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Parametric Complexity Reduction Of The Meixner-Like Model using Genetic algorithms

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Abstract— **This article focuses on modeling and identification of a LTI system represented by Meixner-like functions. The use of the Meixner-like functions is more suitable than the use of Laguerre functions and Kautz functions especially when the system have a slow initial onset or delay. Thus, the order of the model can be low, if the Meixner-like pole is set to its optimal value. Therefore we propose, from input/output measurements, a new method; based on Genetic Algorithms, to estimate the optimal value of the Meixner-like pole. A numerical simulation shows the efficiency of the approach.**

Keywords— **Meixner-like functions; Recursive representation; Meixner-like network; Pole optimization; Genetic Algorithms.**

I. INTRODUCTION

In literature the modeling of Single input Single output (SISO) LTI systems using orthonormal basis functions is particularly attractive in parameter number reduction. Among these orthogonal basis functions we note, first, the Laguerre functions which depend only on a free parameter (pole) and are best suited as they provide the best approximation given the dominant time constant for damped systems [3,5,12,16,17]. Second, for moderately and oscillating systems, the Kautz functions have been employed and which are characterized by two conjugate pole [5,8,20]. Third, the generalized orthonormal basis functions (GOBF) are characterized by a set of poles and can be convenient for highly oscillating systems [9,19,21]. The Meixner-like functions are an extension of Laguerre functions and are suitable when the system have a slow start and an exponential decay towards infinity [2,7,13,14,15]. The Meixner-like functions depend on two parameters, the first is called the order of generalization that determines how fast the functions start and the second is the Meixner-like pole that appears in the error criterion in a nonlinear way and is usually hard to find.

The problem of how to choose the Meixner-like pole is considered in this paper and the optimal identification of which is achieved by exploiting the minimization of the mean square error. In this paper, we propose, from input/output measurements and by our new recursive representation of Meixner-like model, a new method to estimate the optimal value of Meixner-like pole. The proposed method uses the Genetic Algorithms (GA) that are widely adopted in recent years and which are very powerful in stochastic system modeling and have no special requirement on the form of the objective function. The processing of these algorithms can be achieved in many different ways by using some stochastic rules and they can converge to the global optimum with high probability. Genetic Algorithms [10,11] are one of the stochastic optimization algorithms that can express the complex structure problems with its hierarchy and can determine the feasible solving space automatically without giving any superstructure. Then, the global optimum can be seeked automatically in the genetic evolutionary process.

The paper is organized as follows: in section 2, we present the discrete time Meixner-like model. We develop the filter network of the Meixner-like model as well as its new recursive representation. In section 3, we propose a method of the Meixner-like pole optimization by using the Genetic Algorithms method. Section 4 evaluates, through a simulation example of a variable inertia system, the performances of the proposed optimization algorithm as well as the Meixner-like representation in term of approximation quality. Finally, some concluding remarks are made in Section 5.

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II. MEIXNER-LIKE MODEL

A. Preliminaries

The Meixner-like functions are governed by two parameters: the Meixner-like pole ξ and the order of generalization a that determines how fast the functions start. In the special case where $a=0$, the Laguerre functions are obtained. The Meixner-like functions form a complete orthonormal basis in the Lebesgue space $\ell^2\left[0,\infty\right[$ and defined for $a=0,1,2...$ and $n=0,1,2,...$ by [2,13,14,15]:

$$m_n^{(a)}(k,\xi)=(k+1)_a\,(\xi)^k\,\rho(k)\sum_{j=0}^{n}C_{n+1,j+1}^{(a)}\frac{\left(k+a+1\right)_j}{(j+a)!} \qquad (1)$$

With

$$C_{n+1,j+1}^{(a)}=(1-\xi^2)^{a+1/2}\sum_{l=1}^{n}L_{n+1,l+1}^{(a)}(-\xi)^{-l}\begin{pmatrix} l\\ j\end{pmatrix}(\xi^2-1)^j \qquad (2)$$

where $\rho(k)$ is the unit step, $(.)_j$ is the Pochhammer symbol and $L_{n+1,j+1}^{(a)}$ the $(n+1,j+1)$ element of the lower triangular matrix $L^{(a)}$. The matrix $L^{(a)}$ is not explicitly known and can be computed by inversing the Cholesky decomposition of the matrix $W^a\left\{W^a\right\}^T$, where W is a higher bi-diagonal matrix defined by:

$$W=\begin{bmatrix} 1 & \xi & 0 & \\ 0 & 1 & \xi & \ddots \\ 0 & \ddots & \ddots & \ddots \end{bmatrix} \qquad (3)$$

The Z transform of the Meixner-like functions $m_n^{(a)}(\xi,k)$ are given by [7]:

$$M_n^{(a)}(z,\xi)=(1-\xi^2)^{a+1/2}\left(\frac{z}{z-\xi}\right)^{a+1}\sum_{j=0}^{n}L_{n+1,j+1}^{(a)}\left(\frac{1-\xi z}{z-\xi}\right)^j \qquad (4)$$

Due to the completeness of the Meixner-like sequences, the transfer function of all time invariant linear system can be decomposed in the form:

$$G(z)=\sum_{n=0}^{\infty}g_n(\xi)M_n^{(a)}(z,\xi) \qquad (5)$$

with $g_n(\xi)$ are the Fourier coefficients of the decomposition. For an input $U(z)$, the output of the system is then defined by:

$$Y(z)=\sum_{n=0}^{\infty}g_n(\xi)M_n^{(a)}(z,\xi)U(z) \qquad (6)$$

B. Meixner-like Network

In physical applications for filter design, the infinite series (6) is truncated with limited number of Meixner-like stages, e.g. $N+1$, with tolerating some truncation error.

$$Y(z)=\sum_{n=0}^{N}g_n(\xi)X_n^{(a)}(z,\xi)+E(z) \qquad (7)$$

with $X_n^{(a)}\left(z,\xi\right)=M_n^{(a)}\left(z,\xi\right)U\left(z\right)$

The inverse Z-transform of the relation (7) explicit the output in the time domain:

$$y(k)=\sum_{n=0}^{N}g_n\ m_n^{(a)}(k,\xi)*u(k)\ +e(k) \qquad (8)$$

where * is the convolution product and $e(k)$ is the modelling error .

As from (7), the input filtered by Meixner-like functions is expressed $x_n^{(a)}(k)=m_n^{(a)}(k,\xi)*u(k)$ for $n=0...N$, the relation (8) can also be written as follows:

$$y(k)=\sum_{n=0}^{N}g_nx_n^{(a)}(k)+e(k) \qquad (9)$$

The filtering scheme given by Fig. 1 using Meixner-like functions can be easily deduced from relation (7) complemented by the definitions (4). A unit delay has been incorporated into the network to represent strictly causal systems [13,14,15].

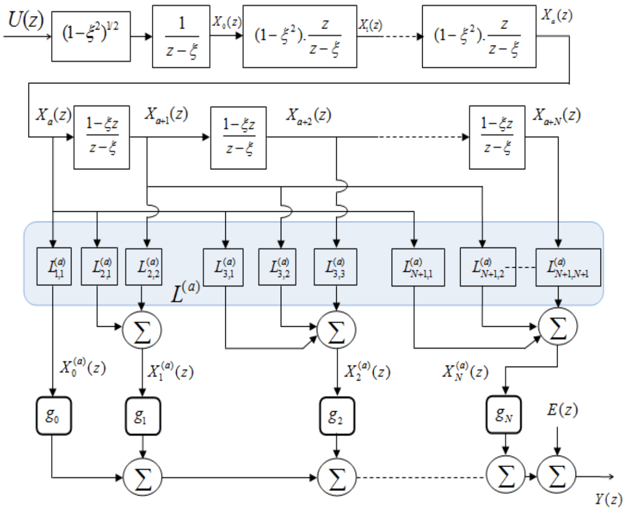

Fig. 1 Meixner-like network

Note that the Meixner-like network can also generate the network of classical Laguerre filters by choosing as generalization order the particular value $a = 0$.

C. *Recursive Representation Of Meixner-like Model*

In practice, especially for the calculation of the control law, it is preferable to substitute for the definition (9) a recursive calculation form of the output. Consider the variables $x_i(k);\ i = 0,\ldots,a+N$ the inverse Z-transform of the variables $X_i(z);\ i = 0,\ldots,a+N$ located before the bloc containing the elements of the matrix $L^{(a)}$ and $x_n^{(a)}(k,\xi) = \sum_{j=0}^{n} L_{n+1,j+1}^{(a)} x_{j+a}(k,\xi)$ the inverse Z-transform of $X_n^{(a)}(z,\xi) = \sum_{j=0}^{n} L_{n+1,j+1}^{(a)} X_{j+a}(z,\xi)$ for $n = 0 \ldots N$.

From Fig. 1 and by considering $x_i(k);\ i = 0,\ldots,a+N$ as state variables, we obtain the following recursive equations:

$$\begin{cases} x_0(k+1) = \xi x_0(k) + \beta u(k) \\ x_1(k+1) = \xi x_1(k) + \xi \beta^2 x_0(k) + \beta^3 u(k) \\ x_2(k+1) = \xi x_2(k) + \xi \beta^2 x_1(k) + \xi \beta^4 x_0(k) + \beta^5 u(k) \\ x_3(k+1) = \xi x_3(k) + \xi \beta^2 x_2(k) + \xi \beta^4 x_1(k) + \xi \beta^6 x_0(k) + \beta^7 u(k) \\ \vdots \\ x_a(k+1) = \xi x_a(k) + \left[\xi \sum_{j=1}^{a} \beta^{2j}\ x_{a-j}(k) \right] + \beta^{2a+1} u(k) \\ x_{a+1}(k+1) = \xi x_{a+1}(k) + \beta^2 x_a(k) - \left[\xi^2 \sum_{j=1}^{a} \beta^{2j} x_{a-j}(k) \right] - \xi\ \beta^{2a+1} u(k) \\ \vdots \\ x_{a+N}(k+1) = \xi x_{a+N}(k) + \left[\beta^2 \sum_{i=0}^{N-1} (-\xi)^{N-i-1}\ x_{a+i}(k) \right] + (-1)^N \left[(\xi)^{N+1} \sum_{j=1}^{a} \beta^{2j} x_{a-j}(k) \right] \\ \qquad + (-\xi)^N\ \beta^{2a+1} u(k) \end{cases}$$

(10)

with $\beta = (1-\xi^2)^{1/2}$.

The system (10) can be written with the recursive representation form as follow [13,14,15]:

$$\begin{cases} X(k+1) = \Gamma X(k) + B u(k) \\ y(k) = C^T \mathbb{X}_N^{(a)}(k) + e(k) \end{cases} \tag{11}$$

where

$$X(k) = \left[x_0(k),\ldots,x_{a-1}(k), x_a(k), x_{a+1}(k),\ldots,x_{a+N}(k) \right]^T \tag{12}$$

$$\begin{aligned} \mathbb{X}_N^{(a)}(k) &= \left[x_0^{(a)}(k),\ldots,x_N^{(a)}(k) \right]^T \\ \mathbb{X}_N^{(a)}(k) &= L^{(a)} \sigma X(k) \end{aligned} \tag{13}$$

with $\sigma = \left[\begin{array}{ccccccc} 0 & \cdots & 0 & 1 & 0 & \cdots & 0 \\ & & & 0 & 1 & & \vdots \\ \vdots & & \vdots & \vdots & & & \\ & & & & & \ddots & 0 \\ 0 & \cdots & 0 & 0 & \cdots & 0 & 1 \\ \multicolumn{3}{c}{\underbrace{\hphantom{0 \cdots 0}}_{(N+1)\times(a)}} & \multicolumn{4}{c}{\underbrace{\hphantom{0 \cdots 0 \cdots 0}}_{(N+1)\times(N+1)}} \end{array}\right]$

$$\Gamma = \begin{bmatrix} \gamma_{0,0} & \cdots & \gamma_{0,a+N} \\ \vdots & \ddots & \vdots \\ \gamma_{a+N,0} & \cdots & \gamma_{a+N,a+N} \end{bmatrix} \tag{14}$$

with

$$\gamma_{i,j} = \begin{cases} 0 & & i < j & \\ \xi & & i = j & \\ \xi \beta^{2(i-j)} & \mathit{for} & j \le i \le a & i,j = 0,\ldots,N+a \\ (-1)^{i-a} (\xi)^{i-a+1} \beta^{2(a-j)} & & j < a < i & \\ (-\xi)^{i-j-1} \beta^2 & & a \le j < i & \end{cases}$$

$$B = \beta \left[b_0,\cdots,b_a,b_{a+1},\cdots,b_{a+N} \right]^T \tag{15}$$

with

$$b_i = \begin{cases} \beta^{2i+1} & & i \le a \\ & \mathit{for} & & i = 0,\ldots,N+a \\ (-\xi)^{i-a}\ \beta^{2a+1} & & i > a \end{cases}$$

$$C = \left[g_0,\cdots,g_N \right]^T \tag{16}$$

The model given by equation (11) is linear with respect to the Fourier coefficients g_i given in (16), which allows to apply the conventional methods of identification such as the Least Squares. As against the model is not linear to the Meixner-like pole ξ. In this regard, we opted for minimizing the normalized mean square error (NMSE) and which will be developed in the next section.

III. OPTIMIZATION OF MEIXNER-LIKE MODEL

A. *Preliminaries*

From (7) , the truncation error $E(z,\xi)$ can be written as:

$$E(z,\xi)=Y(z)-\sum_{n=0}^{N}g_n(\xi)X_n^{(a)}(z,\xi) \tag{17}$$

To ensure the reduction of the parameter number in the Meixner-like model, the pole characterizing the Meixner-like functions has to be optimized. To do so, the Normalized Mean Squared Error (NMSE) is considered.

$$J(\xi)=\frac{\|E(z,\xi)\|^2}{\|Y(z,\xi)\|^2} \tag{18}$$

Since the criterion $J(\xi)$ is non-linear with respect to the Meixner-like pole, its solution can be transformed in to an optimization problem as follow:

$$\min_{\xi}\{J(\xi)\} \tag{19}$$

As mentioned below, the optimization procedure of the Meixner-like pole is based on input/output measurement. Thus for H input/output data, the criterion is given by:

$$J(\xi)=\frac{\sum_{k=1}^{H}\left[y(k)-\left(\sum_{n=0}^{N}\mathrm{g}_n(k)\,x_n^{(a)}(k,\xi)\right)\right]^2}{\sum_{k=1}^{H}[y(k)]^2} \tag{20}$$

B. *Proposed Optimization Technique*

The Meixner-like pole is expressed in the criterion $J(\xi)$ with nonlinear way. In this section we propose to compute the optimum value of Meixner-like pole by using Genetic Algorithms classified as evolutionary optimization methods [1,22]. They allow defining an optimum of a function defined on a data space. Theses algorithms are applied to a wide variety of problems [6] regarding their simplicity and efficiency [4]. After fixing the expression of the objective function to be optimized, probabilistic steps are involved to create an initial population of individuals [18]. The initial population will submit some genetic operations (such as evaluation, mutation, crossover,…) to converge to the optimal solution of the problem. The optimization with GA is summarized by the following steps:

- Initialization: It is usually random and it is often advantageous to include the maximum knowledge about the problem.
- Evaluation: This step is to compute the quality of individual of positive value called "fitness'' to each one. The highest is assigned to the individual that minimizes (or maximizes) the objective function.
- Selection: This step selects a definite number of individuals of the current population. The selection is probabilistic and characterized by a parameter called selection pressure fixing the number of selected individuals.
- Crossover: The genetic crossover operator creates new individuals. From two randomly selected parents, crossover produces two descendants. This step affects only a limited number of individuals established by the crossover rate (Pc) number.
- Mutation: The mutation consists in providing a small disruption to a number (Pm) of individuals. The effect of this operator is to counteract the attraction exerted by the best individuals this allows to explore other areas of the search space.

The optimum value of Meixner-like pole is determined by using the criterion $J(\xi)$ given by (20) as objective function known as fitness.

After determining the objective function to be minimized, we generate, randomly, a population of individuals ξ. Abilities of individuals ξ are evaluated by the criterion $J(\xi)$. Individuals with the highest skills are selected to undergo different genetic operation (crossover, mutation and selection). After a set of generations, the genetic algorithm converges to the optimal value.

The new genetic curve algorithm is constructed following the strategy outlined as:

Algorithm of Meixner-like pole optimization

```
• Choose an order of troncature $N$, a generalization order $a$, a number of individuals $Nind$ and a maximum number of generations $Gen_{\max}$.
• Generate at randomly an initial population of $Nind$ pole $\xi$.
• Repeat until $Gen_{iteration} > Gen_{\max}$
    For every pole do
        Compute the fitness $J(\xi_i)$
    End for
    Apply crossover to produce new vectors
    Apply mutation
    Select better pole $\xi$
    Reinsertion of better pole
    Calculate the NMSE
  End Repeat
```

IV. SIMULATION RESULTS

In order to illustrate the optimisation of the Meixner-like pole considered in this paper, we shall give results for an example where identification is performed on the basis of simulation data. The simulated system is a discrete time system described by the following transfer function [7,13]:

$$G(z) = \left(\frac{z}{z-p} \right)^4 + \left(\frac{z}{z-p^*} \right)^4 \tag{21}$$

with $p = 0{,}9 + 0{,}1j$.

Meixner-like basis functions have been chosen generated by a high-order inner function $N = 6$ with a generalization order $a = 5$. As the choice of the Meixner-like pole can highly influence the accuracy of the identified model, we have applied the proposed optimization algorithm. We have used a data set of input and output signals with length $H = 150$. The input system is a multilevel pseudo random signal given by Fig. 2.

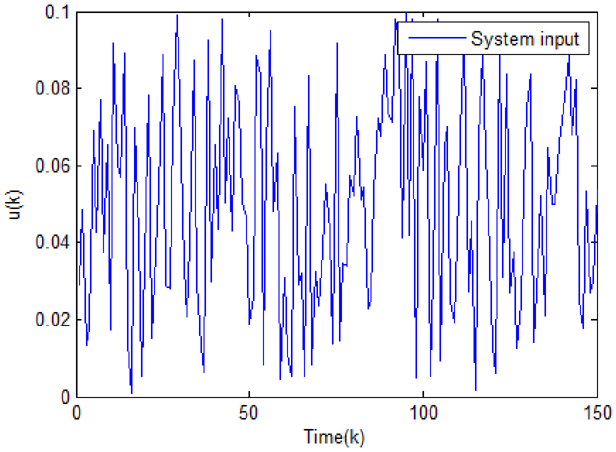

Fig. 2 System input

To evaluate the Meiner-like pole, we choose a value of truncation order equal to $N = 6$, a fixed generalization order $a = 5$, a number of individuals $Nind = 100$ and a maximum number of generations $Gen_{\max} = 20$. From the Fig. 3, where we illustrate the evolution of the Meixner-like pole estimation, we can conclude that the algorithm converges in relatively few iterations to $\xi = 0.8889$.

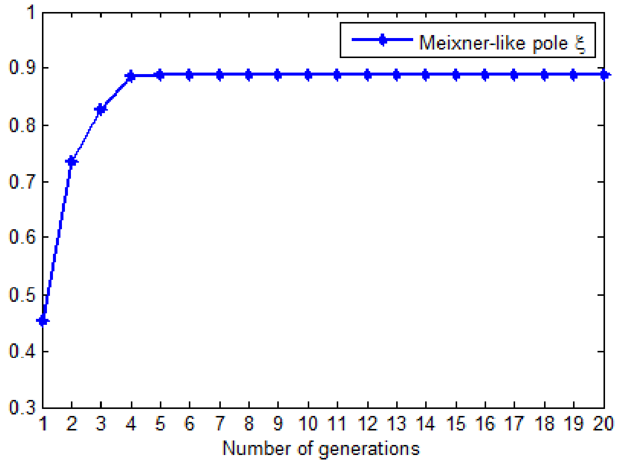

Fig. 3 Meixner-like pole estimation

The modeling performance is shown in Fig. 4 where we illustrate the evolution of the system output and the Meixner-like model output.

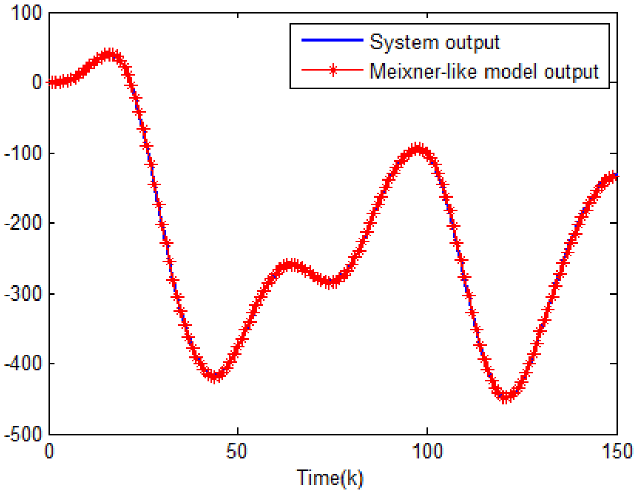

Fig. 4 Validation of the Meixner-like model

It can be observed from Fig. 4 that the model based on the Meixner-like orthonormal basis functions have a good ability to identify the behavior of the system.

V. CONCLUSIONS

The work presented in this paper focuses on modeling and identification of a LTI system represented by Meixner-like functions. We have proposed a new representation of linear discrete-time system. It has been provided by filtering the input system with orthonormal Meixner-like functions. Thus, the order of the model can be low, if the Meixner-like pole is set to its optimal value. Therefore, we have established from the proposed representation a method in which the Meixner-like pole can be set to its optimal value. The optimization was realized by using Genetic Algorithm. This area will be used to synthesize a parametric identification in order to synthesize a law control.

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