Research on Signal Control of Urban Intersection Based on Genetic Algorithms

CHEN Qun
School of Transport Engineering
Central South University
Changsha, China
E-mail: chenqun631@163.com

***Abstract*—In order to study the optimization algorithm of signal timing of urban intersection under real-time traffic flows, with the target of minimizing the total delay of all entrance lanes of all phases and the restrictions of saturation and the minimum green signal time, the non-linear programming model for real-time signal control of urban intersection was constructed and the genetic algorithm for solving the model was proposed. With the model and the genetic algorithm the best scheme of signal timing can be obtained. Through a simulation experiment the application of the model and algorithm was illustrated. The cases of ignoring the restriction of saturation and minimizing the total delay of the key entrance lanes were also analyzed. The analysis results showed that the restriction of saturation should be taken into account and the target function should be constructed on the basis of minimizing the total delay of all entrance lanes of all phases so as to ensure correctness and rationality of the optimizing results.**

Keywords- signal timing; intersection; saturation; non-linear programming; genetic algorithm

I. INTRODUCTION

Intersections are usually the bottleneck of urban traffic, traffic signal is needed to separate the traffic flow which conflicts and intertwines with one another when traffic volume reaches a certain level in order to allow vehicles to run freely through intersections, thus reducing or avoiding traffic congestion. Rational traffic signal timing is of vital importance to achieve effective traffic control. Considering the requirement of concerted control of several intersections, there are relatively fixed cycles in some certain intervals of a day and little chance of dynamic adjustment of the signal cycle. Therefore, studying how to alter the scheme of signal timing according to real-time variation of traffic flows on roads under the fixed signal cycle is critical to realize the optimal signal control of intersections. In this paper, firstly the optimization function was constructed for minimizing the total delay of vehicles, then the non-linear programming model with restrictions of saturation and the minimum green signal time was constructed, and then the genetic algorithm for solving the model was proposed to calculate the best scheme of signal timing.

II. CONSTRUCTION OF NON-LINEAR PROGRAMMING MODEL

With the basic data of real-time traffic volume collected on roads and the target of minimizing the total delay of all entrance lanes of all phases at an intersection, the optimization function is constructed. Webster Signalized Intersection Delay Model [1] is adopted here, so the average delay per vehicle at an intersection is

$$d = \frac{c(1-\lambda)^2}{2(1-\lambda x)} + \frac{x^2}{2q(1-x)} \qquad (1)$$

where the first item of equation 1 stands for the uniform delay (i.e. delay of vehicles under the constant arriving rate), the second item stands for the random delay (delay of vehicles generated by different arriving rates at different periods) [2]. c stands for signal cycle (s), λ stands for green split , q stands for traffic volume（pcu/h）, x stands for saturation.

Take the typical four-phase intersection as an example, the formula to calculate the total delay is

$$D = \sum_{i=1}^{4}\sum_{j=1}^{2}\left\{\left[\frac{c(1-\lambda_i)^2}{2(1-\lambda_i x_{ij})} + \frac{x_{ij}^2}{2q_{ij}(1-x_{ij})}\right]q_{ij}\right\} \qquad (2)$$

where q_{ij} denotes traffic volume（pcu/h）at phase i and entrance lane j; likewise, x_{ij} denotes saturation at phase i and entrance lane j; λ_i denotes green split at phase i.

The objective function is

$$\min\ D \qquad (3)$$

For restrictions, for safety needs of crossing the roads at intersections, the shortest green time at each phase shouldn't be less than a certain value e (here take $e \ge 10$s), so signal timing at each phase must satisfy

$$10 \le t_i \le c - L - 10\times 3 \qquad (i = 1,\cdots 4) \qquad (4)$$

978-0-7695-3804-4/09 $26.00 © 2009 IEEE
DOI 10.1109/ICICTA.2009.55

In formula 4, t_i denotes the effective green time at phase *i*, *c* denotes signal cycle, *L* denotes the total lost time.

The second is the restriction of the maximum saturation. Rational design of intersection and rational cycle during a certain time interval should ensure that saturation at each phase are not too big (for instance a value exceeding 1) under rational signal timing, otherwise traffic congestion might appear at the entrance lanes.

Here assume that the saturation at all phases and all entrance lanes should not be greater than 0.9.

$$\text{Saturation } (x) = \frac{\text{Actual flow } (q)}{\text{Capacity } (N)}$$

$$= \frac{\text{Actual flow } (q)}{\text{Saturation flow } (s)\dfrac{\text{Effective green time } (g_e)}{\text{Signal cycle } (c)}} = \frac{cq}{sg_e} \le 0.9$$

i.e. $g_e \ge \dfrac{cq}{0.9s} = \dfrac{cy}{0.9}$

where y denotes flow ratio. For all phases,

$$t_i = g_{ei} \ge \frac{cy_{i\max}}{0.9} \quad (i = 1,\cdots 4) \tag{5}$$

where $y_{i\max}$ denotes the bigger value of y at phase *i*.

To sum up, all the restrictions are as follows.

$$\begin{cases} t_1 + t_2 + t_3 + t_4 = c - L & (6) \\ 10 \le t_i \le c - L - 10\times 3 \ (i = 1,\cdots 4) & (7) \\ t_i = g_{ei} \geqslant \dfrac{cy_{i\max}}{0.9} \ (i = 1,\cdots 4) & (8) \end{cases}$$

III. Apply Genetic Algorithm to Solve the Model

Apply genetic algorithms [3,4] to solve the above non-linear programming model.

(1) Variable encoding

Adopt real number encoding, code length equals to the number of variables. For example, for a typical four-phase intersection, t_1, t_2, t_3 are the effective green time at 1st, 2nd and 3rd phase respectively and are also the variables to optimize. When signal cycle is fixed and has been given, the effective green time at 4th phase t_4 equals to $T - t_1 - t_2 - t_3 - L$. Code could be expressed as (t_1, t_2, t_3).

(2) Fitness function

The model is an optimization problem to seek the minimal value, so fitness function can be expressed as $F(i) = C_{\max} - O(i)$. $F(i)$ denotes the fitness of individual *i*, $O(i)$ denotes the function value of individual *i*, $C_{\max}$ denotes the estimated maximum of $O(i)$.

(3) Genetic operation

1) Selection

Apply the elite model, reserve the best individual up to the current generation. Use roulette wheel selection operator based on ranking to select the population of next generation. Rank all individuals' fitness value from high to low. The i^{th} individual's survival probability is

$$\Pr ob(i) = a(1-a)^{i-1} \tag{9}$$

$a \in (0,1)$ stands for selection pressure.

After survival possibilities of individuals are calculated, the selection possibilities of individuals can be obtained.

$$p_i = \frac{\Pr ob(i)}{\sum \Pr ob(i)}, \tag{10}$$

then select the offspring according to the roulette method.

2) Crossover

Assume crossover probability P_c, mate all individuals randomly. For each pair, a random number r_1 ($r_1 \in [0,1]$) is produced. If $r_1 < P_c$, perform the crossover operation according to the flowing formulas:

$$P_1' = wP_1 + (1-w)P_2 \tag{11}$$

$$P_2' = (1-w)P_1 + wP_2 \tag{12}$$

w ($w \in [0,1]$) is a randomly produced number, P_1, P_2 are the current individuals, P_1', P_2' are the new ones. Since the constraint set of the model is convex (i.e. linear constraints), the new population created from the initial population produced in the feasible region through the crossover operation are certainly feasible.

3) Mutation

Assume mutation probability P_m. For each individual, a random number r_2 ($r_2 \in [0,1]$) is produced. If $r_2 < P_m$, perform the mutation operation according to the flowing formula:

$$P_1' = P_1 + Me \tag{13}$$

In eq.(13), e is a random direction vector, $e = (e_1, e_2, \cdots, e_n)$, $e_i \in [-1,1]$, $M \in \underset{i}{Max}[0, x_i^u - x_i^l]$ (*i* is the sequence number, x_i^u denotes the i^{th} variable's upper limit value, x_i^l denotes the i^{th} variable's lower limit value). If the newly-produced individual is illegal, then adjust M and produce a random number belonging to $[0, M]$ until the legal individual is obtained.

Processes of signal timing calculation using genetic algorithm are as follows:

Step 1. Initialization. Set the number of population, chromosome length (i.e. the number of variables, in this article the variables are t_1 , t_2 , t_3), iteration number, crossover probability and mutation probability.

Step 2. Apply real number encoding to create the initial population within the feasible region.

Step 3. Compute each individual's function value and fitness value.

Step 4. Rank all the current individuals and select the new population according to the roulette wheel selection operator based on ranking.

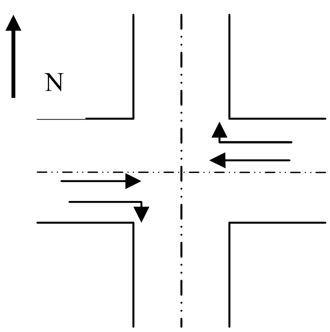
Step 5. Perform the crossover operation for the randomly mated individuals according to the crossover probability.

Step 6. Perform the mutation operation for the individuals according to the mutation probability.

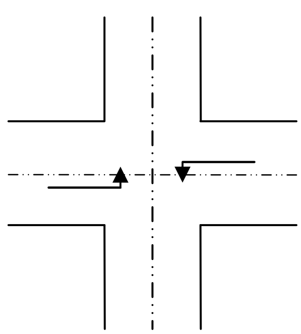
Step 7. Judge whether it reaches the maximal generation. If not, turn to step 3; otherwise, output the best individual (i.e. the optimal scheme of signal timing).

IV. SIMULATION EXPERIMENT

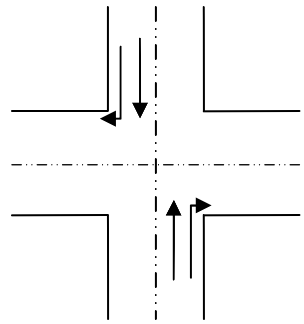
For a certain four-phase intersection, the flow of each phase is shown in the figure1. To make it easier, we suppose straight-going and right-turn share the same phase, which means right-turn is forbidden at red time.



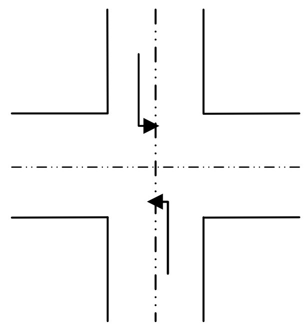
Phase 1



Phase 2



Phase 3



Phase 4

Figure 1 Four-phase intersection

The data matrixes of real-time traffic flows collected are as follows:

$$q=\begin{bmatrix}400 & 240\\ 80 & 120\\ 270 & 240\\ 60 & 60\end{bmatrix}, y=\begin{bmatrix}0.20 & 0.12\\ 0.10 & 0.15\\ 0.18 & 0.16\\ 0.12 & 0.12\end{bmatrix}, s=\begin{bmatrix}2000 & 2000\\ 800 & 800\\ 1500 & 1500\\ 500 & 500\end{bmatrix}$$

where, q denotes traffic flow (unit:pcu/h), y denotes flow ratio, s denotes saturation flow (unit:pcu/h). $q(i, j)$ denotes the flow of the j^{th} entrance lane at the i^{th} phase (i=1,2,3,4; j=1,2). When i=1,2, j=1 denotes the western entrance lane and j=2 denotes the eastern entrance lane; when i=3,4, j=1 denotes the northern entrance lane and j=2 denotes the southern entrance lane. Likewise, the meaning of $y (i, j)$ and $s (i, j)$ can be known. Suppose the signal cycle (c) is 130 s. the total lost time (L) is 10 s and the minimum green time is 10 s at each phase. Suppose that the time of yellow light equals to the lost time at each phase, then the effective green time is just the real green time.

(1) The 1th situation: take constraint of saturation into consideration

The optimized target function is formula (3), the constraint relations are formulas (6), (7), (8).

The minimum green time satisfying the saturation constraint is

$$g_{ei} \geqslant \frac{c y_{i\max}}{0.9} = \frac{130 y_{i\max}}{0.9} \quad (i = 1, \cdots 4)$$

For the 1st phase, y takes the greater number 0.2, then get $g_{e1} \geqslant 29$ s; likewise, for the 2nd phase, $g_{e2} \geqslant 22$ s; for the 3rd phase, $g_{e3} \geqslant 26$ s ; and for the 4th phase, $g_{e4} \geqslant 17$ s .

Take the related data into the constraint relations, after arrangement the results are as follows:

$$\begin{cases} 29 \le t_1 \le 55 \\ 22 \le t_2 \le 48 \\ 26 \le t_3 \le 52 \\ 17 \le 120 - t_1 - t_2 - t_3 \le 43 \end{cases}$$

Write the program of genetic algorithm with MATLAB6.5, the population size takes 300, crossover probability takes 0.7, mutation probability takes 0.1, chromosome length takes 3 (i.e. three variables t_1, t_2, t_3), iteration number takes 50. The calculation results are:

t_1=50.1976（s）， t_2=22.0000（s）， t_3=30.8024（s）, then $t_4 = 120 - t_1 - t_2 - t_3$=17.0（s）. The total delay time is D=5.9237×10^4（s）. The saturation of each approach at each phase are as follows:

$$\begin{bmatrix} 0.5180 & 0.3108 \\ 0.5909 & 0.8864 \\ 0.7597 & 0.6753 \\ 0.9176 & 0.9176 \end{bmatrix}$$ (Rows stand for phases, and columns stand for approaches)

Note: The saturation at the 4th phase is 0.9176 (>0.90), this is a minor error from rounding of the minimum green time calculated through the maximum saturation constraint.

(2) The 2th situation: take no account of the saturation constraint

The optimized target function is function (3). Without considering the saturation constraint, the constraint relations should be:

$$\begin{cases} t_1 + t_2 + t_3 + t_4 = 130 - 10 \\ 10 \le t_i \le 130 - 10 - 10 \times 3 \end{cases}$$

The calculation results after 10 generations of genetic algorithm are

t_1=59.6516（s）， t_2=10.0012（s）， t_3=40.3332（s）, then $t_4 = 120 - t_1 - t_2 - t_3$=10.0140（s）. The total delay time is D=5.4010×10^4（s）, the saturation of each approach at each phase are as follows.

$$\begin{bmatrix} 0.4359 & 0.2615 \\ 1.2998 & 1.9498 \\ 0.5802 & 0.5157 \\ 1.5578 & 1.5578 \end{bmatrix}$$ (Rows stand for phases, and columns stand for approaches)

From the data above, it is shown that there are several data whose saturations are greater than 1.0, which are not correct because random delay will be a negative if the saturation greater than 1.0 is put into the 2th item of the Webster Formula. In fact, the Webster Formula is only available for the delay calculation in the situation that the saturation is smaller than 1.0. Therefore not considering the saturation constraint will lead to a wrong result. The saturation will be great while the total delay would be seemingly smaller. Here, the total delay has fallen to 5.4010 ×10^4（s）（< D=5.9237×10^4（s））only after 10 generations of genetic algorithm.

(3) The 3th situation: take constraint of saturation into consideration but minimize the total delay of key approaches at each phase

The key approach is the approach which has a greater flow ratio at each phase. After 50 generations of genetic algorithm, the results are:

t_1=54.0085（s）， t_2=22.0000（s）， t_3=26.9915（s）, then $t_4 = 120 - t_1 - t_2 - t_3$=17.0000（s）. the total delay of key approaches at each phase are 3.4236×10^4（s）, the total delay D=5.9297×10^4（s）, which is greater than 5.9237×10^4（s） which is got in the1th situation that the optimization target is to minimize the total delay of each approach at each phase. In this example, there is not a distinct difference between flow ratios of two approaches at each phase, otherwise, the optimization results from the two situations will be very different. Therefore, minimizing the total delay of each approach at each phase is more rational.

V. CONCLUSIONS

This paper studied the algorithm of real-time adjusting of signal timing of intersection with change of traffic flows on roads under the relatively fixed signal cycle. When put into practical use, methods discussed in this paper could be adopted for the computers to deal with the collected real-time data of traffic flows which were sent to the computer processing center by data transportation network, and work out a real-time signal control scheme. The cases of ignoring the restriction of saturation and minimizing the total delay of the key entrance lanes were also analyzed. The analysis results showed that the restriction of saturation should be taken into account and the target function should be constructed on the basis of minimizing the total delay of all entrance lanes of all phases so as to ensure correctness and rationality of the optimizing results.

REFERENCES

[1] S. W. Zhou, “Traffic Engineering,” Shanghai: Tongji University Press, 1987: 143-144.

[2] Q. Pu, Y. C. Tan, “Research on Optimization Model of Intersection Signal Control,” Transaction of the Shanghai Railway University, 1999,4: 31-34.

[3] Z. Mikailisway, “Evolution Programs--Combination of Genetic Algorithm and Data Coding,” Beijing: Science Press, 2000: 121-134.

[4] X. P. Wang, L. M. Cao, “Genetic Algorithm--Theoretic, Application and Software Realization,” Xi’an: Xi’an Jiaotong University Press. 2002: 104-122.