ijk}=1, \forall\, i\in C \tag{2}$$

$$\sum_{i\in C} d_i \sum_{j\in N} x_{ijk} \le q, \forall k\in V \tag{3}$$

$$\sum_{j\in C} x_{0jk}=1, \quad \forall k\in V \tag{4}$$

$$\sum_{i\in N} x_{ihk} - \sum_{j\in N} x_{hjk}=0, \forall k\in V\ \forall h\in C \tag{5}$$

$$\sum_{i\in N} x_{i,n+1,k}=1, \quad \forall k\in V \tag{6}$$

$$s_{ik}+T_i+t_{ij}-M(1-x_{ijk})\le s_{jk}\ \forall k\in V, \forall i,j\in N \tag{7}$$

$$a_i \le s_{ik} \le b_i, \forall k\in V\ \forall i\in N \tag{8}$$

$$\sum_{i\in N}\sum_{j\in N} d_{ij}x_{ijk} \le L \quad \forall k\in V \tag{9}$$

$$x_{ijk}\in\{0,1\}, \quad \forall k\in V, \forall i,j\in N \tag{10}$$

Function (1) minimizes total travel distance. Constraint (2) ensures that each customer is visited by exactly one vehicle. Constraints (3) guarantee that the distribution quantity of each vehicle can not over their capacity; restrictions (4) define that at most K vehicles are used. Constraints (5) guarantee that the same vehicle arrives and departs from each client it serves; Restrictions (6) ensures that each vehicle begins at and ends at the depot. Restrictions (7) and (8) are time windows constraints. Restrictions (9) are the maximum distance constraints, L is the upper limit on the total load transported by a vehicle in any given section of the route. Finally, constraints (10) define the nature of the decision variable. If removing the restraint (7) and (8), and set a_i =0 , b_i = ∞ , then The above formulation transformed into VRP

III. THE PROPOSED IGA FOR VRPTW

A. Framework

The IGA starts by computing an initial population, i.e. the first generation. We assume that the initial population contains *popsize* individuals, where *popsize* is an integer. After computing the fitness values of the individuals, we select a pair of individuals, and apply the crossover operator to produce two new (children) individuals. Subsequently, we adopt the swapping mutation and inversion operator to the genotypes of the newly produced children. After determining the fitness of each child individual, the steady-state approach is carried out to obtain the next generation. The algorithm stops if a prespecified number of generations which is denoted as *maxgen* is reached.

B. Coding and fitness function

Like conventional GA for the VRP, a chromosome *I*(*n*) simply is a sequence (permutation) *S* of *n* customer nodes. For instance, there are 10 customer nodes, a randomly generated chromosome is 1 3 6 8 9 5 4 10 2 7, which can be interpreted as *r*=3 feasible routes: 0-1-3-6-0, 0-8-9-5-0, and 0-4-10-2-7-0. If $\bar{k}$ >=*r*, then this chromosome is legal; otherwise, it is illegal.

In order to prevent illegal chromosome entering the next generation in great probability, a penalty function is designed. R is the total distance vehicles traveled of the corresponding chromosome, let $m=r-\bar{k}$, if $r>\bar{k}$, then $m>0$, and $R=R+M\times m$, where M is a very large integer; if $r<\bar{k}$ $m=0$. The fitness function can be expressed as $f=1/(R+M\times m)$. For convenience, capacity of the vehicles is sameness, and the maximum distance that each vehicle can travel are equal, they can be denoted respectively as Q and L.

C. Population initialization

Chromosome $I(n)=\{1,2,\cdots,n\}$ is a one-dimension array, which represents a feasible solution for VRP-SDP. Then 2 integers i_1 and i_2, $i1$, $i2 \in [1,n]$, are generated by random generator. Exchange $I(i_1)$ and $I(i_2)$ to produce a random initial solution, iteration goes on until *popsize* initial solutions are created.

D. Genetic operation

（1）Selection

The selection of parents is done via traditional roulette method in iteration.

(2) Crossover operator

Crossover plays an important role in exchanging information among chromosomes. It leads to an effective combination of partial solutions in other chromosomes and speeds up the search procedure. We try a novel order crossover operator (NOX) in this paper. The NOX operator may be the most useful crossover operator for operating the permutation. It chooses first two crossover points and exchanges the subsection of the chromosome between the two points, and then it fills up in front of the child chromosomes, the other genes of the child chromosome are filled with the other parent's genes that different from the subsection. The NOX operator can generate the child that different from their parent even if the parent are uniform, which better than other crossover operators.

(3) Modified mutation operators

The objective of the mutation is to disrupt the current chromosome slightly by inserting a new gene. Several mutation operators have been proposed for permutation representation. In this research we use swapping mutation and inversion operator together as a modified mutation operator.

a. Swapping mutation

Selects two positions randomly and then swaps the genes on these positions. As shown in Figure 2, from the parent we select two position, $i1$=2 and $i2$=6, and the values on position 2 and 6 will exchange form the parent to produce a child.

b. Inversion operator

Finding out two cutting points within a chromosome randomly and then inverts the substring between these two positions and produce a child. For instance, we select two positions $i1$=3 and $i2$=7in chromosome parents, then inverts the substring between position 3 and 7 to produce the child that shorter the total distance of parent.

E. Replacement scheme

The GA we have developed uses the steady-state approach, in which eligible offspring enter the population as soon as they are produced, with inferior individuals being removed at the same time, so that the size of the population, *popsize,* remains constant.

IV. COMPUTATIONAL EXPERIMENTS

The improved genetic algorithm described in the previous section is coded in Matlab language and applied to the 8-customers vehicle routing problem with time windows. There are five vehicles in the depot, capacity of each vehicle is 8 tons, the demands of the 8 customers are listed in Table 1, the distances between customers and depot is listed in Table 2, and the velocity of each is 50km/h, all the vehicles must return the depot when finish the service. We first consider the minimum the overall length of the tours. The parameters for the proposed algorithm are included as follows: popsize is 40, maxgen is 400, probability of crossover operation is 0.95, probability of mutation is 0.01, and the maximum distance L is 500 kilometers. There are 10 independent trials are carried out to evaluate the average performance. The results of simulations are presented in Table 3, and the convergence results of optimal solution obtained is showed in Figure 4.

TABLE Ⅰ THE DEMANDS AND TIME RESTRICTION OF CUSTOMERS

Customer i	1	2	3	4
Demand(u. tons)	2	1.5	4.5	3
Service time(u. hours)	1	2	1	3
Time windows (u. hours)	[1,4]	[4,6]	[1,2]	[4,7]
Customer i	5	6	7	8
Demand(u. tons)	1.5	4	2.5	3
Service time(u. hours)	2	2.5	3	0.8
Time windows (u. hours)	[3,5.5]	[2,5]	[5,8]	[1.5,4]

TABLE. 2 THE DISTANCE MATRIX OF CUSTOMERS AND DISTRIBUTION CENTER

j \ i	0	1	2	3	4
0	0	40	60	75	90
1	40	0	65	40	100
2	60	65	0	75	100
3	75	40	75	0	100
4	90	100	100	100	0
5	200	50	100	50	100
6	100	75	75	90	75
7	160	110	75	90	75

8	80	100	75	150	100

j \ i	5	6	7	8
0	200	100	160	80
1	50	75	110	100
2	100	75	75	75
3	50	90	90	150
4	100	75	75	100
5	0	70	90	75
6	70	0	70	100
7	90	70	0	100
8	75	100	100	0

TABLE 3 THE AVERAGE COMPUTATIONAL RESULTS OF ITERATING 400

Vehicles	Distances(km)	Percentage of reaching optimal solution	Routes	Computational time(second)
3	910	100%	0-6-4-0; 0-8-5-7-0;0-3-1-2-0	3.1410

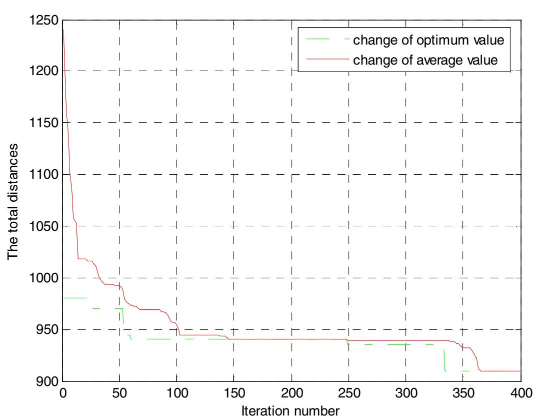

FIGURE 4 THE RESULTS OF MINIMUM VEHICLE NUMBER.

The program was coded in Matlab language and simulations are performed on a personal computer with 3.06MHz Pentium 4 processor and 1G of RAM, the runtime do not exceed three seconds and each individual of populations is the optimal solve when iterating 400 times. The optimal number of vehicles is three and the overall length of the tours is 910, a run of the improved genetic algorithm shows that the best operational plan is

Vehicle 1 : 0—6—4—0, the delivery demand is 7 tons, and full loads ratio is 87.5%, the, the route distance is 265kms.

Vehicle 2: 0—8—5—7—0, corresponding delivery demand is 7 tons, full loads ratio is 87.5%, , the route distance is 405kms.

Vehicle 3: 0—3—1—2—0, the delivery and pick-up demand both are 8 tons and achieve full loads, the route distance is 240kms.

V. CONCLUSIONS

This paper contributed to vehicle routing problem with time windows in the following respects: (a) a mixed integer programming mathematic model of VRPTW was proposed for finding the optimal solutions in details; (b) an improved genetic algorithm to solve the vehicle routing problem with time windows was presented, focusing on total traveled distance minimization, in the operation process, decimal permutation encoding was used to represent solution and penalty function was designed to eliminate illegal solutions, more importantly, the NOX crossover, swapping mutation and inversion operator were used as improved genetic operators to prevent premature convergence and accelerate searching procedure; (c)the effectiveness of the improved genetic algorithm was shown by some numerical examples.

REFERENCES

[1] Dantizing G, Ramser J.The truck dispatching problem.Management Science, 1959,(6):80-91

[2] M.M. Solomon. Algorithms for the vehicle routing and scheduling problems with time window constraints, Operations Research,1987,35 (2): 254-265.

[3] A.W. Chiang, R. Russell, A reactive tabu search metaheuristic for the vehicle routing problem with time windows, INFORMS Journal on Computing, 1997,9 (4):417-430.

[4] J.Y. Potvin, J.M. Rosseau, A parralel route building algorihtm for the vehicle routing and scheduling problem with time windows, European Journal of Operational Research,1993, 66(3) ,331-340.

[5] Y. Rochat, E. Taillard, Probabilistic diversification and intensification in local search for vehicle routing, Journal of Heuristics, 1995,1(1),147-167.

[6] E. Taillard, P. Badeau, M. Gendreau, F. Geurtin, J.Y. Potvin, A tabu search heuristic for the vehicle routing problem with soft time windows, Transportation Science,1997, 31(2):170-186.

[7] J. Schulze, T. Fahle, A parallel algorithm for the vehicle routing problem with time window constraints, Annals of Operations Research,1999,(86):585−607.(special issue).

[8] Kolen, A.W.J., A.H.G. Rinnooy Kan, and H.W.J.M. Trienkens. Vehicle routing and scheduling with time windows. Operations Research, 1987,35(2):266-273.

[9] Desrochers, M., J. Desrosiers and M. Solomon. A new optimization algorithm for the vehicle routing problem with time windows. Operations Research, 1992,40(2): 342-354.

[10] Fisher, M., K.O. Jornsten and O.B. Madsen. Vehicle routing with time windows: two optimization algorithms. Operations Research, 1997,45(3):488-492.

[11] G. Pesant, M. Gendreau, A constraint programming framework for local search methods, Journal of Heuristics,1999,5 (3):255-279.

[12] Baker Barrie M, Ayechew M. A. A Genetic Algorithm for the Vehicle Routing Problem. Computers & Operations Research, 2003, 30(5):787-800.