

# Optimal scheduling of water-supply pump stations based on improved adaptive genetic algorithm

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**Abstract:** Suitable optimal scheduling model is essential to ensuring water-supply pump stations working in high efficiency area. Based on accurate poly-fitted water pump characteristic curves, an optimal scheduling models of water-supply pump stations based on its least shaft power is proposed here. Since the constraints here are mixed with inequality, nonlinear conditions, continuous variables and discrete variables, an improved adaptive genetic algorithm(IAGA) is developed and used here, which is equipped with adaptive crossover and mutation operators based on sigmoid function, as well as adaptive penalty function factor using simulated annealing. The experimental results, comparing with the results based on simple genetic algorithm, show that pump optimal operation can ensure that all the pumps would operate in high efficiency area and meanwhile, reduce electricity energy effectively, which, as a result, implies that IAGA is more suitable for optimal scheduling of water-supply pump station.

**Key Words:** optimal scheduling, constraints, penalty function, improved adaptive genetic algorithm, simulated annealing

## 1 Introduction

Nowadays, as the demand for the urban water increases continuously, how to reduce the water consumption during water supply becomes more and more concerned these days, since the water supply industry management in most of the cities in China relies still on the traditional methods, which causes the waste of water resource and increases electricity consumption [1]. Besides, how to construct a resource-saving and environment friendly society is now China's basic national policy. In water-supply pump station, the electric energy pump unit consumes typically accounts for more than 70% of total electricity consumption [2]. However, thousands of Chinese urban water supply system adopt empirical operation, which makes a great waste of energy [3].



Fig. 1: Pump House of the Qingpu pump stations

Similar problem happens also in a water-supply pump station in Qingpu Shanghai. This station can pump 10,000 tons of water to the allocated region daily. Efficiency optimization can reduce energy consumption greatly. A set of pumps cooperate to pump adequate water to users. Practically, the operators takes only the pressure over the main pipe into consideration, which is usually not the most

energy-saving combination of the on/off states of the pumps [4].

So our work here is to find the optimal combination of pumps enabling the water-supply pump station to run with high efficiency and low cost.

Currently, linear programming [5][6], non-linear programming [7], and dynamic programming [8] have been proposed by many researchers.

However they are actually not able to find the optimal solution, especially when the constraints are mickle and complex. GA is proposed by professor Holland of Michigan University in 1969 [9], which searches the optimal solution by mimicing the biological, namely through process of selection, crossover and mutation between chromosomes. GA has been successfully applied in solving traveling salesman problem, knapsack problem, packing problem, production scheduling and other issues which are NP-hard problem. Currently ,GA has been widely adopted in water-supply pump station [10][11]. However, the traditional genetic algorithm is more suitable for optima problems without constraints and showed a rather low convergence rate when searching in a large space. So here in order to fit in the optimal scheduling problem of water-supply pump station with different kinds of physical and practical constraints, the improved adaptive genetic algorithm is developed in the following passage.

This paper is arranged as follows. In section 2, the optimization model with the considered constraints is illustrated. Section 3 introduced first the simple genetic algorithm (GA) and then the improved adaptive genetic algorithm (IAGA). The implementation of IAGA over a water- supply pump station in Qingpu, Shanghai is showed and compared with the results using GA in Section 4. Section 5 concludes the paper.

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## 2 Modeling of an optimal scheduling of a water-supply pump station

In order to find the optimal combination of the on/off states of the pumps, a suitable objective function is required here [12]. Practically, the combination of the pumps' states only change several times per day. The station should pump more during peak hours and less during sleeping hours of the citizens. So the main cost of the energy here is the total power of the pumps and the shift between the two different states is neglected here.

### 2.1 Pump characteristic curves

There are 3 basic characteristic curves of a pump, describing the relations between head and flow, shaft power and flow and the efficiency and flow [13]. Under a rated speed, the first two curve of a general pump can be poly-fitted with parabolic method. That is:

$$H_N = H_x - S_x Q_N^2 \quad (1)$$

$$N_N = d_0 + d_1 Q_N + d_2 Q_N^2 \quad (2)$$

Where  $H_x, S_x, a, b, c$  are fitting parameters.

According to the geometric characteristics, flow characteristic curve under different speeds can be described as follows:

$$\begin{aligned} \frac{Q}{Q_N} &= \frac{n}{n_0} = S \\ \frac{H}{H_N} &= \left( \frac{n}{n_0} \right)^2 = S^2 \\ \frac{N}{N_N} &= \left( \frac{n}{n_0} \right)^3 = S^3 \end{aligned} \quad (3)$$

Where  $Q, n, S$  and  $H$  are respectively the flow, speed, speed ratio and head of a variable speed pump.

So if we combine first three equations, we can get the head and shaft power of a variable speed pump in function of flow with respect to different speed ratio:

$$H = S^2 H_x - S_x Q^2 \quad (4)$$

$$N = S^3 d_0 + S^2 d_1 Q + S d_2 Q^2 \quad (5)$$

### 2.2 Analysis of the optimal scheduling

Assume, that there are  $m$  variable speed pumps and  $n-m$  general pumps, the aim is to find the most energy-efficient combination of the on-off state of the general pumps as well as the operation states of the variable speed pump. So here we select the least shaft powers of all the pumps as the objective function, which is defined as follows:

$$J = \min \left\{ \sum_{i=1}^m \omega_i (d_{0i} S_i^3 + d_{1i} S_i^2 Q_i + d_{2i} S_i Q_i^2) + \sum_{i=m+1}^n \omega_i (d_{0i} + d_{1i} Q_i + d_{2i} Q_i^2) \right\} \quad (6)$$

Where  $J$  represents the sum of the shaft powers from each pump,  $\omega_i$  stands for the state of pump  $i$  and it equals to 0 or 1.

### 2.3 Model Constraints

Safe operation of the pipeline network is a prerequisite for scheduling. To ensure the safe operation of the water supply network, we must consider the following constraints.

#### 2.3.1 Speed limit of a variable speed pump

Here  $S$  is the speed ratio of variable speed pumps and normally these pumps work under rated speed, which means  $S \in [0, 1]$ . In order to ensure the pump to work efficiently and avoid cavitation erosion, a lower limit  $S_{min}$  should be set. That is:

$$S_{min} \leq S_i \leq 1 \quad i = 1, 2, \dots, m \quad (7)$$

#### 2.3.2 Flow balance of supply and demand in pump station

In a water-supply pump station, the outlet flow quantity  $Q_S$  must be equal to the consumption of users. This constraint can be quantified as:

$$Q_S = \sum_{i=1}^n \omega_i Q_i \quad (8)$$

#### 2.3.3 The flow constraint of a pump

The pump cannot reach considerable efficiency when its flow is too high or too low. The maximum and minimum flows of pump must be considered as the model constraint. For general pump, the form of flow constraint is:

$$Q_{i,min} \leq Q_i \leq Q_{i,max} \quad (9)$$

Where  $Q_{i,min}$  and  $Q_{i,max}$  are the upper and lower flow limits of pump  $i$ .

For general pumps, the lower and upper flow limits are usually listed in the data sheet. However for variable speed pumps, the high efficiency area of the pump is the sector ring ABFE in Figure 1:

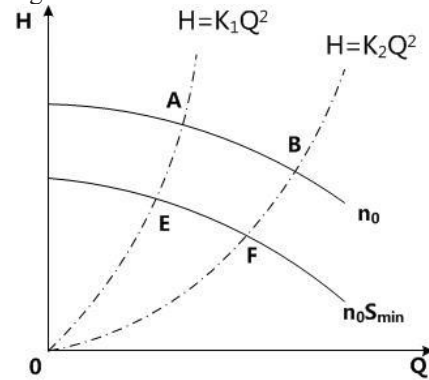


Fig.2: high efficiency area of a variable speed pump

Assume point A and B are the endpoints of a variable speed pump under the highest rated speed  $n_0$ . And point E and F are the pump's endpoints under the lowest rated speed  $n_0 S_{min}$ . So the high efficiency area is the sector ring enclosed by curve AB, EF and the parabolic OA, OB describing the similar conditions of a variable speed pump.

After forming simultaneous equations based on equation [3] and [4] with respect to the mentioned curves in Figure 1, the upper flow limit  $Q_{max}$  and lower flow limit  $Q_{min}$  of each pump can be described as following:

$$Q_{i\min} = \begin{cases} \sqrt{\frac{H_S}{H_{iA}}} Q_{iA}, H_S \geq H_{iE} \\ \sqrt{\frac{H_{iX} S_{i\min}^2 - H_S}{S_{iX}}}, H_S < H_{iE} \end{cases}, i = 1, 2, \dots, m$$

$$Q_{i\max} = \begin{cases} \sqrt{\frac{H_{iX} - H_S}{S_{iX}}}, H_S \geq H_{iB} \\ \sqrt{\frac{H_S}{H_{iB}}} Q_{iB}, H_S < H_{iB} \end{cases}, i = 1, 2, \dots, m \quad (10)$$

### 2.3.4 pressure restraint of pumps in parallel connection

In a pump station, pumps in parallel have the same head. That means:

$$H_1 = H_2 = \dots = H_S \quad (11)$$

Where  $H_S$  is the target lift and the other symbols are already mentioned.

In summary, the operation decision model can be expressed as the following formula:

$$J = \min \left\{ \sum_{i=1}^m \omega_i (d_{0i} S_i^3 + d_{1i} S_i^2 Q_i + d_{2i} S_i Q_i^2) + \sum_{i=m+1}^n \omega_i (d_{0i} + d_{1i} Q_i + d_{2i} Q_i^2) \right\}$$

$$\text{s.t.} \quad Q_S = \sum_{i=1}^n \omega_i Q_i$$

$$H_S = H_1 = H_2 = \dots = H_n \quad (12)$$

$$Q_{\min i} \leq Q_S \leq Q_{\max i}, i = 1, 2, \dots, n$$

$$S_{\min i} \leq S_i \leq 1, i = 1, 2, \dots, m$$

## 3 Design of the improved adaptive genetic algorithm

After modeling the water-supply pump station, a suitable algorithm is now needed to calculate the optimal solution. In order to deal with this nonlinear problem with different types of constraint, genetic algorithm is first adopted and the corresponding procedure for solving this problem is listed in section 3.1.

### 3.1 Optimal procedure with simple genetic algorithm

#### Step 1: Coding the variables

According to the previous assumption that there are  $m$  variable speed pumps and  $(n-m)$  general pumps, so the unknown variables are written as:

$$S_1 S_2 \dots S_m \omega_1 \omega_2 \dots \omega_{n-m} \quad (13)$$

$\omega_i$  represents the working state of a pump with  $\omega_i = 1$  standing for working and  $\omega_i = 0$  for not working.  $S_i$  stands for the speed ratio of pump  $i$ .

In order to fit in the binary code, the speed ratio  $S_i$  should also be substituted by a binary string, whose length is  $l$ , that is:

$$S_i : b_l b_{l-1} b_{l-2} \dots b_2 b_1 \quad (14)$$

Since  $S_i \in [S_{\min}, 1]$ , the accuracy of the binary code is:

$$\delta = \frac{1 - S_{\min}}{2^l - 1} \quad (15)$$

The corresponding decoding equation is:

$$S = S_{\min} + \left( \sum_{i=1}^l b_i \times 2^{i-1} \right) \times \frac{1 - S_{\min}}{2^l - 1} \quad (16)$$

#### Step 2: Initializing the population

Here we randomly create 60 chromosome for the initial population.

#### Step 3: Determining the adapting function

In this article, the total shaft power is chosen as the objective function, which is, the smaller the value is, the better is the result. In order to suit in the adapting function in GA, we let  $eval(X) = F(MAX) - F(X)$ , where MAX is the chromosome with the biggest value of the objective function in the current population.

Here the best chromosome with the biggest adapting function will be saved and then substitute the worst one in the next generation.

#### Step 4: Designing the genetic operator

1) Selection operator

Fitness  $P_j^t$  of each chromosome is needed for the method Roulette Wheel, and it should be in percentage:

$$P_j^t = eval(X_j^t) / \sum_{j=1}^{npop} eval(X_j^t) \quad (17)$$

Where  $npop$  is the size of the population,  $t$  stands for the current generation.

2) Crossover operator

After selecting the chromosomes according to the former operator, each couple of the chromosomes will cross with each other with respect to a random position and the crossover rate. And if the crossover is to be done, then two new chromosomes will be created in this process.

3) Mutation operator

$(npop * p_m)$  number of chromosomes will be selected from the population and randomly one of the gen bits will randomly be chosen to be converted between 0 and 1.

### 3.2 Improved adaptive genetic algorithm

However with this simple genetic algorithm, the inequality constraints are not quantificationally evaluated in the model. Furthermore, the convergence rate also needs to be improved, when facing with new population with different distribution of fit values, as the iteration goes on.

#### 3.2.1 Adaptive crossover and mutation operators

In order to improve the convergence rate as well as the avoid lagging at the initial stage [14], the crossover operator ( $P_c$ ) and the mutation operator ( $P_m$ ) should be enlarged with respect to low fitness value and shrink by large fitness value, so that by large values the convergence rate would be enhanced while by low values, low minimum can be avoided.

Here, the sigmoid curve [15] is introduced to adjust the operators, that is:

$$\varphi(v) = \frac{1}{1 + \exp(-av)} \quad (18)$$

As is showed in Figure 3, the sigmoid function has smoother bottom and top than the cosine and sine functions, which shows a better balance between linear and nonlinear property.

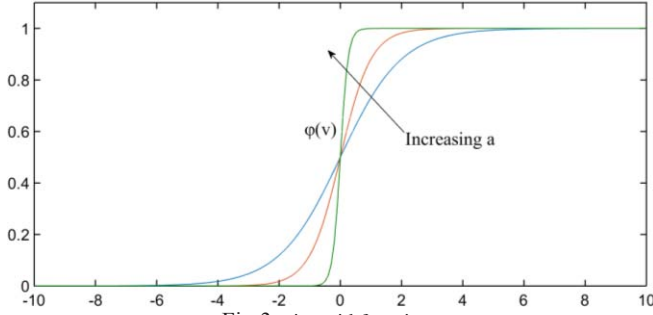


Fig.3: sigmoid function

The corresponding adaptive the crossover operator ( $P_c$ ) and the mutation operator ( $P_m$ ) is presented in equation [19] and equation [20]:

$$P_c = \begin{cases} \frac{P_{c\max} - P_{c\min}}{1 + \exp(A(\frac{2(f' - f_{avg})}{f_{\max} - f_{avg}}))} + P_{c\min} & f' \geq f_{avg} \\ P_{c\max} & f' < f_{avg} \end{cases} \quad (19)$$

$$P_m = \begin{cases} \frac{P_{m\max} - P_{m\min}}{1 + \exp(A(\frac{2(f - f_{avg})}{f_{\max} - f_{avg}}))} + P_{m\min} & f \geq f_{avg} \\ P_{m\max} & f < f_{avg} \end{cases} \quad (20)$$

### 3.2.2 Adaptive penalty function factor

Model described in equation [12] is a mixed and nonlinear programming problem. The decision variable is switch degree of the variable-speed pumps. However, according to equation [12], the variables waiting to be solved are the switch states  $\omega_i$ , the speed ratio  $S_i$ , the flow of each pump  $Q_i$  and the lift of each pump  $H_i$ . In order to reduce the searching region and the searching time cost, we put equation [4] and equation [11] into equation [12], so that  $H_i$  and  $Q_i$  can be substituted with  $S_i$ . The following is the new model for further optimization.

$$J = \min \sum_{i=1}^n \omega_i \left( d_{0i} S_i^3 + d_{1i} S_i^2 \sqrt{\frac{H_{iX} S_i^2 - H_S}{S_{iX}}} + d_{2i} S_i \frac{H_{iX} S_i^2 - H_S}{S_{iX}} \right) \quad (21)$$

In this equation, the variables waiting to be solved are only  $\omega_i$  and  $S_i$ .

Furthermore, in order to enhance the speed and accuracy by the searching process and to ensure the convergence of the algorithm, we introduce the penalty function into the objective function. According to the constraints in equation 11, the main 3 constraints will be processed in the following way:

- 1) The constraint of the speed ratio  $S_i$  is to be taken into consideration through defining the coding region.

- 2) The constraint of the total flow can be transformed into one term of the penalty function:

$$P_1 = \left( \sum_{i=1}^n \omega_i \sqrt{\frac{H_{iX} S_i^2 - H_S}{S_{iX}}} - Q_S \right)^2 \quad (22)$$

- 3) The constraint of the high efficiency region of each pump is converted into the following equation:

$$P_2 = \sum_{i=1}^n (\Delta Q_i)^2 = \begin{cases} \sum_{i=1}^n \left( \sqrt{\frac{H_{iX} S_i^2 - H_S}{S_{iX}}} - Q_{\min i} \right)^2, & \sqrt{\frac{H_{iX} S_i^2 - H_S}{S_{iX}}} < Q_{\min i} \\ 0, & Q_{\min i} < \sqrt{\frac{H_{iX} S_i^2 - H_S}{S_{iX}}} < Q_{\max i} \\ \sum_{i=1}^n \left( \sqrt{\frac{H_{iX} S_i^2 - H_S}{S_{iX}}} - Q_{\max i} \right)^2, & \sqrt{\frac{H_{iX} S_i^2 - H_S}{S_{iX}}} > Q_{\max i} \end{cases} \quad (23)$$

So the new objective function with new terms of penalty function is:

$$F = J + \sigma(P_1 + P_2) = \sum_{i=1}^n \omega_i \left( d_{0i} S_i^3 + d_{1i} S_i^2 \sqrt{\frac{H_{iX} S_i^2 - H_S}{S_{iX}}} + d_{2i} S_i \frac{H_{iX} S_i^2 - H_S}{S_{iX}} \right) + \sigma \left[ \left( \sum_{i=1}^n \omega_i \sqrt{\frac{H_{iX} S_i^2 - H_S}{S_{iX}}} - Q_S \right)^2 + P_2 \right] \quad (24)$$

Since the final solution must be feasible, the penalty factor should be more and more dominant as the iteration goes on. However, at the initial stage, if the penalty factor overwhelms the objective function in equation [20], the algorithm will convergence too early. Traditionally, exterior penalty function method, interior penalty function method or lagrangian multiplier is used to remove the constraints from the model. However the first two methods will cause ill-conditioned problems due to their infinitely enlarged penalty factors. By lagrangian multiplier, such problems will be avoided through using finite penalty factor. But it needs to solve a series of unconstrained minimum problems to approximate the optimal multiplier and the optimal solution. So here the simulated annealing [16] is adopted to let the penalty factor grow as the iteration goes on, that is:

$$\sigma = \alpha \frac{1}{T}, \quad T = \gamma T, \quad \gamma \in [0,1] \quad (25)$$

Where  $\alpha$  is the initial temperature,  $\gamma$  is the cooling parameter.

## 4 Algorithm test and implementation

### 4.1 Algorithm test

Using the classical benchmark test functions in Table 1, the parameters maximum and minimum mutation and crossover rates are adjusted and after comparing with the results from GA showed in Table 2, we can know that with IAGA, we can get better optimal solutions.

Table 1: The classical benchmark test functions

test functions	optimal value	optimal solution	Variable range
$f_1(x_1, \dots, x_n)$	0	$[0]^n$	$[-5.12, 5.12]^n$
$f_2(x_1, x_2)$	0.998	$[-32, -32]$	$[-65.536, 65.536]$
$f_3(x_1, x_2)$	-1.0316	$[-0.0898, 0.7126], [0.0898, -0.7126]$	$[-3, 3], [-2, 2]$

Where:

$$f_1(x) = \sum_{i=1}^n x_i^2 \quad (26)$$

$$f_2(x_1, x_2) = 0.002 + \sum_{i=1}^{25} \frac{1}{j + \sum_{i=1}^2 (x_i - a_{ij})^6} \quad (27)$$

With:

$$[a_{ij}] = \begin{bmatrix} -32 & -16 & 0 & 16 & 32 & -32 & -16 & \dots & 0 & 16 & 32 \\ -32 & -32 & -32 & -32 & -32 & -16 & -16 & \dots & 32 & 32 & 32 \end{bmatrix}$$

$$f_3 = (4 - 2.1x_1^2 + x_1^{4/3})x_1^2 + x_1x_2 + (-4 + 4x_2^2)x_2^2 \quad (28)$$

Here, we apply GA and IAGA with different sets of parameters to these test functions. We search with each algorithm for 500 generation. The results in table 2 are the average fitness value over 500 iterations.

Table 2: Optimization results under different algorithms

function algorithm	$f_1$	$f_2$	$f_3$
GA (PC=0.5, Pm=0.1)	0.001209	1.368662	-1.028704
IAGA (Pcmin=0.5, Pcmax=0.8, Pmmin=0.05, Pmmax=0.1)	0.002022	2.924709	-1.029454
IAGA (Pcmin=0.5, Pcmax=0.8, Pmmin=0.1, Pmmax=0.2)	0.001209	1.015717	-1.030645
IAGA(Pcmin=0.6, Pcmax=1, Pmmin=0.1, Pmmax=0.2)	0.001209	1.010750	-1.030868

From the results in table 2 we can see that, with the simplest test function De Jong J1 ( $f_1$ ), both algorithms with bigger mutation rate can reach the best solution. However when the functions have more extreme points, IAGA overwhelms GA. According to table 2, the last set of IAGA is chosen for optimal scheduling of the water-supply pump station.

## 4.2 Parameter configuration

The model parameters including algorithm parameters and pump parameters are shown in Table 3 and Table 4. Algorithm parameters such as number of cycles, population size, mutation rate, and crossover are selected after several trials. Pump parameters are given according to the real working condition in the pump station. Here a pump station in Nanchang [13] is taken as the research object. This pump station is equipped with 3 centrifugal pumps in type of 14Sh-13, 4 in type of 20Sh-9A and 4 in type of 28SA-10A.

One of the pumps in type of 28SA-10A is a variable speed pump, and the others are general pumps.

Table 3: The pumps parameters

Type of pump	$Q_A$ (L/s)	$Q_B$ (L/s)	$S_{min}$
28SA-10A	420	1400	0.75
14Sh-13	180	440	0.70
20Sh-9A	230	740	0.72

Table 4: IAGA parameters

IAGA parameters	Values	IAGA parameters	Values
Population size	60	Maximum crossover rate	1.0
Number of generation	400	Minimum crossover rate	0.6
Elite count	1	Selection type	Roulette Wheel
Maximum mutation rate	0.2	Initial temperature	1 °C
Minimum mutation rate	0.1	Cooling parameter	0.99

## 4.3 Implementation and discussion

Based on the pump parameters and IAGA parameters given in Table 3 and Table 4, the proposed IAGA optimization schedule model is applied on a water-supply pump station with the real operation data. The algorithm is implemented using MATLAB Programming Language R2014b. The performance of the two algorithms is showed in Figure 3, from which we can see that, it takes GA algorithm over 80 iterations to reach the optimal solution. In comparison, within 10 iterations, the IAGA finds the optimal solution, which is much faster than GA. Furthermore, GA algorithm is more likely to get into a local optimum.

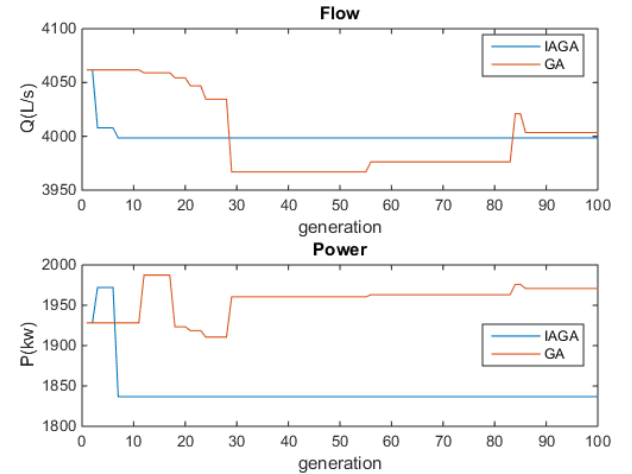


Fig.4: Performance of different algorithms

Table 5 and table 6 show the results from 2 different algorithms according to different dispatch orders ( $H_s, Q_s$ ) from real working conditions. Here  $\omega$  is a binary code of 9 bits, representing the working states of each pump in the water-supply pump station. The first bit stands for the working state of the variable speed pump 28SA-10A, the other 10 bits stands for the 10 general pumps in the order of 3

in type of 14SH-13, 4 in type of 20SH-9A and 3 in type of 28SA-10A.

After optimization with IAGA, we can see that, the total flow is apparently closer to the given orders and furthermore, the energy cost caused by pumps can save averagely 58.25 kw per hour, which according to the electric charge in Nanchang, can help the pump station saving 451,929 RMB in a year.

Table 5: Results from GA

H <sub>s</sub>	Q <sub>s</sub>	$\omega$	S	Total flow	Shaft power
34	900	01100000000	0	872.24	365.1
36	2800	10110000001	0.9380	2805.77	1243.6
37	3600	11111100000	0.8824	3610.39	1687.7
39	4000	11011110000	0.9545	3994.70	1987.4
40	4100	11100000011	0.9757	4096.74	1901.3
42	4300	11110011001	0.8510	4293.71	2170.1

Table 6: Results from IAGA

H <sub>s</sub>	Q <sub>s</sub>	$\omega$	S	Total flow	Shaft power	Saving energy
34	900	1000000000	0.8384	900.52	335.4	29.7
36	2800	1010000001	0.9380	2799.85	1190.3	53.3
37	3600	1000000101	0.8588	3599.20	1576.5	111.2
39	4000	1101000101	0.8173	3998.51	1836.7	150.7
40	4100	1010000101	0.9553	4100.85	1888.4	12.9
42	4300	1111000101	0.8737	4300.02	2056.9	113.2

After optimization with IAGA, we can see that, the total flow is closer to the given orders and furthermore, the energy cost caused by pumps can save averagely 58.25 kw per hour, which according to the electric charge in Shanghai, can help the pump station saving 451,929 RMB in a year.

## 5 Conclusion

In this paper, an optimal pump-scheduling problem using the improved adaptive genetic algorithm (IAGA) has been introduced. We simulated the proposed algorithm with the real data from a water-supply pump station in Nanchang. The results show that the developed pump schedule model based on IAGA can save a great amount of cost for the pump system with compare to the one based on GA and is to be implemented further in the water-supply pump station in Shanghai.

However, in order to save more energy for the whole process, the switch between two states need to be considered for the further research. That is, the optimal solution should not only be based on the new order, but the current switch state should be taken into account as well.

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